

# Advancing High Fidelity Finite Element Model Updating Using Cooperative Game Theory: A Novel Framework for Structural Optimization

Suzana Ereiz<sup>1,2</sup> 0000-0001-8485-0587, Ivan Duvnjak<sup>2</sup> 0000-0002-9921-1013, Javier Fernando Jiménez-Alonso<sup>3</sup> 0000-0002-4592-0375, Marko Bartolac<sup>2</sup> 0000-0002-5330-736X, Janko Koščak<sup>2</sup> 0000-0001-6677-6635, Domagoj Damjanović<sup>2</sup> 0000-0002-3565-1968

<sup>1</sup> Politecnico di Milano, Department of Architecture, Built Environment and Construction Engineering (DABC), Piazza Leonardo da Vinci, 32, Milano 20133, Italy

<sup>2</sup> University of Zagreb, Faculty of Civil Engineering, Fra Andrije Kačića-Miošića 26, Zagreb 10 000, Croatia

<sup>3</sup> Universidad de Sevilla, Escuela Superior de Ingeniería, Avenida Camino de los Descubrimientos s/n, 41092, Sevilla, Spain

email: [suzana.ereiz@polimi.it](mailto:suzana.ereiz@polimi.it), [ivan.duvnjak@grad.unizg.hr](mailto:ivan.duvnjak@grad.unizg.hr), [jfjimenez@us.es](mailto:jfjimenez@us.es), [marko.bartolac@grad.unizg.hr](mailto:marko.bartolac@grad.unizg.hr), [janko.koscak@grad.unizg.hr](mailto:janko.koscak@grad.unizg.hr), [domagoj.damjanovic@grad.unizg.hr](mailto:domagoj.damjanovic@grad.unizg.hr)

**ABSTRACT:** High fidelity finite element model updating plays a critical role in ensuring the accuracy and reliability of structural models for complex infrastructure systems. This study focuses on the application of a cooperative game theory model to update high fidelity finite element model of a pedestrian suspension bridge. By treating the model updating process as a cooperative game model, game theory provides a novel framework for distributing and balancing multiple objectives inherent in this process. The proposed approach is compared with conventional finite element model updating methods to assess its efficiency, accuracy, and robustness. Key performance indicators, such as the reduction in discrepancy between experimental and numerical modal parameters and computational efficiency are evaluated. The cooperative game theory framework is shown to enable an optimized and balanced resolution of conflicting requirements in high fidelity model updating, resulting in improved alignment with observed structural behavior. The primary objective of this research is to demonstrate the potential of game theory as an innovative and effective tool for solving optimization problems in high fidelity FE model updating. The findings are expected to contribute to advancements in structural health monitoring by providing a robust methodology for enhancing the reliability of numerical models.

**KEY WORDS:** High Fidelity Finite Element Model Updating; Cooperative Game Theory; Structural Optimization; Structural Health Monitoring, Dynamic parameters

## 1 INTRODUCTION

High-fidelity numerical modelling has become essential in modern structural engineering for simulating complex physical behavior with a high degree of accuracy. High-fidelity finite element (FE) models are characterized by detailed geometric definitions, fine mesh discretization, and many physically meaningful parameters. These models enable precise structural simulations but also significantly increase computational effort and sensitivity to modelling assumptions. Despite their accuracy, high-fidelity FE models often fail to perfectly represent real structural behavior due to uncertainties in boundary conditions, material properties, and idealizations made during the modelling process. To reduce these discrepancies, the Finite Element Model Updating (FEMU) procedure is employed. FEMU involves adjusting selected model parameters based on experimental data—typically obtained from static tests, dynamic modal analysis, or continuous structural health monitoring—to improve correlation between the numerical model and the actual structural behavior [1]. The updating process becomes particularly challenging for high-fidelity models, where computational demands are high, and the solution space is large. In such cases, effective and reliable optimization strategies are critical. FEMU methods can be broadly classified into direct (non-iterative) and indirect (iterative) approaches [2]. Direct methods update the numerical model by modifying mass or stiffness matrices in a single step but may lack physical

interpretability [3][4]. In contrast, indirect (iterative) methods adjust physical parameters through successive approximations until numerical predictions align with experimental data [5]. One of the most widely used formulations is the Maximum Likelihood Method, which treats FEMU as an optimization problem (Eq. (4)) aimed at minimizing the difference between predicted and measured structural responses (Eq. (1)-(3)) [6].

$$r_t^f(\theta) = |\Delta f_t| = \left| \frac{f_t^{num} - f_t^{exp}}{f_t^{exp}} \right| \quad (1)$$

$$\text{MAC}(\phi_t^{exp}, \phi_t^{num}) = \frac{|(\phi_t^{num})^T \phi_t^{exp}|^2}{((\phi_t^{num})^T (\phi_t^{num})) \cdot ((\phi_t^{exp})^T (\phi_t^{exp}))} \quad (2)$$

$$r_t^m(\theta) = \sqrt{\left( \frac{(1 - \sqrt{\text{MAC}})^2}{\text{MAC}} \right)} \quad (3)$$

These residuals are then combined into a single-objective function (Eq. (4)) using weighting factors:

$$F(\theta) = \sum_{t=1}^{n_r} w_t F_t(\theta)^2 \quad (4)$$

$$F(\theta) = (\sum_{t=1}^{n_f} w_t^f r_t^f(\theta)^2 + \sum_{t=1}^{n_m} w_t^m r_t^m(\theta)^2),$$

$$\theta_l < \theta < \theta_u$$

This discrepancy is mathematically expressed using residuals based on modal parameters—most commonly natural

frequencies (Eq. (1)) and mode shapes (Eq. (3)). The residuals are defined as: optimization problem into a cooperative game structure, enabling a more robust and adaptive solution process suitable for updating complex and high-fidelity FE models.

