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Indirect footbridge damage classification using explainable deep learning: A field-testing study

Zhenkun Li[1, ORCID (0000-0002-1444-6017)], Yifu Lan[1,2, ORCID (0000-0002-8411-6507)], Weiwei Lin[1, ORCID (0000-0001-8486-6538)]
[1]Department of Civil Engineering, Aalto University, 02150 Espoo, Finland
[2]School of Civil Engineering, University College Dublin, D04V1W8 Dublin, Ireland
email: zhenkun.li@aalto.fi, yifu.lan@ucd.ie, weiwei.lin@aalto.fi

ABSTRACT: Structural health monitoring (SHM) has gained significant attention in recent decades due to several structural failures and the increasing maintenance demands from stakeholders. This urgency has been further amplified by the impact of predictive climate changes worldwide. Footbridges, as critical components of modern transportation systems, play a vital role in daily life and therefore require meticulous attention to their health conditions. Traditionally, monitoring footbridge conditions involves installing many sensors directly on the structure, which is often cost-prohibitive in engineering applications. Recent advancements have highlighted the indirect method of bridge health monitoring, where sensors are mounted on passing vehicles rather than the bridge itself. This approach is not only more economical but also easier to implement in practical engineering scenarios. This paper further extends the indirect monitoring method to classify footbridge damage using the responses of shared scooters. Advanced deep learning techniques are utilized to predict the severity of damage to the footbridge based on the vibrations recorded from shared vehicles. The proposed method was validated through field tests involving scooters and a footbridge. Furthermore, to interpret the outputs of the deep learning model, SHapley Additive exPlanations (SHAP) values were calculated, offering insights into the decision-making process of the model.

KEY WORDS: structural health monitoring; footbridge; damage classification; convolutional neural networks; SHAP.

1 INTRODUCTION

The assessment of bridge health has gained significant attention over the past decades due to widespread aging and deterioration. This concern stems from the fact that many of these structures were built in the previous century, with a large proportion having been in service for over 50 years. For instance, in Finland, the Finnish Transport Infrastructure Agency reported that, as of 2023, 882 out of 17,351 highway bridges (5.1%) were in poor condition, with aging structures from the 1960s and 1970s accumulating a growing maintenance backlog [1]. The European Commission has noted that bridges constructed after 1945 were typically designed for a lifespan of 50 to 100 years. In 2001, it was reported that bridges in France, Germany, and the UK showed deficiency rates of 39%, 30%, and 37%, respectively [2]. These figures highlight the urgent need for effective health monitoring of in-service bridges, which can provide critical information on their condition and support informed decision-making by stakeholders.

Traditional bridge inspections rely heavily on human vision, requiring engineers to conduct on-site visits and determine whether maintenance is needed [3]. However, as modern bridge construction becomes more extensive and complex, this approach faces several limitations, including being labor-intensive, inefficient, and time-consuming. At the beginning of this century, structural health monitoring (SHM) systems gained popularity [4]. These systems involve installing various sensors on bridges to continuously collect different types of data. In practice, however, this approach has proven to be expensive. It typically involves a one-to-one setup, where the monitoring system is customized for a specific bridge and cannot be easily transferred to others. Moreover, the cost of installing numerous sensors can be high. As the number of aging and newly built bridges continues to rise, there is an increasing need for cost-effective and scalable monitoring technologies.

In 2004, Yang et al. [5] proposed the indirect method, where sensors are installed on passing vehicles instead of the bridge itself. This approach is based on the vehicle-bridge interaction (VBI) process. During this interaction, the dynamic characteristics of the bridge are transferred to the vehicles equipped with sensors, allowing the vehicles to act as moving sensors that collect information about the bridge. In this pioneering study, the bridge was simplified as a simply supported beam, and the vehicle was modeled using a spring-mass system. Under these assumptions, the authors demonstrated that the fundamental frequency of the bridge could be extracted from the vehicle's response, laying the groundwork for future research in this area.

In recent studies, researchers have further investigated the extraction of bridge modal shapes and damping ratios from vehicle response [6,7]. For example, Yang et al. [8] proposed using the Hilbert Transform to extract mode shapes from filtered vehicle responses, while González et al. [9] introduced a method for retrieving damping ratios by minimizing the errors in identified road roughness between the front and rear axles. In 2018, Yang et al. [10] introduced the concept of contact-point (CP) response, which represents the response at the interface between the vehicle and the bridge. This response was found to be independent of vehicle characteristics, making it useful for identifying bridge properties from vehicle data [11]. In addition to vehicle influence, road roughness is another major source of interference when identifying bridge dynamic parameters from vehicle accelerations. This issue can be mitigated by using residual CP responses between vehicle axles,

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which eliminates the effects of road roughness [12–14]. However, in practical applications, modal parameters often show limited sensitivity to structural damage and may be significantly affected by operational conditions. Moreover, most existing studies on the indirect method have focused on road bridges. Although footbridges play an essential role in modern transportation and logistics, they are rarely equipped with SHM systems and have received little attention in indirect monitoring research.

