

Topological Higher-Order Features for Brain-Computer Interfaces

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ABSTRACT: Integrating features from the way different brain signals mutually interact has been recently used to understand and improve brain-computer interfaces (BCIs). However, most existing approaches have focused on pairwise network features, which neglect possibly existing inter-actions involving more than two brain areas. Increasing evidence suggests that higher-order (HO) interactions arising from the simultaneous coordination of multiple groups of regions, bring complementary information for brain states discrimination.

Here, we investigated whether higher-order brain network features extracted from EEG signals can improve the classification of simple motor imagery-based BCI tasks. To this end, we developed a method that combines spectral graph embedding and topological data analysis to extract HO structures from functional connectivity networks estimated using imaginary coherence between EEG signals.

We evaluated the ability of these features to discriminate motor imagery (MI) and resting-state EEG activity in a typical 1D cursor task control. Results showed that MI significantly increases the involvement of sensorimotor electrodes in HO triadic schemes as compared to resting states. These changes were particularly present in *beta* band frequency ranges. Furthermore, we found that in 16 out of 19 participants, such HO features achieved higher accuracy than both local power spectra and connectivity. These findings suggest that topological characterizations of functional brain networks capture behaviorally relevant aspects of neural organization and offer a promising direction for incorporating higher-order interactions into EEG-based BCI applications.

INTRODUCTION

Brain-computer interfaces (BCIs) rely on the extraction of informative features from brain signals in order to discriminate between different mental states. In motor imagery (MI) BCIs in particular, decoding performance strongly depends on the ability to capture neural patterns that reliably distinguish task-related activity from resting-state dynamics. Traditionally, most BCI pipelines rely on local spectral features, such as power spectral density in sensorimotor rhythms, or on pairwise functional connec-

tivity measures that describe interactions between pairs of brain regions[1, 9, 10, 11].

While these approaches have demonstrated their potential, classification accuracy often remain too variable within and between subjects [11, 12, 13]. Furthermore, these state-of-the-art features provide only a partial description of the collective organization of neural activity. Brain processes often involve coordinated activity across multiple neural populations simultaneously, suggesting that relevant information may be contained in interaction patterns that extend beyond pairwise relationships. In recent years, increasing attention has therefore been given to higher-order interactions (HOI), which aim to characterize dependencies involving groups of nodes rather than isolated pairs [2, 14, 15, 16].

Several methodological frameworks have been proposed to investigate HOI in brain networks. Topological approaches model neural systems using hypergraphs or simplicial complexes and apply tools from algebraic topology to describe higher-dimensional structures such as cliques, simplices, and topological cavities [3]. In parallel, statistical approaches grounded in information theory attempt to quantify multivariate dependencies that cannot be reduced to pairwise associations, for instance through measures of redundancy and synergy [4]. Although these approaches provide valuable insights into the collective organization of neural systems, their interpretation in terms of functional brain processes remains challenging, and their direct usefulness for BCI decoding tasks has only been partially explored. [17]

In particular, most BCI feature extraction pipelines remain focused on node-level or pairwise descriptors, such as spectral power or connectivity strength. This raises the question of whether higher-order representations derived from functional connectivity networks can capture additional information relevant for discriminating brain states in BCI settings.

While imaginary coherence provides informative pairwise interactions between EEG channels, it does not directly describe coordinated patterns involving multiple channels simultaneously. A central question of this study is therefore whether higher-order information can be inferred from pairwise connectivity structures and whether such representations provide additional discriminative value for motor imagery decoding. In this work, we investigate whether higher-order brain network features

can improve motor imagery classification in an EEG-based BCI context. The analysis is based on EEG recordings acquired during a motor imagery training protocol in which participants controlled a one-dimensional cursor by imagining right-hand movements versus remaining at rest.

Starting from functional interactions between EEG channels, we derive higher-order descriptors intended to capture coordinated patterns involving multiple channels simultaneously. We then evaluate whether these representations improve the discrimination between motor imagery and resting-state activity compared with traditional features such as power spectral density and node strength.