The performance of this approach is highly dependent on the proper selection of the weighting factors and, which balance the influence of different types of residuals. However, determining these weights is non-trivial—typically requiring trial-and-error procedures, sensitivity analyses, and expert judgment [7–11]. This process is especially inefficient and unreliable when applied to high-fidelity FEMU, where each function evaluation involves high computational cost. To overcome these limitations, this paper proposes a novel formulation of the single-objective FEMU problem using Cooperative Game Theory (CGT). In the proposed approach, each residual is modelled as a player in a cooperative game. Instead of manually assigning weights, the optimization seeks a compromise solution by introducing a weighted objective function and a super-criterion that captures collective performance. This eliminates the need for manual tuning of weighting factors and enables a more automated and adaptive updating process. The optimization is performed using the Harmony Search (HS) algorithm—a population-based metaheuristic known for its balance between exploration and exploitation, and for its computational efficiency in solving nonlinear problems [12]. The proposed CGT-based method is applied to the updating of a high-fidelity finite element model of a pedestrian suspension bridge, providing a relevant and demanding benchmark for testing performance. The results are compared to those obtained using conventional optimization algorithms, highlighting the benefits of the proposed approach in terms of accuracy, robustness, and efficiency.

The paper is structured as follows. Section 2 introduces the cooperative game theory model applied to single-objective optimization in the context of high-fidelity finite element model updating (FEMU). Section 3 describes the case study structure, including numerical modelling and experimental testing that define the target structural behavior. Section 4 presents the FEMU process, including sensitivity analysis and comparison between conventional and game theory-based optimization. Section 5 discusses the results, and Section 6 provides concluding remarks based on the findings.

## 2 COOPERATIVE GAME THEORY MODEL FOR HIGH FIDELITY FINITE ELEMENT MODEL UPDATING

Game theory (GT) is a mathematical framework used to model decision-making, conflict, and cooperation among multiple agents, or "players" [13]. Recent trends in optimization highlight the transformation of classical optimization problems into game-theoretic formulations [14]. Within this framework, the fundamental components include players, strategies, utility, information, and equilibrium [15]. In the context of optimization, objective functions can be interpreted as players, with their design variables acting as strategies, and their respective function values as utilities [16]. Cooperative game models are particularly suitable for complex engineering problems, where conflicting objectives need to be aligned into a compromise solution through a negotiation model or a super-criterion [17]. This study applies the cooperative game theory (CGT) approach to high-fidelity finite element model updating

(FEMU)—a process characterized by computationally intensive models and the need for precise alignment with experimental data. CGT has been successfully combined with various soft computing techniques in the literature: Dhingra and Rao [18] integrated CGT with fuzzy set theory; Xie et al. [19] developed a four-step GT-based multi-objective method; Monfared et al. [20] formulated Pareto-optimal equilibrium (POE) points via two-player games; and Cheng and Li [21] incorporated genetic algorithms into the CGT framework. Annamdas and Rao [22] proposed a modified CGT model using Particle Swarm Optimization (PSO), which is adapted in this work for single-objective optimization.

### 2.1 Single-Objective optimization using Cooperative Game Theory

To update high-fidelity FEM models without explicitly analysing the impact of weighting factors, this study uses a single-objective optimization approach based on the CGT model introduced by Annamdas and Rao [22]. In this adapted method, Harmony Search (HS) is used instead of PSO to reduce computational cost [23]. The procedure includes four main steps:

- definition of the objective function;
- minimization, maximization, and normalization of objective function residuals;
- formulation of a weighted objective function;
- optimization of the weighted objective function.

Initially, the HS algorithm is used to minimize the objective function and obtain optimal residual values -  $f_t(\theta_t^*)$ . Next, maximization via HS yields the worst-case residuals -  $f_t(\theta_t^{**})$ . These results are used to normalize the residuals:

$$f_{nt}(\theta) = \frac{f_t(\theta) - f_t(\theta_t^*)}{f_t(\theta_t^{**}) - f_t(\theta_t^*)} \quad (5)$$

The normalized values are then used to define a weighted objective function:

$$F_{w,t} = K_1 f_{n1}(\theta) + K_2 f_{n2}(\theta) + \dots + K_{k-1} f_{n(k-1)}(\theta) + (1 - K_1 - K_2 - \dots - K_{k-1}) f_{nk}(\theta) \quad (6)$$

with the constraints

$$0 \leq K_t \leq 1, \sum_{t=1}^k K_t = 1 \quad (7)$$

To ensure that residuals are as far as possible from their worst-case values, a super-criterion is introduced:

$$SC = \prod_{t=1}^k [1 - f_{nt}(\theta)] \quad (8)$$

The final optimization problem is thus defined as:

$$F_{w,t}(\Phi) = FK - SC, \quad \Phi = [\theta_1 \ \theta_2 \ \dots \ \theta_n \ K_1 \ K_2 \ \dots \ K_{k-1}]^T \quad (9)$$

Minimizing  $F_{w,t}(\Phi)$  yields the optimal set of design parameters and weighting factors, effectively enhancing the correlation between the high-fidelity FEM predictions and experimental observations.

