DOI: 10.3217/978-3-99161-057-1-191

Vehicle speed estimation using convoluted reciprocity for bridge structural monitoring

Daniel Cantero[1,] [0000-0001-8947-8569]
[1]Structural Engineering, NTNU Norwegian University of Science and Technology, Trondheim, Norway
email: daniel.cantero@ntnu.no

ABSTRACT: The estimation of vehicle speed is a critical first step in deriving vehicle weight from bridge responses. Various strategies have been developed to extract the speed of passing vehicles, primarily relying on sensors that capture signals with features related to the vehicle's axles. These signals are processed through diverse methods; however, existing strategies often fail to perform optimally across different structural configurations. To address these challenges, the convoluted reciprocity (CR) relationship was recently proposed, which was verified numerically and validated experimentally in a laboratory setting. In this document, the novel speed estimation strategy based on CR is applied to an operational bridge using signals from the SCSHM benchmark. The results confirm that CR provides a robust speed estimation method for cases when the signals lack individual axle features.

KEY WORDS: Speed; Vehicle; Bridge Monitoring; Convoluted Reciprocity; SCSHM.

1 INTRODUCTION

Instrumented bridges can be utilised for weighing vehicles as they pass over them, enabling the estimation of Gross Vehicle Weight (GVW) based on the integral of the recorded signal [1]. When integrated into Bridge Weigh-in-Motion (BWIM) systems, such instrumentation allows for the identification of individual axle loads and axle spacing. This technology has gained significant attention in recent years ([2], [3], [4], [5]) due to the valuable site-specific traffic data it provides, as well as its potential applications in structural health monitoring [6].

A crucial first step in any weighing solution based on bridge responses is determining the speed of the passing vehicle. This requires signals that provide reliable speed estimation. Standard approaches include FAD (Free of Axle Detectors) and NOR (Nothing on the Road) methods, which utilise specific signal features to estimate vehicle speed, axle count, and axle spacing [4]. Additionally, alternative methods have successfully employed acoustic signals generated by tyres passing over bridge expansion joints to determine vehicle speeds [7].

A recent study by the author [8] investigated various speed estimation strategies using strain sensors, demonstrating that existing methods perform well in most practical scenarios. However, a consistent theoretical framework for speed estimation was lacking. The standard approach relies on the cross-correlation of signals at different bridge locations, which is effective when distinct axle signatures are present but fails in certain cases, particularly for simply supported bridges. To address this limitation, [8] introduced the Convoluted Reciprocity (CR) relationship, developing a novel speed estimation method. This approach does not require signals to exhibit distinct axle features, making it more widely applicable. The Convoluted Reciprocity framework was theoretically derived, numerically verified, and experimentally validated in [8].

This document aims to present the limitations of the correlation-based method for speed estimation and to illustrate the effectiveness of the novel Convoluted Reciprocity (CR) approach. First, the shortcomings of the correlation method are visually demonstrated. Then, the CR concept is introduced and verified through numerical examples. Finally, the method is validated using real bridge measurements from the publicly available SCSHM dataset [9].

2 THE PROBLEM WITH CORRELATION

Arguably, the most common strategy for estimating the speed of a passing vehicle using bridge responses is based on correlating signals recorded at two separate locations along the bridge. The key idea is to determine the time lag that maximises the correlation between the signals, which indicates the time taken by the vehicle to travel between the sensor locations. Given the known sensor distance, the vehicle speed can then be directly estimated. This method assumes that the signals are shifted versions of each other and has been successfully applied in many cases. However, its effectiveness relies on the presence of distinct peaks corresponding to individual axles in the signals, which is not always guaranteed.

To illustrate this, synthetic bridge responses are used from a numerical simulation of a two-axle vehicle traversing a bridge, modelled using the open-source VBI-2D tool [10]. The vehicle's axles have equal weights and are spaced 5 m apart. The bridge has a span of 20 m, with strain measured at sensors located at ¼ and ¾ of the span, denoted as S25 and S75, respectively. The vehicle travels at a constant speed of 20 m/s (72 km/h). For verifying purposes, only the quasi-static response is simulated, excluding dynamic effects and noise.

