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Assessment method for torsional performance of high-rise buildings based on period ratio

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ABSTRACT: The torsional performance of a structure significantly impacts the safety and service life of high-rise buildings. Due to deviations between real structures and design models, it is essential to evaluate the torsional performance of in-service high-rise buildings. This paper proposes a method for evaluating the torsional performance of in-service high-rise buildings based on the measured period ratio. By abstracting the high-rise building as an equivalent cantilever beam model with unidirectional eccentricity, the free vibration equation of the structure is derived, and the relationship between the period ratio, stiffness ratio, eccentricity, and radius of gyration is analyzed. The results indicate that changes in the period ratio can reflect the torsional performance of the structure. Based on the Latin Hypercube Sampling (LHS) and Kernel Density Estimation (KDE) methods, the probability density function and cumulative distribution function of the period ratio are established, and a four-level classification method for torsional performance is proposed. Application to a 40-story high-rise building validates the method. The research results provide a new theoretical basis and practical guidance for evaluating the torsional performance of in-service high-rise buildings.

KEY WORDS: Torsional performance assessment; Period ratio; High-rise buildings; Structural health monitoring.

1 INTRODUCTION

The torsional effect of high-rise buildings under external loads is a significant factor affecting their service performance. As building height increases, the dynamic response of structures to horizontal excitations such as wind loads and seismic actions becomes more complex, and the torsional effect significantly intensifies [1-7]. Torsional vibrations can lead to the redistribution of internal forces within the structure, exacerbate damage to local components, and even trigger overall instability. For instance, during the 1995 Kobe earthquake, several high-rise buildings experienced asymmetric damage due to torsional effects, further underscoring the importance of torsional control [8-9]. Therefore, the assessment and optimization of torsional performance have become central issues in the design and safety maintenance of high-rise buildings.

Accurately assessing the torsional performance of high-rise buildings is crucial for ensuring their service safety. Currently, domestic and international codes primarily evaluate the torsional performance of structures indirectly through parameters such as the period ratio. Article 3.4.3 of the Chinese code *Code for Seismic Design of Buildings* (GB 50011-2010) [10] explicitly stipulates that the period ratio (the ratio of torsional period to translational period) should not exceed 0.9 to avoid significant torsional irregularity; Article 4.3.5 of the Chinese code *Technical Specification for Concrete Structures of Tall Buildings* (JGJ 3-2010) [11] further specifies that strengthening measures are required when the period ratio exceeds 0.85. Similarly, the American ASCE 7-22 code also limits torsional effects through modal participation mass ratios [12]. However, these methods are largely based on theoretical models during the design phase and struggle to reflect the time-varying characteristics of torsional performance during service due to material degradation, load variations, and accidental eccentricities [13-14].

To bridge the gap between theoretical models and real structures, there is an urgent need to obtain actual dynamic parameters of in-service buildings through Structural Health Monitoring (SHM) technology [15-17]. The period ratio, as a key indicator reflecting the torsional stiffness and mass distribution of a structure, can be dynamically updated through long-term monitoring data, thereby providing a more accurate assessment of the trends in torsional performance. However, there is currently a lack of research on the evaluation of in-service torsional performance based on measured period ratios.

This paper proposes a method for evaluating the torsional performance of in-service high-rise buildings based on measured period ratios, encompassing the following main aspects: Firstly, a theoretical relationship between the period ratio and the stiffness ratio, eccentricity, and radius of gyration is established using an equivalent cantilever beam model. Secondly, a probabilistic distribution of the period ratio is constructed using Latin Hypercube Sampling (LHS) and Kernel Density Estimation (KDE), and a four-level torsional performance classification standard is proposed. Finally, the engineering applicability of the method is validated using a 40-story reinforced concrete frame-shear wall structure as an example. The research results provide new theoretical foundations and practical guidance for the assessment of torsional performance in in-service high-rise buildings.

2 CLASSIFICATION METHOD FOR TORSIONAL PERFORMANCE LEVELS

2.1 Analysis of influencing factors of period ratio

To facilitate analytical derivation and computational modeling, a series of idealized assumptions are implemented in the structural characterization of the target high-rise building. Specifically, the structural system is postulated to exhibit unidirectional eccentricity along the Cartesian coordinate system - that is, a deliberate offset is introduced between the center of mass and the center of stiffness exclusively in the X direction, while maintaining perfect spatial coincidence of these two critical centers in the orthogonal Y direction. This intentional asymmetrical configuration is adopted to isolate and

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investigate the torsional effects induced by eccentricity along a single principal axis, thereby simplifying the coupled lateral-torsional vibration analysis.

