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Nonparametric identification of structural nonlinear behavior based on extended Kalman particle filter and Chebyshev polynomial model

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ABSTRACT: Describing the damage initiation and development of engineering structures during strong dynamic loadings such as earthquake is one of the most important topics in structural condition monitoring and identification. Structural nonlinear restoring force (NRF) can not only directly describe the initiation and development process of nonlinear behavior of the structure during strong dynamic loadings but also can be used to evaluate the energy dissipation of structural members or substructures. However, it is hard to measure structural dynamic responses at all degree of freedoms (DOFs) of a structure in practice, and to model the NRF with an accurate parametric mathematical model in advance due to the variability and individuality of structural materials and types. In this study, a Chebyshev polynomial model as a nonparametric model is employed to model the NRF of a structure and structural stiffness, damping, mass and NRF are identified based on the extended Kalman particle filter (EKPF) algorithm by using acceleration measurements at limited DOFs during the known external excitation. Then, two multi-degree-of-freedom (MDOF) numerical models equipped with different types of magnetorheological (MR) dampers are used as numerical examples to validate the performance of the proposed approach. Identified results show that the proposed method is effective for identifying the nonlinear MDOF structures with different nonlinearity with limited noise-polluted acceleration measurements.

KEY WORDS: Nonlinear restoring force; Extended Kalman particle filter; nonparametric identification; Chebyshev polynomial; MR damper.

1 INTRODUCTION

During the service of engineering structures, when subjected to severe loads, the structural characteristics may change abruptly or gradually, resulting in stiffness deterioration and increased damping. The problem of precisely detecting parameter changes has piqued civil engineering researchers' interest. Understanding changing structural parameters is crucial for designing, maintaining, and reinforcing structures, as well as selecting post-disaster rescue routes. When civil engineering structures are subjected to extraordinarily significant external excitations, such as earthquakes on buildings or heavy cars on bridges, they frequently exhibit nonlinear behavior.

Identification of structural parameters and nonlinear restoring force (NRF) of nonlinear structural systems using partial acceleration measurements from structural health monitoring (SHM) is crucial for structural condition assessment and damage identification[1, 2]. In the past decades, many researchers have developed many parameters and NRF identification methods of nonlinear system. It is difficult to measure the acceleration responses of all degrees of freedom in practical engineering, some methods based on Kalman filter (KF)[3, 4], extended Kalman filter (EKF)[5, 6], unscented Kalman filter (UKF) [7-9], and particle filter (PF) [10, 11] were proposed to tackle the problem. However, conventional methods are only suitable for nonlinear hysteresis model parameters that are known. Due to the diversity and individuality of nonlinear behaviors, it is crucial to propose a general nonparametric identification method for nonlinear behaviors that does not rely on nonlinear hysteresis models.

The idea of nonparametric identification of nonlinear behavior was first proposed by Masri and his collaborators[12, 13]. Based on the equivalent linear theory and least squares method, identified methods of the structural nonlinear restoring force were proposed by Xu et al. [14, 15] using external excitation and complete dynamic response information and verified the feasibility of the proposed method through dynamic test data of a multi-degree-of-freedom shear frame model equipped with a magnetorheological (MR) damper. Xu and his cooperators proposed nonparametric identification method of the NRF in the presence of the known or unknown input, where the NRF was expressed using different polynomial models[16-19]. Some researchers regarded the nonlinear restoring force as an unknown virtual input, proposing different NRF identification methods[20-22]. The effectiveness of the proposed method was verified by numerical simulation and experiment. However, as far as the author knows, there is no extended Kalman particle filter (EKPF) method that is suitable for nonparametric identification of structural nonlinear behavior under non-Gaussian measurement noise without the need of the known parametric model of nonlinear behavior.

Since the standard PF algorithm takes the transition probability of the system state as the importance density function, it does not use the updated observations. Therefore, the generated particle samples are concentrated at the tail of the posterior probability distribution, resulting in a large randomness in the selection of particles, which affects the filtering results. When there is a peak in the likelihood distribution, the prediction state is distributed at the tail of the likelihood distribution, which has a particularly serious impact on the filtering accuracy. The EKPF uses EKF as the posterior probability density function, which solves the problem of particle degradation in PF algorithm and improves the filtering

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accuracy. For the EKF part of EKPF, when Kalman filter is applied in practice, model error, noise error and calculation error may cause the prediction error covariance matrix and gain matrix to weaken the modified state estimation with the increase of iteration times, which leads to filter divergence. Therefore, the fading memory filtering (MF) technology can be used for EKF to increase the proportion of new data, reduced the proportion of old data and the negative impact of old data on filtering [16].