For a bridge with fixed-fixed boundary conditions, the resulting signals are shown in Figure 1(a). The peaks corresponding to each axle are clearly distinguishable. Although the signals are not perfect shifted versions of each other, the correlation method performs accurately. By

DOI: 10.3217/978-3-99161-057-1-191

computing the correlation and identifying the time lag that maximises it, the exact travel time between sensor locations is obtained, as illustrated in Figure 1(b). Consequently, the estimated speed in this case is precise.

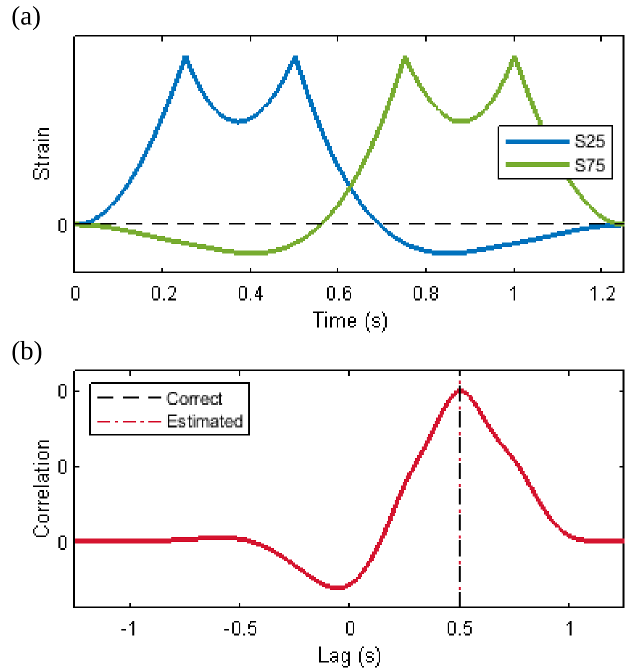

Figure 1. (a) Simulated fixed-fixed beam responses due to a 2-axle vehicle; (b) Cross-correlation of signals. Vertical lines indicate the time lag giving maximum correlation and the one that should have been obtained for correct speed estimation.

In contrast, for a simply supported bridge, the responses do not exhibit distinct peaks, leading to poor speed estimates using the correlation method. Figure 2(a) presents the simulated bridge responses for the same vehicle and bridge configuration but with simply supported boundary conditions. Applying the correlation method in this case (Figure 2(b)) results in a significantly inaccurate speed estimate. Specifically, the estimated speed is 47.62 m/s, corresponding to a 138% error.

Therefore, the correlation method is poorly suited for simply supported bridges due to the absence of distinct axle features. However, even in cases where clear axle features are present, such as fixed-fixed bridges, the method does not always guarantee perfect results. The speed estimation based on maximum correlation may still be imprecise, depending on the vehicle configuration and span length. This limitation is not new, and various correction strategies exist, but they are either specific to certain vehicle types or rely on signal processing techniques. A more detailed analysis can be found in [8]. Nevertheless, until now, no theoretically sound alternative had been established. This gap is addressed by the speed estimation method based on Convoluted Reciprocity.