The entire high-rise building is abstracted as an equivalent cantilever beam element with prescribed geometric and material properties, as schematically illustrated in Figure 1. The rationale behind this modeling approach stems from multiple considerations: Firstly, the cantilever beam provides a mathematically tractable framework for solving equations of motion. Secondly, the unidirectional eccentricity assumption enables parametric investigation of torsion-translation coupling mechanisms without introducing unnecessary computational complexity from bidirectional interactions.

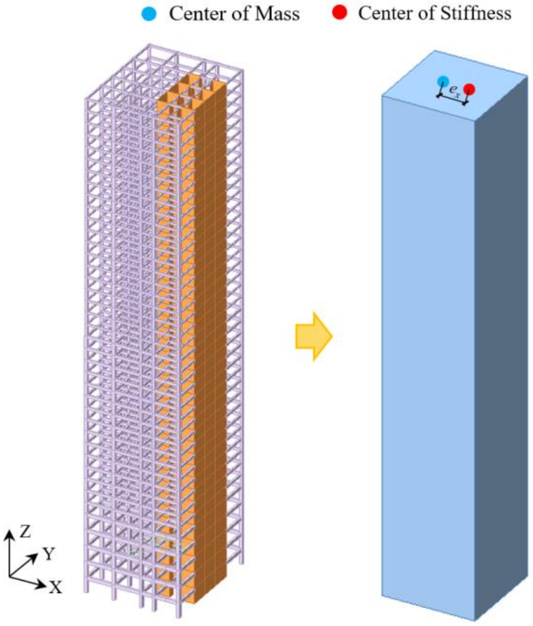

Figure 1. Schematic diagram of structural eccentricity.

The undamped free vibration motion equation of the structure is

$$\begin{bmatrix} M & \\ & J_c \end{bmatrix}\begin{bmatrix} \ddot{u}_y \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} K_y & e_x K_y \\ e_x K_y & K_\theta \end{bmatrix}\begin{bmatrix} u_y \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

where M is the mass of the high-rise building; J_c is the mass moment of inertia about the center of mass, $J_c = Mr^2$, and r is the radius of gyration of the structure relative to the center of mass; u_y is the translational displacement in the Y direction; θ is the torsion angle of the high-rise building; K_y is the translational stiffness in the Y direction, $K_y = \sum_{i=1}^{n} K_{yi}$, and K_{yi} is the horizontal lateral stiffness of the structural member in the Y direction; K_θ is structural torsional stiffness, $K_\theta = \sum_{i=1}^{n} K_{xi} {x_i}^2 + \sum_{i=1}^{n} K_{yi} {y_i}^2$, K_{xi} and K_{yi} are the horizontal lateral stiffness of the structural member in the X and Y directions respectively, and x_i and y_i are the distances from the component to the mass center, respectively; e_x is the eccentricity in the X direction.

Assume the response of the structure is

$$u_y(t) = A_y \sin(\omega t + \alpha) \quad (2)$$

$$\theta(t) = A_\theta \sin(\omega t + \alpha) \quad (3)$$

Substituting equations (2) - (3) into equation (1) and simplifying them, the circular frequency of the structure can be obtained. Assuming $\omega_1 < \omega_2$,

$$\begin{cases} \omega_1 = \sqrt{\dfrac{(K_\theta + r^2 K_y) - \sqrt{(K_\theta - r^2 K_y)^2 + 4r^2 {e_x}^2 {K_y}^2}}{2Mr^2}} \\ \omega_2 = \sqrt{\dfrac{(K_\theta + r^2 K_y) + \sqrt{(K_\theta - r^2 K_y)^2 + 4r^2 {e_x}^2 {K_y}^2}}{2Mr^2}} \end{cases} \quad (4)$$

Defining stiffness ratio $S_y = K_\theta / K_y$, equation (4) is transformed into the following form:

$$\frac{T_2}{T_1} = \frac{\frac{2\pi}{\omega_2}}{\frac{2\pi}{\omega_1}} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{(S_y + r^2) - \sqrt{(S_y - r^2)^2 + 4r^2 {e_x}^2}}{(S_y + r^2) + \sqrt{(S_y - r^2)^2 + 4r^2 {e_x}^2}}} \quad (5)$$

From equation (5), it can be seen that the ratio of the second-order period to the first-order period $\frac{T_2}{T_1}$ depends on S_y, r and e_x.