### 3 CASE STUDY ON REAL STRUCTURE

Suspension bridges, though efficient for spanning long distances and visually striking, are prone to damage mechanisms such as corrosion and fatigue, especially in their main cables and hangers. Due to limitations in traditional local damage detection methods, global vibration-based approaches combined with high-fidelity finite element model updating (FEMU) have proven to be a promising solution for monitoring such complex structures.

#### 3.1 Description of the structure

To evaluate the proposed model updating methodology presented in the previous section, a pedestrian suspension bridge over the Drava River in Osijek (Figure 1.) was selected as the case study. Constructed in 1980, the bridge features a single span of 209.5 m, suspended by a parabolic cable anchored behind 24 m high steel pylons.

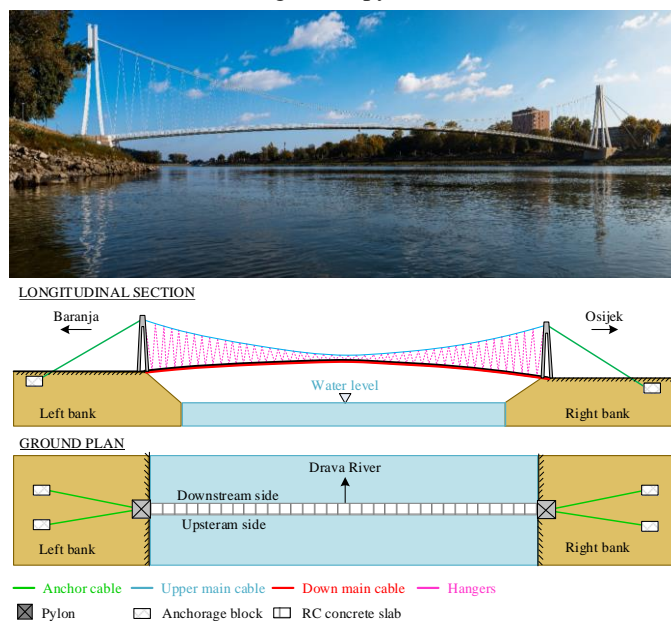


Figure 1. a) View on the bridge from the right bank b) Longitudinal section and the ground plane of the bridge

The 5 m wide pedestrian deck is composed of 50 prefabricated concrete slabs of three different types, characterized by reinforced longitudinal and transverse ribs. The slabs are supported by inclined hangers ( $\phi$  21 mm) on one side and longitudinally movable connections ( $\phi$  28 mm) on the other, allowing for limited displacement and load redistribution.

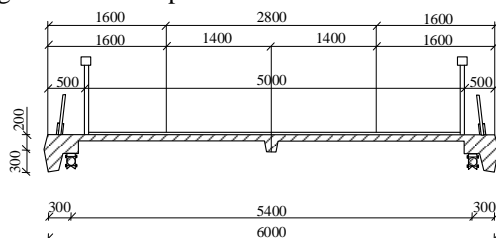


Figure 2. Cross section of the bridge (all dimensions are in millimetres)

The structural system includes two  $\phi$  61 mm pre-tensioned cables on each side, anchored at the base of the pylons to reduce deck deformations and mitigate vibrations (Figure 2.). During

the 1990s war, the bridge sustained damage to its hangers and several slabs. It was subsequently rehabilitated to its original state. In 2009, an asphalt layer was added and the slab connections repaired. Further rehabilitation work in 2022 included replacement of upper cable connections, reprofiling of slab beams, sealing of joints, corrosion protection renewal, and repair of the handrail. This real-world example, with its complex structural behavior and history of interventions, provides an ideal scenario for applying and validating advanced model updating techniques within a structural health monitoring context.

#### 3.2 Initial numerical model

An initial finite element (FE) model of the pedestrian suspension bridge was developed using ANSYS software, consisting of 20,787 elements (Figure 3.). The structural components were modelled as follows: main and transverse beams, handrails, and rigid joints with BEAM188 elements; concrete slabs with SHELL181 elements; cables and hangers with LINK180 elements; and interconnections via COMBIN14 spring-damper elements. Boundary conditions were applied to restrict translations at anchor points and pylons in all directions.