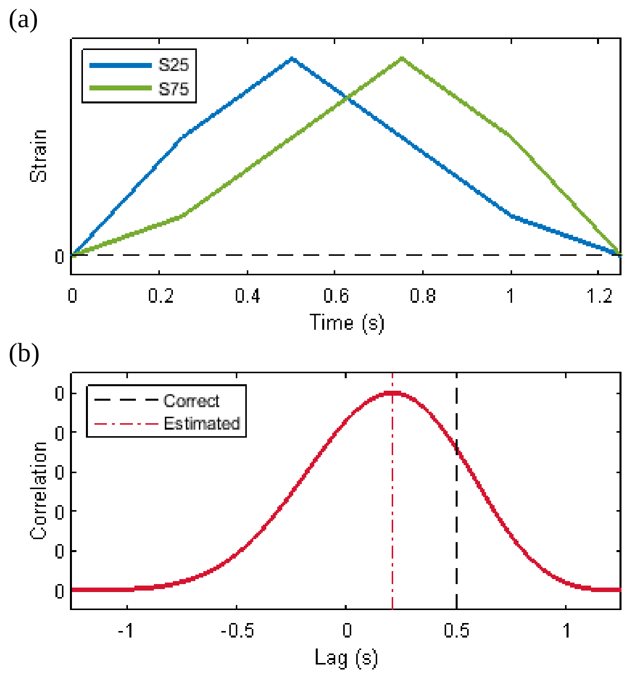

Figure 2. (a) Simulated simply supported beam responses due to a 2-axle vehicle; (b) Cross-correlation of signals. Vertical lines indicate the time lag giving maximum correlation and the one that should have been obtained for correct speed estimation.

3 CONVOLUTED RECIPROCITY

This section introduces the concept of Convoluted Reciprocity and its application to speed estimation, originally presented in [8]. Here, a step-by-step example is provided to further explain and verify the method. The verification is performed using VBI-2D [10] under ideal conditions, considering perfect quasi-static signals without noise or dynamic disturbances. The analysis follows the same case as presented in Section 2, ensuring direct comparison with the correlation-based approach.

In [8], it was shown that a relationship exists between bridge responses at two different locations (A and B) for two different vehicle passages, say Vehicle 1 (V1) and Vehicle 2 (V2). This relationship is expressed in Eq. (1).

$$S_{A,V1}(t) \times S_{B,V2}(t) = S_{B,V1}(t) \times S_{A,V2}(t) \tag{1}$$

where $S_{i,j}$ represents the measured load effect at location *i* due to the passage of vehicle *j*, and × denotes the convolution operation. This relationship follows from the fact that any bridge response to a passing vehicle can be expressed as the convolution of the vehicle's forcing function with the corresponding influence line. By taking advantage of the commutative property of convolution, the expression is derived (see [8] for a detailed derivation). This result establishes a reciprocal relationship between signals recorded at different sensor locations and vehicle passages when convolved together, leading to the adopted term Convoluted Reciprocity (CR). The expression can be further simplified to:

$$CR_{AB}(t) = CR_{BA}(t) \tag{2}$$

This relationship is rather powerful, as it connects any two load effects for any two vehicle passages. It holds under the

DOI: 10.3217/978-3-99161-057-1-191

standard assumption that vehicles travel at a constant speed. Using this relationship, the speed of an unknown vehicle passage can be estimated if the response of a reference event is known. In practice, signals from a vehicle event with a known speed are stored as a reference, enabling the speed estimation of subsequent vehicle passages.

For example, Figure 3 presents the signals for a five-axle truck with axle spacing and load distribution as specified in [11], representing a typical European configuration for a fully loaded articulated five-axle truck. Since the vehicle speed is known for this reference event, the signals can be transformed into the spatial domain. This event serves as a calibration or reference event, providing a basis for speed estimation of other vehicle passages.

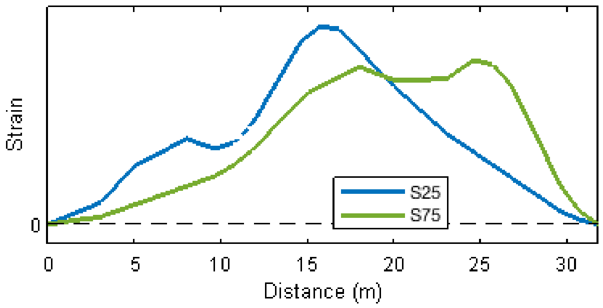

Figure 3. Simulated simply supported beam response due to a 5-axle truck.