Over the past decade, advancements in computer science algorithms and hardware have significantly enhanced deep learning techniques, particularly through the widespread use of neural networks. Technologies such as large language models (LLMs) have transformed many aspects of daily life and have also made a notable impact on SHM for bridges. Researchers have increasingly applied machine learning and deep learning methods to assess bridge health using structural responses [15]. In the context of the indirect method, vehicle responses typically consist of three components: vehicle dynamics, road roughness, and bridge vibrations [16,17]. This complexity makes it challenging to isolate bridge-specific information for use as damage indicators. However, deep learning models are sensitive to subtle signal variations, making them well-suited for detecting damage-related changes in vehicle responses. For example, Li et al. [18] applied support vector machines and Mel-frequency cepstral coefficients (MFCCs) to predict the severity of bridge damage based on vehicle responses. Unlike earlier studies that focused only on low-frequency signals [19], this research also explored high-frequency responses. In addition, Corbally and Malekjafarian [20] used convolutional neural networks (CNNs) to classify the type, location, and severity of bridge damage using drive-by data. In their study, particle swarm optimization was employed to fine-tune the vehicle model, and the resulting simulated data were used to train the CNN. Laboratory experiments validated the framework, demonstrating its ability to accurately detect and classify damage in most cases. These studies highlight the promising role of deep learning in bridge health monitoring. However, most research to date has focused on road bridges, with footbridges largely overlooked [21]. Additionally, deep learning models are often applied without adequate explanation of how features are selected or interpreted, limiting the transparency and broader adoption of these approaches.

In this paper, an explainable deep learning-based method is proposed to detect and classify damage in footbridges. Shared scooters equipped with smartphones are used to assess the health condition of footbridges when they pass the footbridge structures and collect dynamic data. 2D CNN is employed to extract key features from the time-frequency representations (TFRs) of the scooter's response as it moves over the footbridge. A field test is conducted to validate the effectiveness of the proposed method. The structure of the paper is as follows: Section 2 introduces the fundamental scheme of the proposed method and architecture of the used 2D CNNs. Section 3 describes the field test setup and discusses the results. Finally, Section 4 presents the conclusions of the study.

2 PROPOSED METHODOLOGY

2.1 Data collection

In this study, smartphones were mounted on a scooter to record vibration data. The scooter first crossed the bridge multiple times when it was in a healthy state to establish a baseline. After the bridge had been in use for several months or years and potential damage had developed, the scooter was used again to collect vibration data. These recordings, from both healthy and possibly damaged states, were used to train neural networks for predicting the bridge's health condition.

2.2 Data processing

Before feeding scooter data into the 2D CNNs, signals are preprocessed in 3 steps: (1) synchronization: two smartphones on different scooter parts are synchronized using Unix time to align data collection; (2) channel formation: only vertical and pitch accelerations, the most relevant signals, are retained. Signals from misaligned sensors are combined to extract these components; (3) segmentation: only data collected while the scooter is on the footbridge is kept, removing unrelated signals before and after crossing. This ensures clean, relevant input for 2D CNN training.

2.3 2D CNN

CNNs are widely used for extracting damage-sensitive features from signals. In this study, the 2D CNNs based on a simplified Visual Geometry Group (VGG)-16 [22] architecture (with two instead of four fully connected layers) were developed to analyze scooter vibrations for footbridge monitoring. When sensors are mounted on a scooter, fewer measurement points are available, but key inputs: vertical body acceleration ($\ddot{z}_s$), angular acceleration ($\ddot{\theta}_s$), and front wheel acceleration ($\ddot{z}_t$), can be collected using two smartphones. The 2D CNN uses TFRs of scooter vibrations as input. To standardize input size, signals are first truncated to a uniform time length (5 s in this study), then transformed into 2D representations using methods like short-time Fourier Transform [23]. Each CNN channel uses 2D kernels (kernel_size=3) with zero padding to keep the input size. The max-pooling (kernel_size = 2, stride = 2) is utilized to extract key features. Activation function was selected as rectified linear unit (ReLU), and the Cross-Entropy (CE) loss was employed [24]. The architecture of the 2D CNNs are shown in Table 1.

Table 1. Architecture of the 2D CNNs.