MATERIALS AND METHODS

Dataset: We used the dataset introduced by Corsi et al. (2020), consisting of recordings from 19 healthy right-handed BCI-naïve participants performing a motor imagery BCI task. Participants were instructed to imagine sustained right-hand grasping movements (MI condition) or remain at rest (REST condition) in order to control a one-dimensional cursor. The experimental protocol comprised four recording sessions per participant. In each session, the BCI testing phase contained six runs of MI/REST trials, yielding 114 available trials per session. The present study focused on one selected session per participant, from which all available MI/REST runs were included for analysis. [5]. which all available MI/REST runs were included for analysis. EEG was recorded with 74 electrodes, from which 72 channels were retained after exclusion of T9 and T10. Signals were acquired at 1000 Hz and subsequently downsampled to 250 Hz. Ocular and cardiac artifacts were reduced using Independent Component Analysis (ICA). Data were segmented into 7-second trials, noisy epochs were rejected through quality control, and signals were average-referenced before further analysis. For the present work, feature extraction was performed on the final 5 seconds of each trial, corresponding to the most stable task-engagement period.

Imaginary coherence networks:

Functional interactions between EEG channels were estimated using imaginary coherence (ImCoh), a frequency-domain connectivity measure based on the imaginary part of coherency between two signals [6]. For two channels $x_i(t)$ and $x_j(t)$, ImCoh at frequency f is defined as:

$$\text{ImCoh}_{ij}(f) = \frac{\text{Im}(S_{ij}(f))}{\sqrt{S_{ii}(f)S_{jj}(f)}}.$$

Here, $S_{ij}(f)$ denotes the cross-spectrum between channels i and j , while $S_{ii}(f)$ and $S_{jj}(f)$ denote their respective auto-spectra. Because the cross-spectrum is a complex-valued quantity, it contains information about the relative phase relationship between the two signals at frequency f . By focusing on its imaginary component, this measure is particularly sensitive to consistent non-zero phase-lag interactions, while attenuating zero-lag couplings that may arise from volume conduction or

common reference effects.

For each trial and frequency, ImCoh matrices were computed between all pairs of EEG channels, resulting in frequency-resolved connectivity networks for both MI and REST conditions.

Spectral embedding: Each connectivity matrix was mapped to a low-dimensional space using Laplacian spectral embedding. More specifically, from the connectivity matrix, a graph Laplacian was constructed, and its leading eigenvectors were used to define the coordinates of the nodes in a reduced-dimensional Euclidean space [7]. In the present study, a two-dimensional embedding was used, as our analysis focused on triangular higher-order motifs (2-simplices).

This transformation converts the connectivity graph into a geometric representation in which each EEG channel is represented as a point in the plane. Channels showing stronger functional connectivity tend to be placed closer to each other, whereas weakly connected channels are positioned farther apart. In this way, the spatial distribution of the points reflects the relational organization of the original network.

The embedding preserves the main structural properties of the connectivity graph while enabling subsequent geometric and topological analysis of the interaction patterns. Higher-dimensional embeddings could naturally be considered in future work to investigate more complex higher-order motifs involving larger groups of nodes.

Topological feature extraction: Topological data analysis was applied to the embedded point clouds. In particular, we constructed α -complexes [8], which are simplicial complexes built from a set of points according to their geometric proximity. Starting from isolated points, nearby points are connected as the scale parameter α increases, progressively forming edges, triangles, and higher-dimensional simplices. More specifically, a simplex is included when its vertices are sufficiently close to share a common ball of radius related to α , which defines the inclusion criterion. This provides a structured representation of the shape and organization of the point cloud while remaining computationally efficient through the associated Delaunay triangulation. A simplicial complex is a geometric structure composed of vertices, edges, triangles, and higher-dimensional elements used to represent relationships among multiple points.

Within this structure, a k -simplex represents a joint interaction among $k + 1$ nodes; for example, a triangle corresponds to a 2-simplex. To characterize these higher-order structures at the node level, we introduced node-wise simplex participation features derived from the α -complexes.