Applying the Convoluted Reciprocity (CR) relationship in the time domain, we estimate the speed of the unknown vehicle by iteratively testing different speed guesses and evaluating the CR relationship. For each guessed speed, the signals from Figure 3 are transformed accordingly into the time domain. By applying the relationship in Eq. (2), together with the signals in Figure 2(a), results are obtained for each speed guess. Figure 4 presents the left-hand side and right-hand side of Eq. (2) for different guessed speeds. The speed estimate that results in a match between both curves corresponds to the actual speed of the unknown vehicle event.

To systematically quantify the differences between both sides of Eq. (2), we evaluate the norm of their difference (see Eq. (3)). The study in [8] explored various norm choices and suggested that the 1-norm could be a suitable option. However, other p-norms may also yield good results. The best speed estimate can be determined by finding the speed that minimises the norm of the difference.

$$\|CR_{AB} - CR_{BA}\| = 0 \tag{3}$$

Figure 5 presents the norm of the difference between both sides of Eq. (2) for a range of guessed speeds. The minimum of this norm corresponds to the speed that best matches the actual speed of the unknown vehicle event. This example was conducted using ideal quasi-static responses, free from noise and disturbances. As a result, the speed estimation is exact, confirming the validity of the methodology. However, when applying this approach to real signals affected by noise and disturbances, the accuracy of the estimated speed may be impacted.

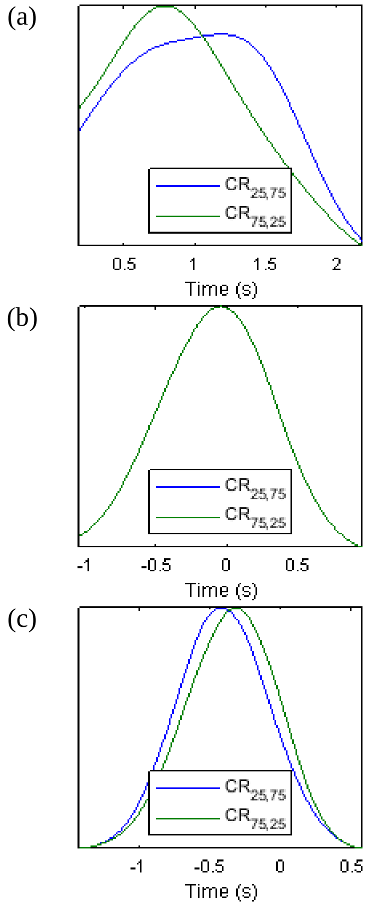

Figure 4. Detail of the CR for various speed guesses: (a) 10 m/s; (b) 20 m/s; (c) 30 m/s.

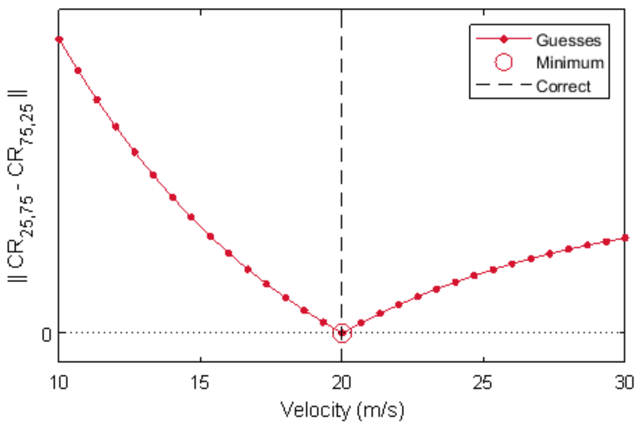

Figure 5. Norm values for a range of speed guesses.

4 VALIDATION USING SCSHM

The CR method for speed estimation was verified in the previous section using ideal quasi-static bridge responses. This section focuses on validating the method under realistic conditions by applying it to data from a real bridge, where signals include noise and dynamic effects. The CR approach is tested using bridge responses from the SCSHM dataset [9], specifically for cases where the signals do not exhibit clear axle features.