For eccentric structures, each vibration mode of the structure is composed of the superposition of translational vibration mode and torsional vibration mode. Substitute equation (4) into equations (1) - (3) to solve the ratio of torsional amplitude to translational amplitude $\frac{A_\theta}{A_y}$ in the vibration mode.

By substituting ω_1,

$$\frac{A_\theta}{A_y} = \frac{(\eta_y - r^2) - \sqrt{(\eta_y - r^2)^2 + 4r^2 {e_x}^2}}{2r^2 e_x} \quad (6)$$

By substituting ω_2,

$$\frac{A_\theta}{A_y} = \frac{(\eta_y - r^2) + \sqrt{(\eta_y - r^2)^2 + 4r^2 {e_x}^2}}{2r^2 e_x} \quad (7)$$

Equations (6) and (7) are the ratios of the amplitudes of the torsional direction to the translational direction in the first-order vibration mode and the second-order vibration mode, respectively.

For in-service high-rise buildings, the radius of gyration r remains constant; stiffness ratio S_y varies with the increase in service years; due to variations in live load distribution during the service life of the structure, as well as changes in stiffness of individual components over the same period, leading to shifts in the stiffness center, therefore the eccentricity e_x varies throughout the entire service life of the structure. The relationship between the torsional displacement ratio, stiffness ratio and eccentricity is shown in Figure 2.

As can be seen from Figure 2, the curve of the period ratio is divided into two parts: the rising section on the left side and the falling section on the right side. According to the calculations, for the rising section on the left side, the first-order vibration mode is torsion, and the second-order vibration mode is Y-direction translation. For the falling section on the right side, the first-order vibration mode is Y-direction translation, and the second-order vibration mode is torsion. Regarding the eccentricity, as the eccentricity increases, the ratio of the second-order period to the first-order period decreases.

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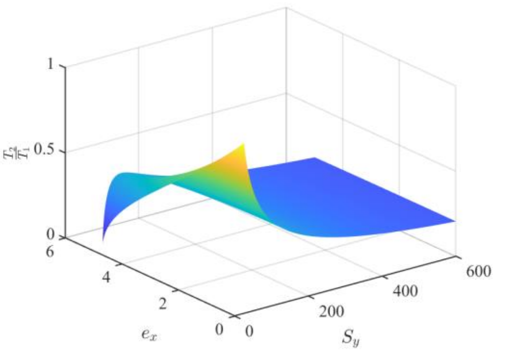
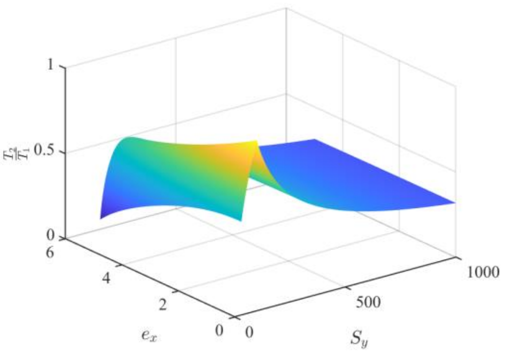
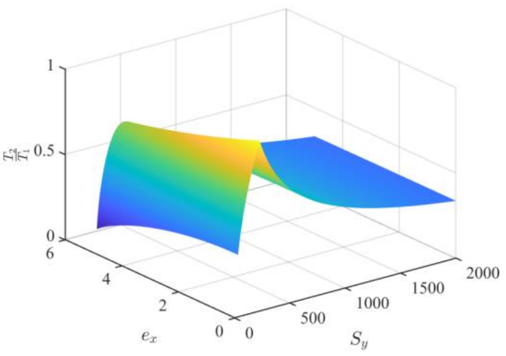