For each node i and frequency f , we define the participation coefficient as

$$T_i(f) = \sum_{\sigma \in \mathcal{S}_k(f)} \mathbf{1}(i \in \sigma),$$

where $\mathcal{S}_k(f)$ denotes the set of k -simplices at frequency f , and $\mathbf{1}(i \in \sigma)$ is an indicator function equal to 1 if

node i belongs to simplex σ , and 0 otherwise. This measure quantifies how strongly each channel participates in higher-order interaction structures.

In this work, we specifically focus on 2-simplices (triangles), which capture simultaneous interactions among triplets, or triads, of EEG channels.

Comparison with node-centered features: To evaluate the contribution of these topological descriptors, we compared them with two standard node-centered representations.

First, power spectral density (PSD) [11, 18] features were computed from selected sensorimotor EEG channels using Welch’s method. For a signal $x_i(t)$ recorded at channel i , the PSD is defined as

$$P_i(f) = |\mathcal{F}\{x_i(t)\}|^2,$$

where $\mathcal{F}$ denotes the Fourier transform.

Second, node strength was extracted from the ImCoh networks [19] as the sum of connectivity weights incident to each node. For node i , the strength is defined as

$$s_i(f) = \sum_j w_{ij}(f),$$

where $w_{ij}(f)$ denotes the imaginary coherence weight between nodes i and j at frequency f .

Classification: All feature representations were evaluated for the classification of MI versus REST trials using a Linear Discriminant Analysis (LDA) classifier. The model was trained and evaluated using a stratified train-test split in order to preserve the class distribution across the two conditions. Classification performance was assessed using accuracy as the primary evaluation metric. To ensure a fair comparison between feature representations, we applied a feature selection step in which the five most discriminative node-level features were retained for each method. This selection was performed independently for each participant using all available sessions, with the objective of identifying subject-specific features that remained stable across sessions rather than optimizing session-specific classification performance. Features were ranked according to the average magnitude of the difference between MI and REST conditions across sessions, together with their variability, allowing us to favor channels whose activity patterns showed both strong discrimination and low variability over time. Notably, the selected nodes were predominantly located over sensorimotor regions, which are known to play a central role in motor imagery tasks.

RESULTS

Figure 2(a) illustrates examples of higher-order network activity derived from the triangle participation features for two representative sensorimotor channels (C1 and FC5) for Subject 01 during Session 04. These electrodes lie over the sensorimotor cortex. The triangle participation spectrum reveals systematic differences between MI and REST conditions across several frequency ranges. In

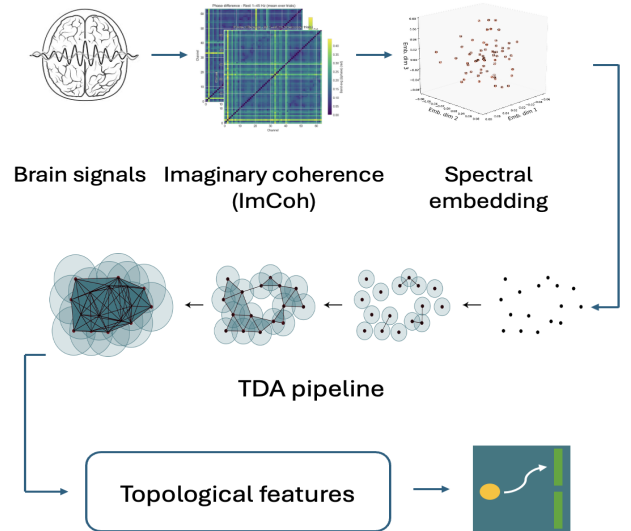


Figure 1: Pipeline for extracting higher-order topological features from EEG signals for motor imagery BCI decoding.

particular, MI trials tend to exhibit higher triangle participation values compared to REST in the beta frequency band, suggesting that motor imagery is associated with stronger higher-order interaction patterns within sensorimotor regions. This observation is coherent with the findings reported in the original dataset study by Corsi et al. (2020) [5], where beta-band connectivity patterns were also shown to play an important role during motor imagery tasks and BCI learning.