DOI: 10.3217/978-3-99161-057-1-191

4.1 The SCSHM dataset

The dataset introduced in [9] corresponds to a nine-span bridge with a total length of 291 m, located in Manitoba, Canada. The bridge carries two lanes of traffic, one in each direction, and its superstructure consists of four lines of I-girders supporting a reinforced concrete (RC) deck. Of particular interest is Span 2, a simply supported span of 22.71 m, which is instrumented with strain gauges and thermocouples at various locations. The signals are sampled at 200 Hz, and only events exceeding a predefined strain threshold are recorded.

For this study, key sensors are those located at the ends of the span (Sections AA and EE), where strain gauges attached to the deck capture signals with clear individual axle features. These signals were used in [9] to accurately estimate vehicle speeds, and these estimates will be taken as the reference speeds in this work. Additionally, two other instrumented sections, FF (at approximately ¼ span) and BB (at approximately ½ span), contain multiple strain gauges on each girder. Here, only the strain measured at the soffit will be considered. The dataset includes both strain measurements and vehicle photographs from several monitoring campaigns, totalling over 3,000 heavy vehicle crossing events. For further details on the instrumentation setup, refer to [9].

4.2 Example for one event

As an example, this section estimates the speed of a vehicle passage using the signals from a reference event with a known speed. Specifically, the analysis focuses on vehicles travelling in the westbound lane, where strain gauges located on girder G2 are considered (channel 10 in section FF and channel 22 in section BB). The reference event corresponds to file 04/E00003 (event 3 in folder 04), with a recorded speed of 17.2 m/s (61.92 km/h) and a GVW of 433.3 kN. The signals used for the CR-based speed estimation are shown in Figure 6. For speed estimation, these signals are first transformed into the spatial domain using the known speed of the reference vehicle.

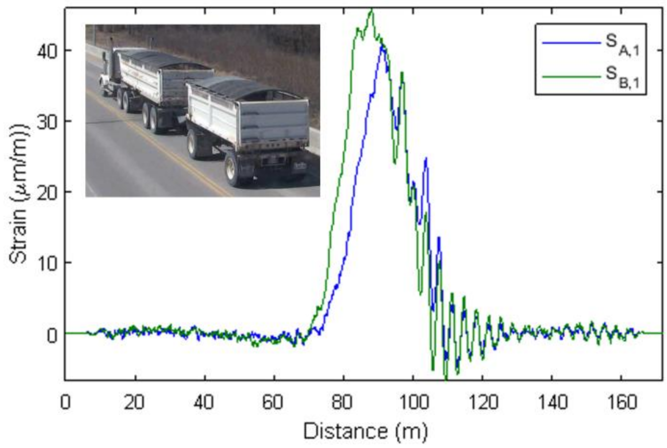

Figure 6. Signals of reference event (04/E00003).

The goal is to estimate the speed of an event with an unknown speed using the CR method. The event under analysis is 05/E00001, corresponding to a truck with a GVW of 319.9 kN. The signals for this event are shown in Figure 7. The dataset provides a recorded speed of 14.5 m/s (52.2 km/h), which will be used as the reference value for validation.

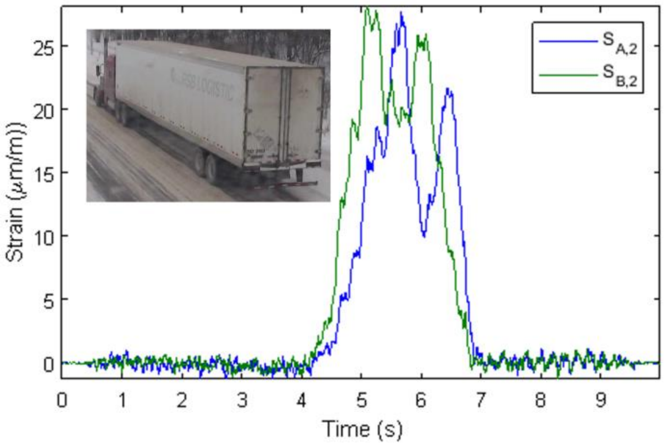

Figure 7. Signals of event to determine its speed (05/E00001).