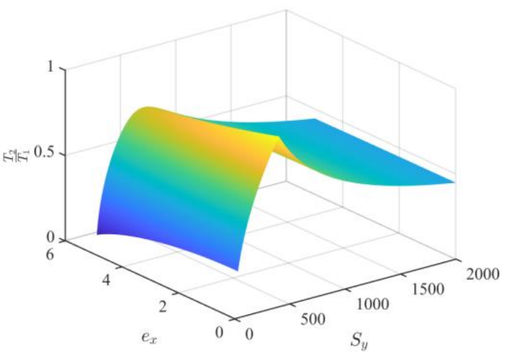
(a) $r = 5\text{m}$



(b) $r = 10\text{m}$



(c) $r = 15\text{m}$



(d) $r = 20\text{m}$

Figure 2. Period ratio curve.

2.2 Performance level classification method

As mentioned in Section 2.1, for in-service high-rise buildings, the radius of gyration r remains constant, while the eccentricity e_x and stiffness ratio S_y may vary. Regarding eccentricity, the Chinese code *Code for Seismic Design of Buildings* (GB 50011-2010) defines torsional irregularity in Clause 3.4.3 as follows: "Under the specified horizontal forces with accidental eccentricity, the ratio of the maximum value to the average value of the elastic horizontal displacement (or story drift) of the lateral force-resisting members at both ends of a story is greater than 1.2." Additionally, Clause 3.4.5 of the Chinese code *Technical Specification for Concrete Structures of Tall Buildings* (JGJ 3-2010) stipulates: "Under the specified horizontal seismic forces considering the influence of accidental eccentricity, the maximum horizontal displacement and story drift of the vertical members of a story should not exceed 1.2 times the average value of the story for Class A high-rise buildings, and should not exceed 1.5 times the average value of the story. For Class B high-rise buildings, mixed structures exceeding the height of Class A, and complex high-rise buildings referred to in Chapter 10 of this specification, the maximum horizontal displacement and story drift should not exceed 1.2 times the average value of the story, and should not exceed 1.4 times the average value of the story." Both code provisions mention the concept of "accidental eccentricity". Accidental eccentricity refers to the incomplete coincidence of the mass center and the stiffness center of a structure due to factors such as construction errors, material inhomogeneity, and uncertain load distribution, which results in additional torsional effects under external loads. To account for this uncertainty, seismic codes typically specify the introduction of an accidental eccentricity in calculations, generally taken as 5% of the structural plan dimensions. Therefore, it can be assumed that during the service life of the structure, the eccentricity follows a normal distribution, where the mean value of the eccentricity is the eccentricity of the design model. When the eccentricity is the mean value plus 5%, the distribution function of the eccentricity approaches 1 (here set to 0.999). This can be expressed in the following formula:

$$\mu_{e_x} = e_{xM} \tag{8}$$

where e_{xM} is the structural eccentricity obtained from the design model.

Assuming the distribution function of the eccentricity e_x is F_{e_x}, converting it to a standard normal distribution allows for the determination of the variance of the eccentricity e_x.

$$F_{e_x}(e_{xM} + a \cdot 5\%) \approx 0.999 \tag{9}$$

$$\sigma_{e_{xM}} = \frac{e_{xM} + a \cdot 5\% - e_{xM}}{3} = \frac{a \cdot 5\%}{3} = 0.01667a \tag{10}$$

$${\sigma_{e_x}}^2 = 0.000278a^2 \tag{11}$$

where a is the length of the structure in the X direction.

Assuming the initial stiffness ratio of the structure is the same as that in the structural design model or digital twin model, the degradation function of the stiffness ratio follows an exponential function. That is, the variation of the stiffness ratio with service years, denoted as $S_y(t)$, is as follows:

$$S_y(t) = S_{yM} e^{-0.005t} \tag{12}$$

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where S_{yM} is the structural stiffness ratio calculated by the structural design model or digital twin model; t is the service time in years.