To evaluate whether these higher-order features contribute to decoding performance, we compared classification accuracy obtained using three feature representations: power spectral density (PSD), node strength derived from imaginary coherence (ImCoh), and the proposed higher-order interaction (HOI) features. Figure 2(b) presents the per-subject classification accuracy obtained with a Linear Discriminant Analysis (LDA) classifier for all participants in Session 04. Overall, HOI-based features achieve higher or equal classification performance compared to both PSD and node strength for most subjects.

Specifically, HOI features outperform the baseline representations in the majority of participants (16 out of 19 subjects), while also exhibiting relatively stable performance across individuals. In contrast, PSD and node strength features show larger inter-subject variability and lower accuracy for several participants. These results indicate that higher-order topological descriptors capture discriminative information that is not fully represented by local spectral activity or pairwise connectivity measures, and that they provide complementary features for motor imagery decoding in EEG-based BCI systems.

DISCUSSION

These preliminary results provide initial evidence that

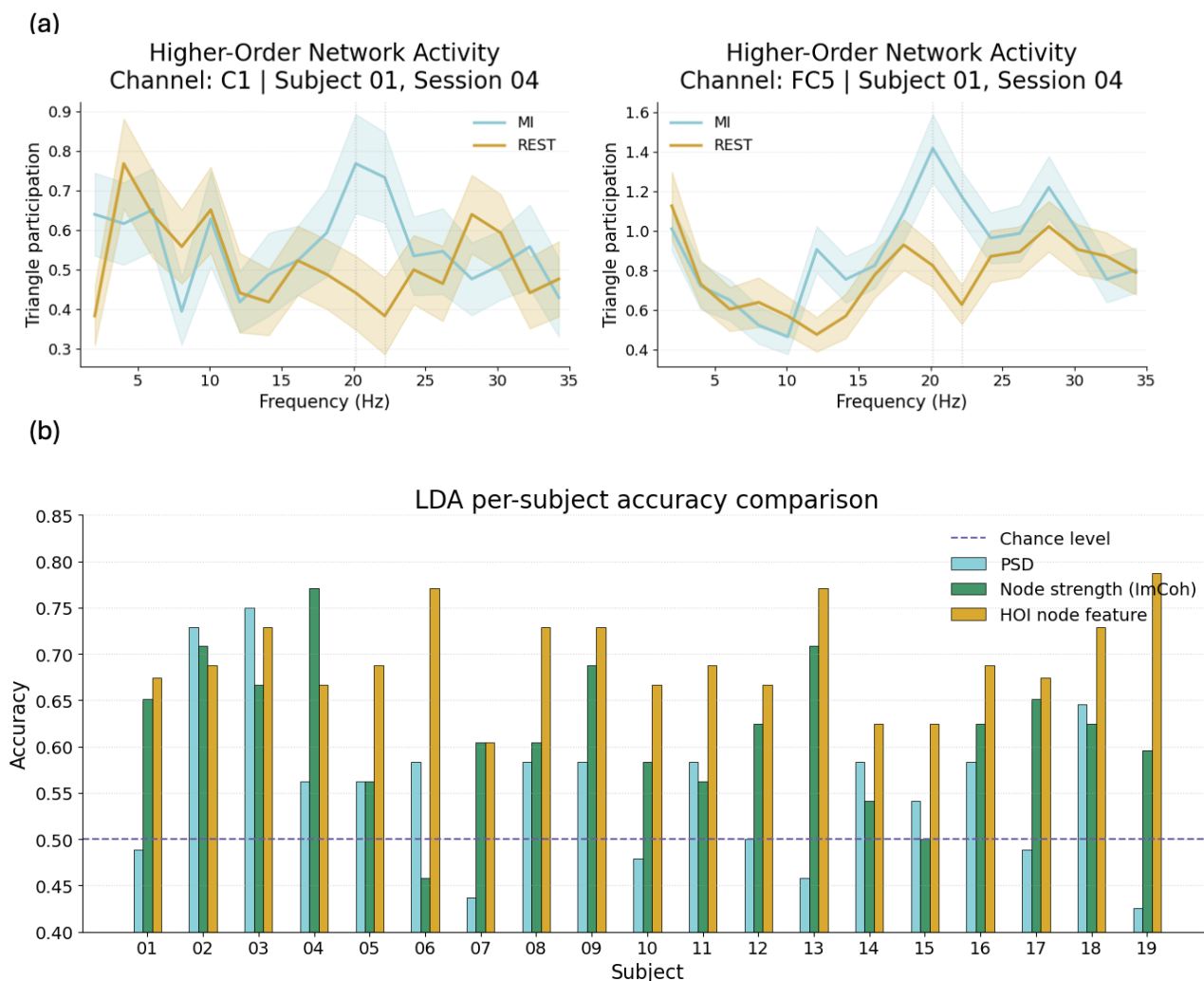


Figure 2: Higher-order network activity and classification performance. (a) Mean triangle participation as a function of frequency for MI and REST conditions in two sensorimotor channels (C1 and FC5). (b) Per-subject classification accuracy obtained with Linear Discriminant Analysis (LDA) using PSD, node strength (ImCoh), and higher-order interaction (HOI) features.

higher-order features derived from phase-based representations, specifically imaginary coherence (ImCoh) in this study, can discriminate motor imagery (MI) from resting-state (REST) activity. The simplex profiles reveal systematic differences between the two conditions, with consistently higher levels of higher-order activity during MI than during REST across several frequencies. At first glance, this pattern may appear counterintuitive, as resting-state activity is often associated with large-scale distributed coordination across brain networks, whereas MI is typically considered a more localized task-driven process centered on sensorimotor regions.

The observed increase in simplex counts during MI likely reflects the specific type of higher-order interactions captured by the proposed framework. Rather than describing slowly varying functional integration across distant regions, these features emphasize simultaneous phase coordination across multiple channels. In contrast to classical functional connectivity measures, which may capture delayed or indirect interactions, the present approach high-

lights time-aligned coordination patterns among groups of electrodes. Under this interpretation, motor imagery may induce stronger multi-channel coordination within sensorimotor networks than unconstrained resting activity.