Using the CR relationship, the estimated speed must satisfy Eq. (2). To apply this method, the speed of the reference vehicle is assumed, and the signal is transformed back into the time domain by assuming a different speed. As an example, this calculation is repeated for three different assumed speeds, and the results are plotted in Figure 8.

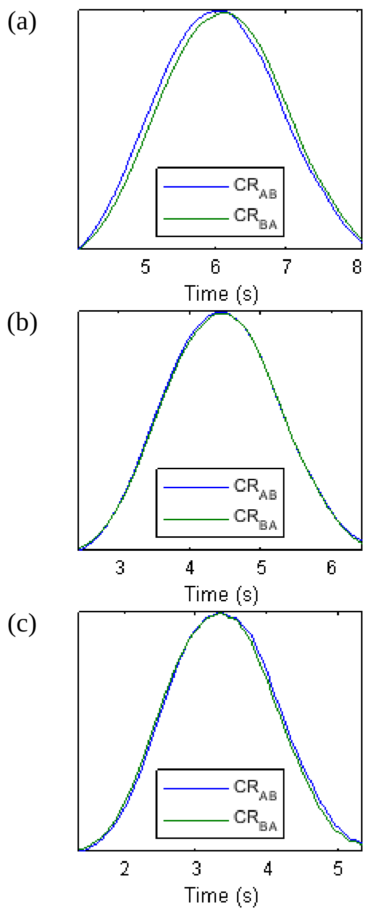

Figure 8. Detail of the CR for various speed guesses: (a) 40 km/h; (b) 50 km/h; (c) 60 km/h.

This process can be repeated for many more guesses. To evaluate the similarity between both CRs, the norm of the difference between both could be used. For this study, we adopted the 1-norm. Only a few guesses are shown below in

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Figure 9 for visualization purposes. More guesses can be easily computed to refine the final speed estimation. The speed that gives the minimum norm is considered as the estimated speed of the event. Increasing the number of guesses, one obtains an estimated speed of 14.44 m/s (51.98 km/h), which for this case is almost a perfect match (-0.38% error).

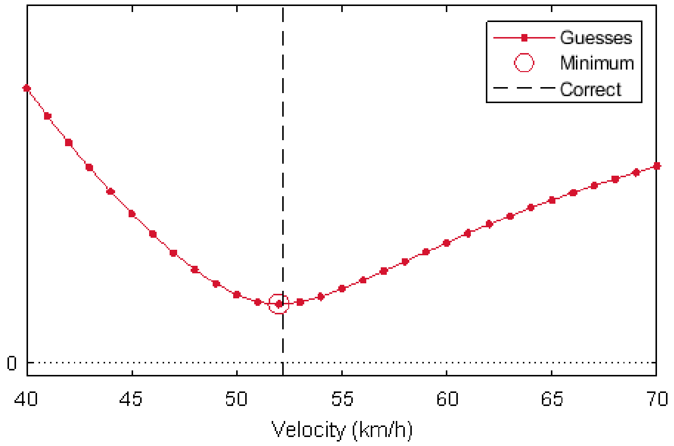

Figure 9. Norm values for a range of speed guesses.

4.3 Database analysis

This section applies the CR method to estimate the speed of all the valid events available in folder 05 in the database. Note that only events with a single vehicle are considered. Also, events with missing pictures are removed. Therefore, a total of 2149 events are analysed below.

To apply the CR method to a real bridge, we need to use a reference event for each lane. Speed estimation for each lane is done separately. The bridge response is different for each lane, so we need to define one reference event for each lane. When processing, we can identify which lane the vehicle is traveling on simply from the maximum response values across different girders. Once this is detected, the CR method is applied for the corresponding reference vehicle.

Figure 10(a) shows all the speed estimation errors in terms of difference to those provided by the database. The same results can be visualized in a histogram in Figure 10(b). Overall, one can see that most of the speed estimates fall within the 5% band, which is reasonable for the goal of estimating the GVW of passing vehicles. However, there are some instances where much higher errors are observed.