Since t is a deterministic variable (assuming t ranges between 0 and 50), S_y is a deterministic function of t. Consequently, the distribution of S_y is contingent upon the distribution of t. Assuming that t follows a uniform distribution, the probability density function of S_y can be derived as follows:

$$f_{S_y}(S_y) = f(x) = \begin{cases} \frac{4}{S_y}, & S_{yM} e^{-0.005\times 50} \le S_y \le S_{yM} \\ 0, & \text{others} \end{cases} \quad (13)$$

A Latin Hypercube Sampling (LHS) of e_x and S_y is performed, where e_x follows a normal distribution and S_y adheres to the distribution specified in equation 13. LHS is a statistical method for generating a near-random sample of parameter values from a multidimensional distribution. It is a form of stratified sampling that ensures that the entire range of each variable is represented in the sample. The method divides the distribution of each variable into intervals of equal probability and selects one sample from each interval. This approach guarantees that the samples are more evenly distributed across the range of possible values than in simple random sampling, leading to more precise and reliable results, especially in the context of computer simulations and sensitivity analyses. LHS is particularly useful when dealing with complex models that require significant computational resources, as it can reduce the number of simulations needed to achieve a given level of accuracy.

The period ratio of the sample is calculated according to equation (5), and the Probability Density Function (PDF) F of the period ratio is obtained by fitting using Kernel Density Estimation (KDE). KDE is a non-parametric statistical method used to estimate the probability density function of a random variable. Unlike parametric methods, KDE does not require any assumptions about the data distribution (e.g., normal distribution) and instead estimates the density function directly from the data itself. The core idea is to treat each data point as the center of a "kernel function" (e.g., Gaussian kernel, uniform kernel, etc.) and then sum all these kernel functions to form a smooth density curve.

After obtaining the probability density function of the period ratio $\frac{T_2}{T_1}$, it can be integrated to derive the distribution function of the period ratio $\frac{T_2}{T_1}$. Based on this distribution function, the torsional performance can be classified into four levels, as shown in Figure 3. Specifically, $\left.\frac{T_2}{T_1}\right|_1 = F^{-1}(0.20)$, $\left.\frac{T_2}{T_1}\right|_2 = F^{-1}(0.35)$, $\left.\frac{T_2}{T_1}\right|_3 = F^{-1}(0.50)$, $\left.\frac{T_2}{T_1}\right|_4 = F^{-1}(0.65)$, $\left.\frac{T_2}{T_1}\right|_5 = F^{-1}(0.80)$, $\left.\frac{T_2}{T_1}\right|_6 = F^{-1}(0.95)$. The ordinate in Figure 3 represents the closeness coefficient, which is a concept within the Fuzzy Analytic Hierarchy Process (FAHP). FAHP is a decision analysis method that integrates fuzzy mathematics with the Analytic Hierarchy Process (AHP), designed to address issues of uncertainty and fuzziness. In FAHP, the closeness coefficient quantifies the proximity of alternatives to the ideal solution. The closeness coefficient indicates the optimality of the alternatives, with values approaching 1 signifying more ideal solutions. This coefficient offers decision-makers a straightforward metric for ranking alternatives and is extensively applied in multi-criteria decision-making scenarios.

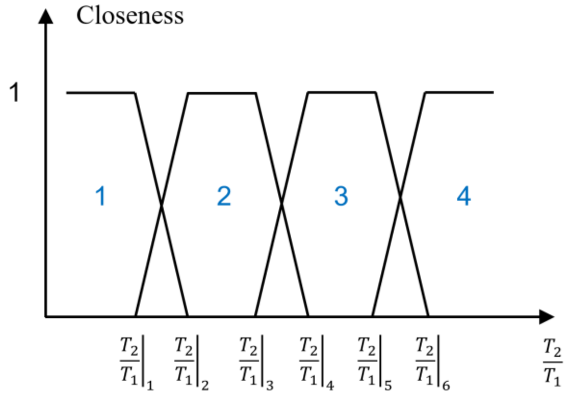

Figure 3. Membership function of torsional performance.

3 ENGINEERING APPLICATION

3.1 *Project overview*

The high-rise building under investigation is a 40-story reinforced concrete frame-shear wall structure, with a total height of 150 meters. The lower section (1st to 5th floors) has a story height of 4.8 meters, while the upper section (6th to 40th floors) maintains a uniform height of 3.6 meters per story. In the longitudinal direction, the structure comprises five spans measuring 8.4 m, 5.7 m, 5.7 m, 5.7 m, and 8.4 m respectively, resulting in a total length of 33.9 meters. Transversely, the building features four spans with dimensions of 8.4 m, 8.4 m, 3.9 m, and 7.5 m, accumulating to a width of 28.2 meters. The structural design incorporates a floor dead load of 5.0 kN/m² and a live load of 2.0 kN/m². Detailed specifications regarding component dimensions and material properties are provided in Table 1, while Figure 4 illustrates the structural layout of the high-rise building. A comprehensive three-dimensional finite element model was developed using midas Gen software, with the complete model visualization presented in Figure 5.