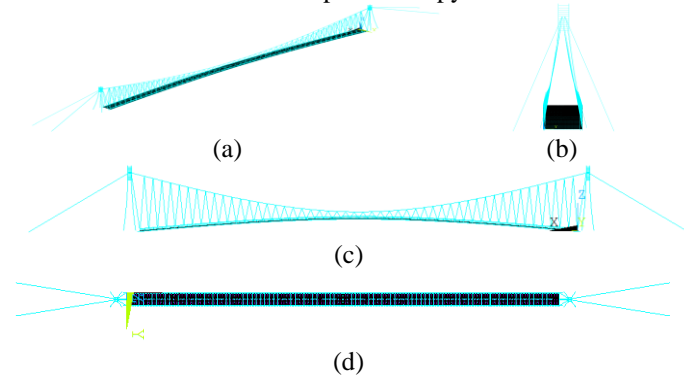


Figure 3. Initial numerical model of pedestrian suspension bridge over Drava River a) 3D view b) y-z plane c) x-z plane d) x-y plane

Material properties and cross-sectional dimensions were assigned based on project documentation, and initial tensile forces in cable elements were derived from previous experimental measurements. Hangers were grouped into four categories based on mean axial force values and standard deviations (ranging from 30.4 kN to 52.1 kN), while the upper main cables were divided into four groups with forces between 4744 kN and 4852 kN. The lower main cables were assigned a prestress force of 1300 kN. A numerical modal analysis was performed to extract natural frequencies and mode shapes (Figure 4).

$$\begin{array}{lll} \phi_1^{num} - \text{torsional} & \phi_2^{num} - \text{vertical} & \phi_3^{num} - \text{torsional} \\ f_1^{num} = 0.335 \text{ Hz} & f_2^{num} = 0.428 \text{ Hz} & f_3^{num} = 0.641 \text{ Hz} \end{array}$$

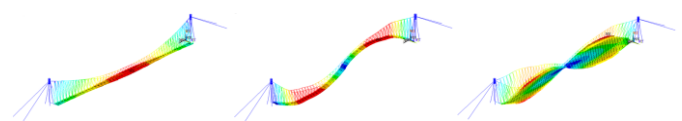


Figure 4. First three numerically determined natural frequencies ( $f_t^{num}$ ) and mode shapes ( $\phi_t^{num}$ ) of pedestrian suspension bridge over Drava River ( $t=1, \dots, 3$ )



These results served as a reference for subsequent model updating and were compared with experimentally obtained modal parameters to assess the model's initial accuracy.

### 3.3 Experimental campaign

A comprehensive experimental campaign was conducted to identify the dynamic properties of the pedestrian suspension bridge and its key structural components. The investigation included determination of axial forces in all hangers (Figure 5.) and main anchor cables, natural frequencies of the down main cables and pylons, as well as dynamic parameters of characteristic edge and span slabs.



Figure 5. Calculated force values in the hangers on the upstream and downstream side of the bridge

Global dynamic properties of the entire bridge were determined under ambient excitation from pedestrian walking. Axial forces in the hangers were determined using the resonant vibration method [24] by measuring the natural frequency of each hanger following a manual excitation. These frequencies were correlated to tensile force using string vibration theory [25]. The results showed highest force values in the mid-span hangers and noticeable deviations between upstream and downstream pairs. A similar procedure was used to determine force magnitudes in the main anchorage cables on both banks, with calculated values showing good agreement with historical measurements and design data. Dynamic testing of the down main cables and pylons was performed using impulse excitation with a rubber hammer. Natural frequencies were identified using frequency domain decomposition based on acceleration measurements in orthogonal directions. To assess local behavior, dynamic parameters of a representative edge slab and a central span slab (Figure 6.) were identified through ambient vibration testing induced by random pedestrian walking [26].

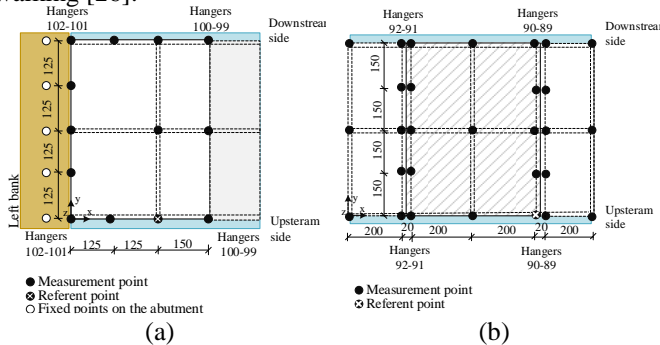


Figure 6. Arrangement of the measurement points on characteristic slab (a) edge (b) span

Acceleration responses were recorded at 13 and 29 measurement points, respectively, and modal properties were extracted using FDD. For global structural identification, vertical excitation due to pedestrian traffic was used to excite the entire structure. Acceleration was measured at 100 nodes (50 upstream and 50 downstream) in two directions, resulting in 200 degrees of freedom. Natural frequencies, mode shapes, and damping ratios (Figure 7.) were extracted using Enhanced Frequency Domain Decomposition (EFDD).

$$\phi_1^{exp} - \text{Torsional} / X; f_1^{exp} = 0.337 \pm 0.011 \text{ Hz}; \zeta_1^{exp} = 3.83 \pm 1.23$$