The results below show that the histogram is not centred around zero value, indicating an underlying bias related to the selected reference vehicles. In this calculation, the reference events correspond to normal events traversing at normal operational speeds. In practice, it should be possible to calibrate the calculation by obtaining signals from a reference event with very slow speeds, one for each lane. We can make the reference vehicle passages occur at slow speeds, making these events almost perfectly quasi-static. The unknown events will have dynamics, introducing some error, which is the dispersion observed in the results. Furthermore, the separation between section FF and BB is rather small; results would improve with sensors placed further apart.

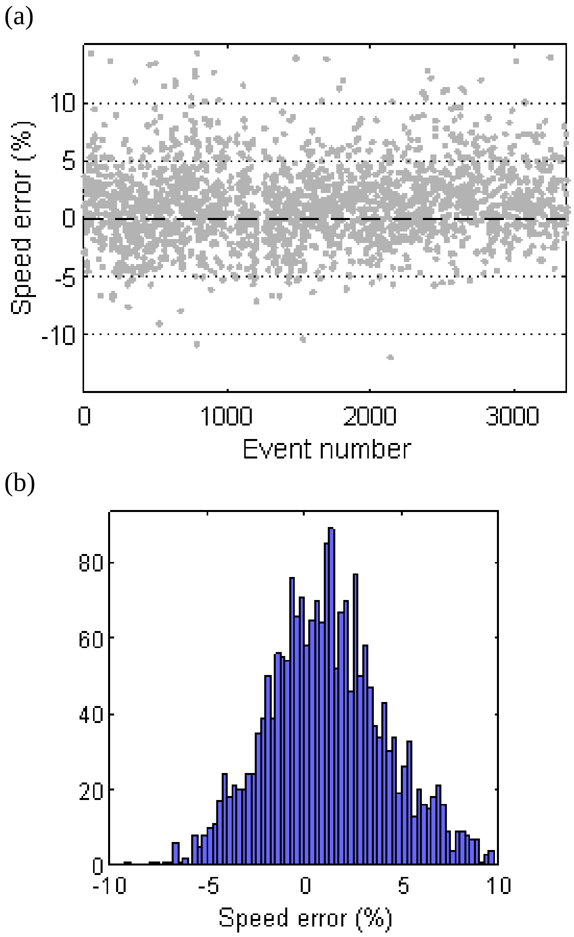

Figure 10. Single vehicle events speed estimation. (a) Estimation errors for each event; (b) Histogram of estimation errors.

Not shown here, but the possibility of improving the performance by signal processing the results was explored. Low-pass filtering and moving average filtering were explored to try to reduce the contributions of bridge dynamics and noise. However, the CR method seems to be rather robust, with only marginal reductions in errors observed. On one hand, this shows that the idea is robust and can be applied directly to unprocessed signals. On the other hand, this indicates that there is no easy way of improving the performance of the method simply by pre-processing the signals.

5 CONCLUSION

In general, when the goal is to estimate the speed of passing vehicles, signals with individual axle features should be used. In those cases, the standard correlation method provides satisfactory results. Nonetheless, this method is not theoretically sound nor valid for all bridge configurations.

This document has presented a methodology to estimate the speed of passing vehicles supported by the convoluted reciprocity relationship. First, an ideal numerical example is used to verify the concept. Then, the method is applied to the measured single vehicle events available in the SCSHM dataset. The reported speed estimation errors show some scatter, but most of them are within a 5% error band.

The methodology presented here enables the speed estimation for a wider range of possibilities. It is not strictly

DOI: 10.3217/978-3-99161-057-1-191

necessary to have signals with clear individual axle features. Therefore, the methodology can use sensors that capture the global behaviour of the bridge. This opens the possibility of having vehicle weighting capabilities on existing monitoring systems, with other load effects, or installing them on bridges that do not have local responses that would show individual axle features. Sometimes bridges do not have locations with responses that have clear individual axle features, or the bridge has an existing installation with the original intention of SHM and no sensors with axle features. CR opens the possibility of estimating the speed also in those cases.

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