This interpretation is supported by the classification results. When HOI features are used, classification accuracy generally improves compared to traditional node-centered metrics such as power spectral density and node strength. Although the performance gain remains moderate, HOI features outperform the baseline representations for most participants and show relatively stable performance across subjects. This suggests that higher-order structures capture complementary information that is not fully represented by local spectral activity or pairwise connectivity measures, and that they may provide useful features for motor imagery decoding in EEG-based BCI systems.

Further work is required to validate these findings and refine their interpretation. Future studies should evalu-

ate the proposed approach across larger datasets, additional recording sessions, and other cognitive or motor tasks. Moreover, comparisons with alternative connectivity and information-theoretic measures may help clarify the neural mechanisms underlying the extracted higher-order structures. Such investigations will be important to assess the generality and practical relevance of higher-order representations for EEG-based brain-computer interface applications. From an applied perspective, the moderate computational cost of the proposed pipeline and the stability of the extracted subject-specific features suggest that this framework could also be relevant for future online BCI implementations. In particular, after an initial calibration phase, such higher-order descriptors could potentially be integrated into real-time decoding pipelines for adaptive motor imagery control.

CONCLUSION

In this work, we investigated the use of higher-order interaction (HOI) features derived from phase-based functional connectivity for motor imagery decoding in an EEG-based BCI context. By combining imaginary coherence (ImCoh) networks, spectral graph embedding, and topological data analysis, we extracted simplex-based descriptors that capture coordinated activity patterns across multiple EEG channels.

Our results demonstrate that these higher-order features reveal clear differences between motor imagery and resting-state conditions and improve classification performance compared to traditional representations such as power spectral density and node strength. In particular, HOI features achieve higher accuracy for the majority of subjects while maintaining stable performance across participants. These findings indicate that higher-order structures capture discriminative information that is not fully represented by local spectral activity or pairwise connectivity measures.

Overall, this study shows that topological higher-order descriptors provide an effective representation of functional brain networks and constitute a valuable feature space for motor imagery decoding in EEG-based brain-computer interfaces.

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