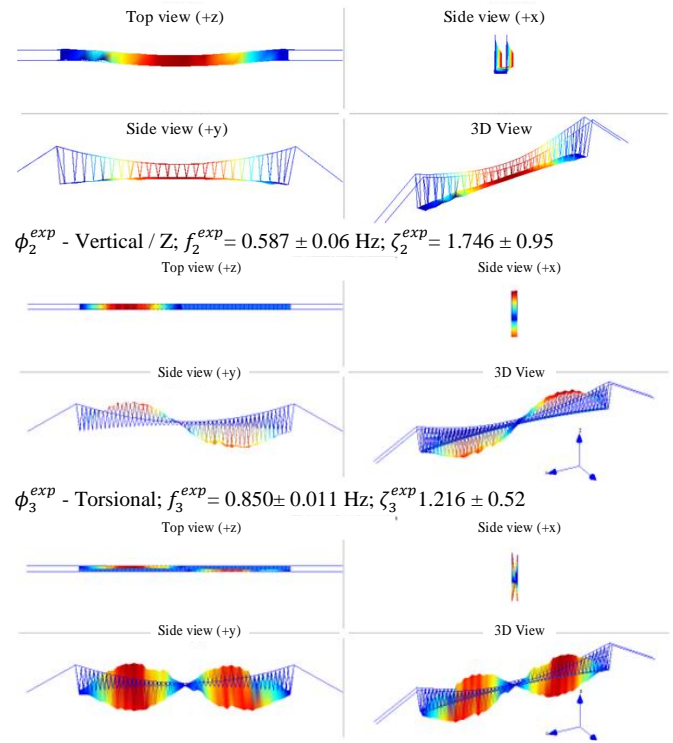


Figure 7. First three experimentally determined natural frequency ( $f_t^{exp}$ ), damping ratio ( $\zeta_t^{exp}$ ) with their standard deviation ( $\sigma_t^f, \sigma_t^z$ ) and mode shapes ( $\phi_t^{num}$ ) of pedestrian suspension bridge over Drava River ( $t=1, \dots, 3$ )

The analysis revealed distinct global mode shapes consistent with the expected behavior of a suspension bridge, providing essential input for finite element model updating (FEMU). These experimentally obtained dynamic parameters form the basis for calibrating and validating the numerical model, as described in the following chapter.

### 3.4 Comparison of Numerical and Experimental Results

A comparison between the initial numerical model and experimental results was conducted to evaluate model accuracy. Natural frequencies and mode shapes were compared using relative differences and the MAC coefficient (Table 1.). While the initial model showed acceptable agreement, some deviations indicated the need for refinement. To improve accuracy, finite element model updating (FEMU) was performed using two approaches: a conventional multi-objective (MO) optimization method and a Cooperative Game Theory (CGT)-based method.

Table 1. Comparison of the pedestrian suspension bridge modal parameters predicted by initial numerical model and its actual modal parameters based on the absolute relative difference between the natural frequency values ( $\Delta f_t$ ) and modal assurance criterion MAC ( $\phi_t^{exp}, \phi_t^{num}$ )

Mode shape t	$f_t^{num}$ [Hz]	$f_t^{exp}$ [Hz]	$ \Delta f_t $ [%]	MAC ( $\phi_t^{exp}, \phi_t^{num}$ ) [/]
1	0.335	0.337	0.597%	0.995
2	0.569	0.587	3.163%	0.967
3	0.862	0.850	1.392%	0.960
4	1.170	1.013	13.419%	0.937
5	1.142	1.150	0.701%	0.845
6	1.530	1.400	8.497%	0.870
7	1.694	1.663	1.830%	0.964
8	1.791	1.925	7.482%	0.802
9	2.061	2.188	6.162%	0.974
10	2.582	2.475	4.144%	0.967
11	2.661	2.737	2.856%	0.953
12	2.881	3.037	5.415%	0.812
13	3.197	3.313	3.628%	0.943

## 4 FINITE ELEMENT MODEL UPDATING

### 4.1 Sensitivity Analysis

To identify the most influential parameters for the model updating process, a sensitivity analysis was performed using the ratio of modal strain energy (MSE) associated with each physical parameter to the total MSE of the structure. Initially, 17 parameters were considered, but based on the analysis results (Figure 8.), 13 were selected for updating. The selected parameters include material properties (e.g., Young's modulus of concrete and handrails), connection stiffnesses, and cable pretension forces (down main cables, hangers, and upper main cables).