Layers	Output shape	Kernel size	Activation
Input	$3 \times 402 \times 257$	-	-
Conv 2D	$64 \times 402 \times 257$	3	ReLU
Conv 2D+MaxPooling	$64 \times 201 \times 128$	3	ReLU
Conv 2D	$128 \times 201 \times 128$	3	ReLU
Conv 2D+MaxPooling	$128 \times 100 \times 64$	3	ReLU
Conv 2D	$256 \times 100 \times 64$	3	ReLU
Conv 2D	$256 \times 100 \times 64$	3	ReLU
Conv 2D+MaxPooling	$256 \times 50 \times 32$	3	ReLU
Conv 2D	$512 \times 50 \times 32$	3	ReLU
Conv 2D	$512 \times 50 \times 32$	3	ReLU
Conv 2D+MaxPooling	$512 \times 25 \times 16$	3	ReLU
Conv 2D	$512 \times 25 \times 16$	3	ReLU
Conv 2D	$512 \times 25 \times 16$	3	ReLU
Conv 2D+MaxPooling	$512 \times 12 \times 8$	3	ReLU
Flattened	49152	-	-
Fully connected	4	-	-

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3 FIELD TESTS AND DISCUSSIONS

3.1 Field test setups

Field tests were carried out using scooters equipped with two smartphones to validate the proposed footbridge damage detection method (see Figures 1 and 2). Smartphone 1 (iPhone 12) was mounted on the scooter body, and Smartphone 2 (iPhone 8) on the front wheel. The scooter passed the footbridge multiple times under similar road roughness. A short acceleration zone was used for the scooter to reach a top speed of 20 km/h, which can be powered by an electric motor without human pedaling force. MATLAB Mobile was used on smartphones to collect data with sampling frequency of 100 Hz. The footbridge investigated in the field tests is shown in Figure 3. To simulate experimental damage cases (EDCs), different masses (people standing at the center) were used, which is a practical approach and has been validated in prior studies [25–27]. The added masses were 55 kg, 125 kg, and 185 kg for damage cases 1, 2, and 3, respectively. EDC 0 refers to the undamaged case with no added mass.

Figure 1. Scooter with two smartphones.

Figure 2. Scooter with a rider.

Figure 3. Experimental damage cases.

All EDCs and corresponding scooter runs are detailed in Table 2. Impulse excitation was applied by having a person jump on the bridge to assess the effect of added mass. A smartphone placed at the 1/4 span of the footbridge recorded the resulting vibrations. For EDC 0, accurate natural frequencies cannot be obtained with this method, as at least one person is required to apply the excitation. Instead, an alternative approach using the bridge's free vibration after scooter crossing is utilized. Frequency values for all EDCs are listed in Table 2, where $f_{b1} - f_{b4}$ represent the first four frequencies of the explored footbridge. It can be seen that when more people are standing on the footbridge, the first two frequencies of the footbridge f_{b1} and f_{b2} decrease apparently. However, for the third and fourth frequencies f_{b3} and f_{b4}, the frequency values sometimes remain unchanged due to the measuring accuracy. Even though the frequency changes can be observed from the first two frequencies, the change ratio can be minor, say 1.22% for the fundamental frequency in EDC 1 compared to that of EDC 0. It can be challenging to determine the damage condition only based on the changes in frequencies. Therefore, the following will investigate the use of data-driven methods for damage detection of the footbridge using scooter vibrations.

Table 2. Footbridge frequencies in all EDCs.

EDCs	People mass	f_{b1}	f_{b2}	f_{b3}	f_{b4}	Runs
EDC 0	0 kg	4.028 Hz	4.468 Hz	10.486 Hz	11.316 Hz	124
EDC 1	55 kg	3.979 Hz	4.443 Hz	10.486 Hz	11.304 Hz	65
EDC 2	125 kg	3.955 Hz	4.431 Hz	10.486 Hz	11.304 Hz	63
EDC 3	185 kg	3.918 Hz	4.370 Hz	10.437 Hz	11.304 Hz	60

3.2 Results and discussions

For analysis, 63 runs were randomly selected from EDC 0, yielding a total of 251 runs in the experimental dataset. Of these, 70% were used for training and 30% for testing. The CNN configurations matched those used in simulations. Hyperparameters were set as follows: batch size = 32, optimizer = Adam, learning rate = $1e^{-6}$, weight decay = $1e^{-5}$, loss function = CE loss, activation = ReLU, and number of epochs = 400. CE loss and damage prediction accuracy are shown in Figure 4.

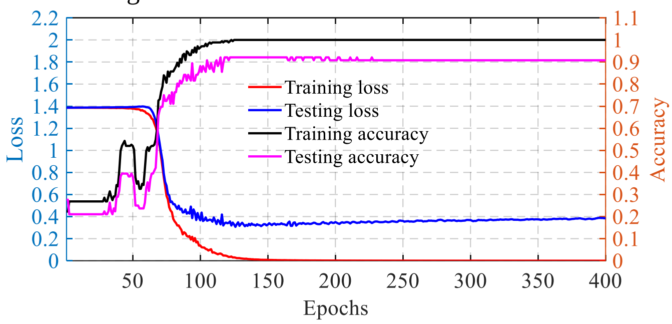

Figure 4. Loss and accuracy using 2D CNNs.