Table 1. Basic information of beams, columns, and shear walls.

Component type	Floor	Size	Material
Column	1-5	900mm×900mm	C60
	6-20	800mm×800mm	C60
	21-40	700mm×700mm	C60
Beam	1-5	500mm×1000mm	C60
	6-20	400mm×800mm	C60
	21-40	300mm×600mm	C60
Coupling Beam	1-5	500mm×1000mm	C60
	6-20	400mm×800mm	C60
	21-40	300mm×600mm	C60
Shear Wall	1-5	Thickness 500mm	C60
	6-20	Thickness 400mm	C60
	21-40	Thickness 300mm	C60

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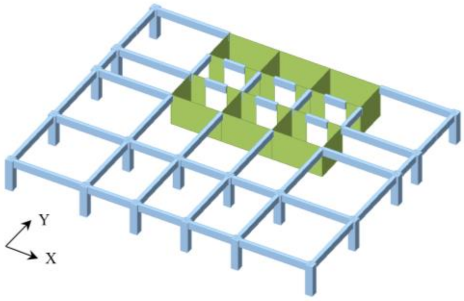


Figure 4. Layout plane of the high-rise building.

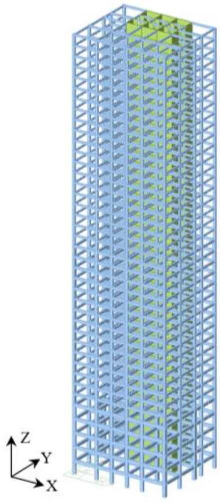


Figure 5. Three-dimensional finite element model.

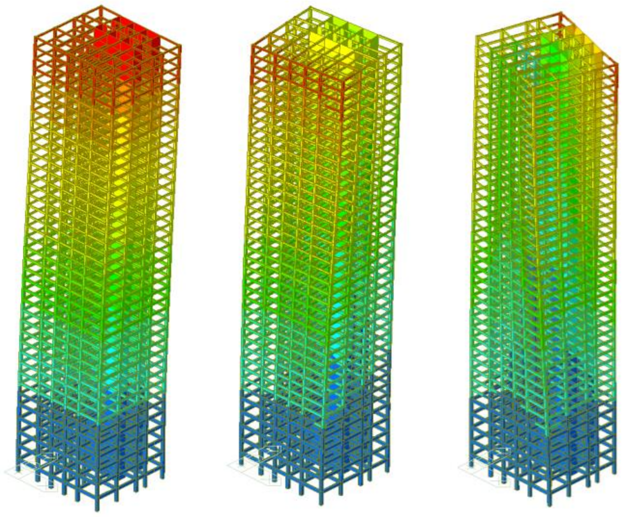
It can be observed that the structure does not exhibit eccentricity in the X direction, but does have eccentricity in the Y direction. The modal analysis of the structure was conducted using midas Gen, and the results are presented in Table 2 and Figure 6. The first mode shape corresponds to translational motion in the Y direction with a period of 4.338 seconds. The second mode has a period of 3.424 seconds and is primarily characterized by translational motion in the X direction, with a component of torsional motion. The third mode shape is torsional, with a period of 2.321 seconds.

Table 2. Vibration period of the structure.

No.	Period(s)	Direction
1	4.338	Y
2	3.424	X
3	2.321	T

3.2 Torsional performance levels

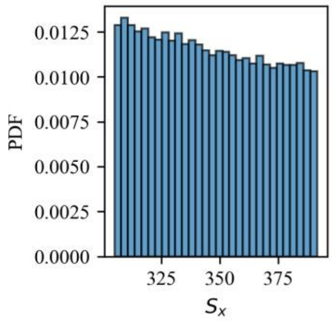
This paper separately discusses the methodology for classifying the torsional performance levels of structural service based on the ratio of the third-order period to the second-order period $\frac{T_3}{T_2}$. Initially, the radius of gyration r of the structure is obtained based on the finite element model. For the high-rise building examined in this study, $r = 13.50\text{m}$.