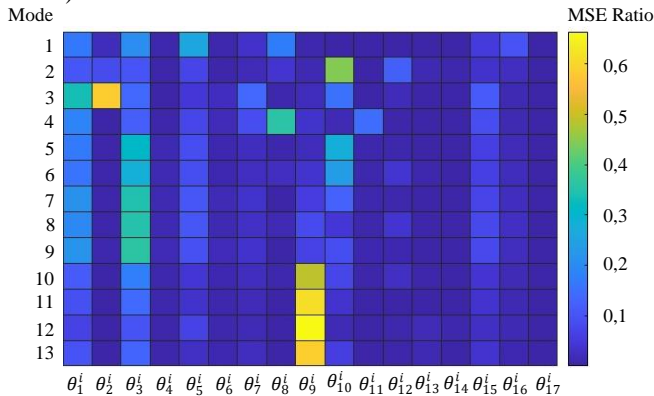


Figure 8. Results of sensitivity analysis performed on the pedestrian suspension bridge finite element model for initial selected ( $\theta_{1,...,17}$ ) 17 updating parameters

To ensure physical feasibility, each parameter was constrained within predefined lower and upper bounds. Following parameter selection, the optimization problem was structured by partitioning the residuals of natural frequencies and mode shapes. Using a sorting-based approach [14], the influence of each parameter was quantified, and two strategy spaces were defined: for natural frequency,  $S_f = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{11}\}$  and for mode shape,  $S_{ms} = \{\theta_{10}, \theta_{12}, \theta_{13}\}$ . This

selection guided the subsequent model updating process to achieve more accurate and efficient calibration of the FE model.

### 4.2 Solution of the MO FEMU problem based on the conventional optimization method

To assess the computational efficiency of the proposed CGT-based model for high-fidelity FEMU of complex structures, a benchmark analysis was conducted using a conventional multi-objective optimization method since previous research [23] has confirmed its effectiveness and accuracy. The Harmony Search (HS) algorithm was adopted for this comparison. The optimization process was implemented by coupling ANSYS for FE analysis with MATLAB for optimization. Key HS parameters were population size (PS = 50), maximum iterations ( $I_{max} = 100$ ), objective function tolerance ( $10^{-4}$ ), pitch adjustment rate (PAR = 0.3), and harmony memory consideration rate (HMCR = 0.9). The resulting Pareto front of the two objective function residuals is shown in Figure 9, highlighting the “knee point” as the most balanced solution. This optimal solution corresponds to a set of updated model parameters that improved the accuracy of the numerical model. The total computational time required to reach this solution using HS was approximately 192,783 seconds, providing a reference for evaluating the performance of the CGT approach.

### 4.3 Solution of the FEMU problem based on the CGT model

Following its proven efficiency and accuracy on a laboratory-scale bridge model, the Cooperative Game Theory (CGT) model was applied to solve the high-fidelity FEMU problem of a complex pedestrian suspension bridge. The optimization began from an initial strategy vector  $\theta_{initial\_PSBO}^0 = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$  and iterations were carried out until the convergence criterion  $\xi = 0.001$  was met. Cooperation weights were set symmetrically  $w_{11} = w_{22} = w_{12} = w_{21} = 0.5$  based on the established rules. Upon convergence, the optimal parameter set  $\theta_{CGT\_PSBO}^* = [0.9997 \ 0.8917 \ 1.0585 \ 1.0305 \ 0.8686 \ 1.2281 \ 1.7530 \ 1.0488 \ 0.8219 \ 1.0221 \ 1.0086 \ 0.9969 \ 1.0020]$  showed strong alignment with physical properties, leading to significantly improved correlation with experimental data. The CGT model completed the optimization in 89,758 seconds, demonstrating both computational efficiency and robustness in handling the multi-objective FEMU problem for a real-world, large-scale structure.

## 5 DISCUSSION

To evaluate the performance of the proposed Cooperative Game Theory (CGT) method for multi-objective finite element model updating (FEMU), a comparative analysis was conducted against a conventional Harmony Search (HS) multi-objective optimization approach. Two main criteria were considered: solution accuracy and computational time. As illustrated in Figure 9, the solution obtained using the CGT method closely matches the optimal solution (“knee point”) identified by the conventional HS method. Importantly, this level of accuracy was achieved with significantly lower computational effort. The CGT model required 89,758 seconds, compared to 192,780 seconds for the HS algorithm—demonstrating a reduction in computational time of over 50%, without compromising result quality. This efficiency is

achieved through the direct identification of the knee point using game theory principles, eliminating the need to compute the entire Pareto front, as required in conventional methods.

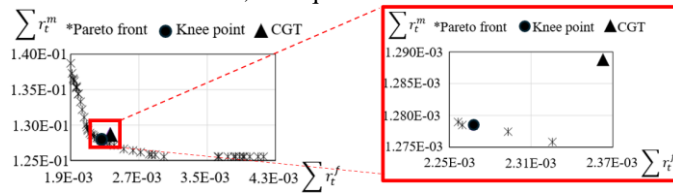


Figure 9. Comparison of the “knee” point obtained based on the Pareto front (conventional method) with the position of the optimal solution obtained using CGT model

Furthermore, Table 2 and Table 3 presents the updated natural frequencies and MAC values for both methods. The CGT approach (Table 3.) yields comparable accuracy in terms of relative frequency differences and mode shape correlation (MAC factors), confirming its robustness and suitability for high-fidelity FEMU of complex structures such as suspension bridges.