Figure 4 showed that the 2D CNN achieved early and sharp drops in both training and testing losses. The training loss nearly reached zero, and despite a minor rise in testing loss after 150 epochs, testing accuracy remained consistently above 90%.

To further interpret these findings, Shapley Additive Explanations (SHAP) were used to explain the 2D CNN's predictions. SHAP values reveal each feature's contribution to the model's output [28,29]. One sample from each EDC was analyzed to show how the 2D CNN classified bridge conditions. Figure 5 displays the SHAP values and predicted probabilities

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for each TFR image. Red pixels (positive SHAP values) support the model's prediction, blue pixels (negative SHAP values) oppose it, and grey pixels have little impact.

The SHAP value images reveal that most significant features, those with strong positive or negative contributions, are concentrated in the 0–30 Hz range. This aligns with the fact that the bridge's natural frequencies, identified through impulse excitation (Table 2), also lie below 30 Hz. In EDC 0, for instance, image 2 shows that key features appear between 0.5–3.5 s as distinct non-horizontal lines and points, corresponding to peaks in the footbridge's frequency response. The 2D CNN confidently classified this sample as EDC 0 with a 99.98% probability. Images 4 and 5 contain blue regions indicating features that helped the model rule out EDCs 2 and 3. However, image 3 shows some overlap with EDC 1, resulting in a small 0.01% probability being assigned to that class. In EDC 1 (image 8), time-varying features near 1.5 s played a critical role, especially in distinguishing it from EDC 2. Because of the resemblance to EDC 0 around the same time, the model assigned a minor 0.08% probability to EDC 0. For EDC 2, distinct features were noted at 2.5 s and 4 s, while in EDC 3 (image 20), key contributions appeared around 4 Hz and 20 Hz. Although EDC 3 shared similar patterns with EDC 0 between 1.5–2.5 s, additional higher-frequency features around 1.5 s enabled the 2D CNN to correctly identify it as EDC 3.

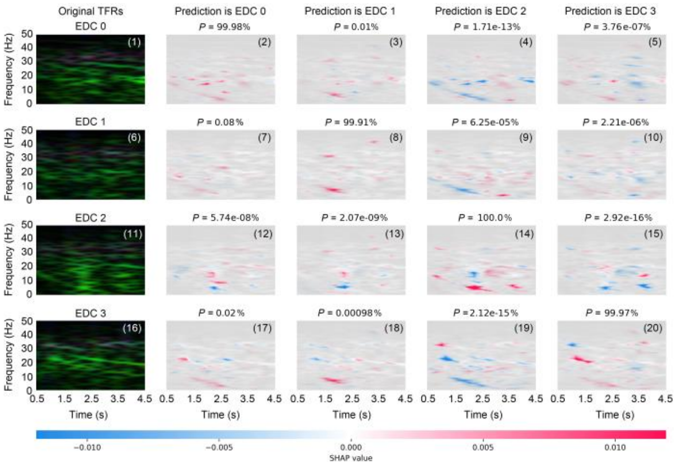

Figure 5. Explanation of 2D CNNs (*P*: Probability).

These observations confirm that the 2D CNN identifies damage-sensitive features based on both time and frequency information. Therefore, using TFRs as input to a 2D CNN provides good damage detection performance in the indirect method with vehicle-mounted sensors. This advantage stems from the inherently non-stationary dynamics of VBI systems, where both vehicle and bridge frequencies shift during interaction [30–32]. Therefore, by preserving time-varying characteristics, the 2D CNN in this study can effectively identify key features in scooter vibrations.

4 CONCLUSIONS AND FUTURE WORK

This paper proposes a method for detecting and classifying footbridge damage by analyzing scooter vibrations collected via smartphones and processed through explainable deep learning. Specifically, TFRs of scooter vibrations were used with a 2D CNN to assess damage severity. The method was validated through real-world field tests. It was found that the 2D CNN can accurately predict the damage severity of the footbridge by using the TFRs of scooters. The 2D CNN's superior performance is linked to its ability to capture the non-stationary characteristics of VBI responses. SHAP analysis confirmed that damage-sensitive features vary over time in the scooter's vibrations.

Future work will focus on enhancing the practicality and robustness of the proposed method by exploring alternative smartphone placements (e.g., on the standing slab or handlebar), considering the behaviors of the drivers, incorporating influential factors such as temperature, road roughness variations, and pedestrian presence, and reducing reliance on labeled data through unsupervised learning. Furthermore, the authors understand that using standing people on the footbridge to simulate synthetic damage scenarios can not fully represent the real damage in practical engineering. In our future studies, we would like to test the proposed method on other bridges with real damage to evaluate the generalization.

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