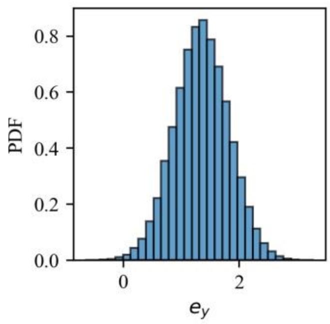
(a) First order (b) Second order (c) Third order

Figure 6. Structural vibration mode.

First, values of e_y and S_x are sampled using LHS, as illustrated in Figure 7 (a) and (b), respectively, with $\mu_{e_y} = 1.35\text{m}$, ${\sigma_{e_y}}^2 = 0.2211$, and $S_{xM} = 391.63$. The $\frac{T_3}{T_2}$ values for the samples are then calculated using equation (5), as shown in Figure 7 (c).



(a) S_x



(b) e_y

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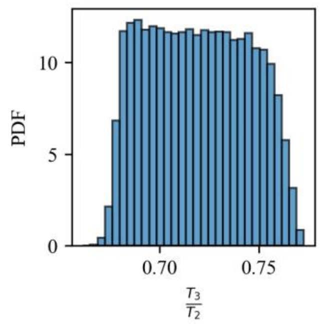

(c) $\frac{T_3}{T_2}$

Figure 7. Latin Hypercube Sampling of parameters.

The KDE was applied to fit Figure 7 (c), resulting in the probability density distribution function of the period ratio, as illustrated in Figure 8. Subsequently, the cumulative distribution function of the period ratio was obtained by integrating the probability density function, which is depicted in Figure 9. The membership function of the in-service torsional performance of the structure is shown in Figure 10.

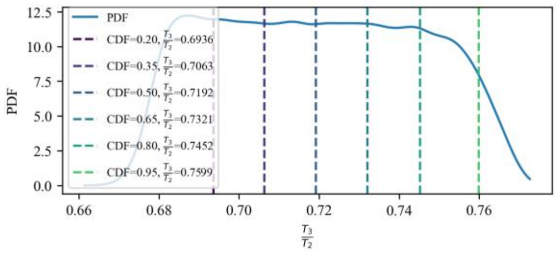

Figure 8. Probability density function of $\frac{T_3}{T_2}$.

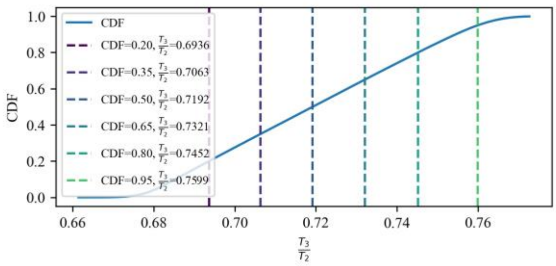

Figure 9. Cumulative distribution function of $\frac{T_3}{T_2}$.

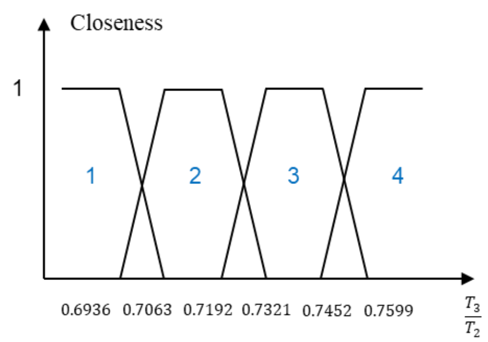

Figure 10. Membership function of torsional performance.

4 CONCLUSIONS

This paper proposes a method for evaluating the torsional performance of in-service high-rise buildings based on the measured period ratio, establishing a relationship between period ratio, stiffness ratio, eccentricity, and radius of gyration. Using Latin Hypercube Sampling and Kernel Density Estimation, a four-level classification method for torsional performance was developed and validated through a 40-story reinforced concrete frame-shear wall structure. The findings provide a new theoretical and practical approach for assessing torsional performance, bridging the gap between design models and real-world structural behavior.

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