Table 2. Correlation between experimental and updated natural frequencies and mode shapes using conventional HS

Mode t	$f_t^{exp}$ [Hz]	$f_t^{upd,HS}$ [Hz]	$\Delta f_t^{HS}$ [%]	$MAC_t^{HS}$ [/]
1	0.337	0.335	-0.46	0.997
2	0.587	0.596	1.53	0.985
3	0.850	0.842	-0.94	0.984
4	1.013	1.025	1.18	0.954
5	1.150	1.142	-0.70	0.986
6	1.400	1.386	-1.01	0.982
7	1.663	1.634	-1.74	0.957
8	1.925	1.896	-1.51	0.972
9	2.188	2.215	1.23	0.996
10	2.475	2.427	-1.94	0.987
11	2.737	2.692	-1.64	0.993
12	3.037	3.054	0.56	0.964
13	3.313	3.258	-1.65	0.993

Table 3. Correlation between experimental and updated natural frequencies and mode shapes using CGT model

Mode t	$f_t^{exp}$ [Hz]	$f_t^{upd,CGT}$ [Hz]	$\Delta f_t^{CGT}$ [%]	$MAC_t^{CGT}$ [/]
1	0.337	0.334	-0.89	0.997
2	0.587	0.597	1.70	0.984
3	0.850	0.843	-0.82	0.984
4	1.013	1.025	1.18	0.954
5	1.150	1.141	-0.78	0.987
6	1.400	1.384	-1.14	0.982
7	1.663	1.635	-1.68	0.957
8	1.925	1.898	-1.40	0.971
9	2.188	2.223	1.60	0.996
10	2.475	2.425	-2.02	0.987
11	2.737	2.692	-1.64	0.993
12	3.037	3.042	0.16	0.964
13	3.313	3.269	-1.33	0.992

These results validate the CGT method as a computationally efficient and accurate alternative to traditional optimization

approaches for model updating in structural engineering applications.

## 6 CONCLUSION

This research presents a novel and efficient framework for high-fidelity finite element model updating (FEMU) by leveraging the principles of Cooperative Game Theory (CGT). The proposed approach was applied to a real-world pedestrian suspension bridge, providing a rigorous testbed to evaluate the effectiveness of the method in handling the complexity and precision demands of high-fidelity finite element analysis. Key contributions and findings include:

- By formulating FEMU as a cooperative game, the method enables a targeted and efficient resolution of conflicting objectives, such as matching both natural frequencies and mode shapes, without the need to compute the entire Pareto front. This aspect is especially beneficial in complex, high-fidelity models with many interdependent parameters.
- Compared to the conventional Harmony Search (HS) method, the CGT approach achieved equivalent or better accuracy with a reduction in computational time of over 50%. This demonstrates that high-fidelity analysis does not necessarily come at the cost of efficiency when advanced optimization strategies are applied.
- The CGT-based method improved the correlation between the numerical and experimental modal parameters, confirming its suitability for high-fidelity finite element analysis where accuracy and detail are critical. The updated model captured the structural behavior of the bridge with remarkable precision, addressing discrepancies in natural frequencies and mode shapes.
- The method was validated using extensive experimental data from a real suspension bridge, including local and global dynamic characteristics. The updated model reflects the true structural behavior with a high level of fidelity, even in the presence of structural uncertainties and historical modifications.

In summary, the CGT-based FEMU framework proves to be a robust, accurate, and computationally efficient solution tailored to the needs of high-fidelity finite element analysis. Its adaptability and performance make it a promising tool for advancing structural health monitoring in complex and intelligent infrastructure systems.

## ACKNOWLEDGMENTS

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No [101151734].

## REFERENCES

- [1] J. Mottershead and M. Friswell, *Finite Element Model Updating in Structural Dynamics*, 1995.
- [2] M. I. Friswell and J. E. Mottershead, *Finite Element Model Updating in Structural Dynamics*, Springer, Dordrecht, Netherlands, 1995.
- [3] M. Baruch, “Optimization procedure to correct stiffness and flexibility matrices using vibration tests,” *AIAA Journal*, vol. 16, pp. 1208–1210, 1978. <https://doi.org/10.2514/3.61032>.
- [4] H. Jensen and C. Papadimitriou, “Bayesian finite element model updating,” 2019. [https://doi.org/10.1007/978-3-030-12819-7\\_7](https://doi.org/10.1007/978-3-030-12819-7_7).
- [5] W.-M. Li and J.-Z. Hong, “Research on the iterative method for model updating based on the frequency response function,” *Acta Mechanica*

- Sinica*, vol. 28, pp. 450–457, 2012. <https://doi.org/10.1007/s10409-012-0063-1>.
- [6] T. Marwala, *Finite-Element-Model Updating Using Computational Intelligence Techniques*, Springer London, London, 2010. <https://doi.org/10.1007/978-1-84996-323-7>.
- [7] S. Ereiz, I. Duvnjak, and J. F. Jiménez-Alonso, “Review of finite element model updating methods for structural applications,” *Structures*, vol. 41, pp. 684–723, 2022. <https://doi.org/10.1016/j.istruc.2022.05.041>.
- [8] D. S. Jung and C. Y. Kim, “Finite element model updating on small-scale bridge model using the hybrid genetic algorithm,” *Structure and Infrastructure Engineering*, vol. 9, pp. 481–495, 2013. <https://doi.org/10.1080/15732479.2011.564635>.
- [9] J. W. Park, S. H. Sim, and H. J. Jung, “Displacement estimation using multimetric data fusion,” *IEEE/ASME Transactions on Mechatronics*, vol. 18, pp. 1675–1682, 2013. <https://doi.org/10.1109/TMECH.2013.2275187>.
- [10] S. Kim, N. Kim, Y.-S. Park, and S.-S. Jin, “A sequential framework for improving identifiability of FE model updating using static and dynamic data,” *Sensors*, vol. 19, p. 5099, 2019. <https://doi.org/10.3390/s19235099>.
- [11] J. Naranjo-Pérez, J. F. Jiménez-Alonso, A. Pavić, and A. Sáez, “Finite-element-model updating of civil engineering structures using a hybrid UKF-HS algorithm,” *Structure and Infrastructure Engineering*, vol. 17, pp. 620–637, 2021. <https://doi.org/10.1080/15732479.2020.1760317>.
- [12] D. Manjarres, I. Landa-Torres, S. Gil-Lopez, J. Del Ser, M. N. Bilbao, S. Salcedo-Sanz, and Z. W. Geem, “A survey on applications of the harmony search algorithm,” *Engineering Applications of Artificial Intelligence*, vol. 26, pp. 1818–1831, 2013. <https://doi.org/10.1016/j.engappai.2013.05.008>.
- [13] J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, 5th ed., Princeton University Press, Princeton, 1953.
- [14] S. Ereiz, J. F. Jiménez-Alonso, I. Duvnjak, and A. Pavić, “Game theory-based maximum likelihood method for finite-element-model updating of civil engineering structures,” *Engineering Structures*, vol. 277, 115458, 2023. <https://doi.org/10.1016/j.engstruct.2022.115458>.
- [15] S. Ereiz, I. Duvnjak, and J. F. Jiménez-Alonso, “Structural finite element model updating optimization based on game theory,” *Materials Today: Proceedings*, 2022. <https://doi.org/10.1016/j.matpr.2022.04.401>.
- [16] M. Jin, X. Lei, and J. Du, “Evolutionary game theory in multi-objective optimization problem,” *International Journal of Computational Intelligence Systems*, vol. 3, pp. 74–87, 2010. <https://doi.org/10.1080/18756891.2010.9727754>.
- [17] S. Özyildirim and N. M. Alemdar, “Learning the optimum as a Nash equilibrium,” *Journal of Economic Dynamics and Control*, vol. 24, pp. 483–499, 2000. [https://doi.org/10.1016/S0165-1889\(99\)00012-3](https://doi.org/10.1016/S0165-1889(99)00012-3).
- [18] A. K. Dhingra and S. S. Rao, “A cooperative fuzzy game theoretic approach to multiple objective design optimization,” *European Journal of Operational Research*, vol. 83, pp. 547–567, 1995. [https://doi.org/10.1016/0377-2217\(93\)E0324-Q](https://doi.org/10.1016/0377-2217(93)E0324-Q).
- [19] N. Xie, N. Shi, J. Bao, and H. Fang, “Analysis and application of multi-object decision design based on game theory,” *6th World Congress on Structural and Multidisciplinary Optimization*, 2005.
- [20] M. S. Monfared, S. E. Monabbati, and M. Mahdipour Azar, “Bi-objective optimization problems with two decision makers: refining Pareto-optimal front for equilibrium solution,” *OR Spectrum*, vol. 42, pp. 567–584, 2020. <https://doi.org/10.1007/s00291-020-00587-9>.
- [21] F. Y. Cheng and D. Li, “Genetic algorithm and game theory for multiobjective optimization of seismic structures with/without control,” in *11th World Conference on Earthquake Engineering*, Pergamon, Oxford, England, 1996, pp. 1–8.
- [22] K. K. Annamdas and S. S. Rao, “Multi-objective optimization of engineering systems using game theory and particle swarm optimization,” *Engineering Optimization*, vol. 41, pp. 737–752, 2009. <https://doi.org/10.1080/03052150902822141>.
- [23] J. F. Jiménez-Alonso, J. Naranjo-Perez, A. Pavić, and A. Sáez, “Maximum likelihood finite-element model updating of civil engineering structures using nature-inspired computational algorithms,” *Structural Engineering International*, pp. 1–13, 2020. <https://doi.org/10.1080/10168664.2020.1768812>.
- [24] M.-H. Nguyen, T.-D.-N. Truong, T.-C. Le, and D.-D. Ho, “Identification of tension force in cable structures using vibration-based and impedance-based methods in parallel,” *Buildings*, vol. 13, p. 2079, 2023. <https://doi.org/10.3390/buildings13082079>.
- [25] G. Nugroho, H. Priyosulistyo, and B. Suhendro, “Evaluation of tension force using vibration technique related to string and beam theory to ratio of moment of inertia to span,” *Procedia Engineering*, vol. 95, pp. 225–231, 2014. <https://doi.org/10.1016/j.proeng.2014.12.182>.
- [26] J. S. Jensen, D. M. Frangopol, and J. W. Schmidt, *Bridge Maintenance, Safety, Management, Digitalization and Sustainability*, CRC Press, London, 2024. <https://doi.org/10.1201/9781003483755>.