In this paper, a model-free identification method for structural parameters and nonlinear restoring force under limited acceleration observation is proposed by using EKPF algorithm. Based on the equivalent linear theory and EKPF algorithm, the structural parameters, unknown dynamic response measurements and nonlinear locations are identified under limited acceleration observations. Based on the identification value, the Chebyshev polynomial model is used to characterize the nonlinear restoring force of the structure, and the nonparametric identification of the NRF is realized. To verify the feasibility of the proposed method, two four-degree-of-freedom shear frame models are established, and MR dampers with different numbers and different parameter models are introduced to simulate the nonlinear behavior of shear frame structures. Considering the influence of measurement noise in the observed acceleration signal, the structural stiffness, damping coefficient, mass, unknown dynamic response and NRF are identified. The feasibility of the proposed method is verified by comparing the identification results with the real values.

2 EXTENDED KALMAN PARTICLE FILTERING ALGORITHM

2.1 The state-space equation

In general, the nonlinear dynamic system of structures in civil engineering can be described as,

$$\begin{cases} \boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}) + \boldsymbol{r}_{k-1} \\ \boldsymbol{y}_k = h(\boldsymbol{x}_k) + \boldsymbol{v}_k \end{cases} \tag{1}$$

where, the function $f(\cdot)$ and $h(\cdot)$ represent the state transition function and the measurement model function of the system respectively. k is the number of time steps, $\boldsymbol{x}_k$ is the state value of step k, $\boldsymbol{y}_k$ is the observation value of step k, $\boldsymbol{r}_k$ is the process noise of step k, $\boldsymbol{v}_k$ is the observation noise of step k. Equation (1) describes the recursive relationship between the structural state vector and the structural response over time.

2.2 Bayesian theorem and Monte Carlo simulation

For the state space equation assumed by the formula (1), let $\boldsymbol{x}_{0:k} = \{x_0, x_1, ..., x_k\}$, $\boldsymbol{y}_{1:k} = \{y_1, y_2, ..., y_k\}$, and given $\boldsymbol{x}_{0:k}$, when the measurement sequence $\boldsymbol{y}_k$ is independent of each other, the prediction and update can be written recursively by the Bayesian formula,

$$p(x_k \mid y_{1:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid y_{k-1}) dx_{k-1} \tag{2}$$

$$p(x_k \mid y_{1:k}) = \frac{p(y_k \mid x_k) p(x_k \mid y_{1:k-1})}{\int p(y_k \mid x_k) p(x_k \mid y_{1:k-1}) dx_k} \tag{3}$$

For nonlinear models, the above analytical formulas are often unable to be obtained, and it is also very difficult to solve them integrally. Therefore, the Monte Carlo simulation is considered to realize the recursion of Bayesian filter. The Monte Carlo simulation regards the problem to be solved as a random variable. By establishing a probability model and sampling a large number of samples, the integral value is regarded as the mathematical expectation of the random variable, and then the problem to be solved is estimated. That is to say, for the integrand $f(x)$, it can be decomposed into the product of the state variable $g(x)$ and its probability density function $p(x)$, then the integral of $f(x)$ can be regarded as the mathematical expectation of $g(x)$.

2.3 The sequential importance sampling

According to the Monte Carlo simulation, if we can sample from the posterior probability density function $p(x_{0:k} \mid y_{1:k})$ and get the sample set $\left\{x_{0:k}^{(i)}, \omega_k^{(i)}\right\}_{i=1}^{N}$, then $p(x_{0:k} \mid y_{1:k})$ can be approximated by the sum of discrete samples, that is, the approximate solution formula of $g(x_{0:k})$ can be written as,

$$E\left(g(x_{0:k})\right) = \sum_{i=1}^{N} \omega_k^i g\left(x_{0:k}^i\right) \tag{4}$$

However, in practice, it is very difficult to extract samples from the posterior probability distribution. Therefore, the importance sampling method is introduced to extract samples from the importance density function $q(\mathrm{x}_k^{(i)} | \mathrm{x}_{k-1}^{(i)}, y_k)$. Then the mathematical expectation of $g(x)$ can be written as,

$$\begin{aligned} E[g(x_{0:k})] &= \int g(x_{0:k}) \frac{p(x_{0:k} \mid y_{1:k})}{q(x_{0:k} \mid y_{1:k})} q(x_{0:k} \mid y_{1:k}) dx_{0:k} \\ &= \sum_{i=1}^{N} \hat{\omega}_k^{(i)} g\left(x_{0:k}^{(i)}\right) \end{aligned} \tag{5}$$

where $\hat{\omega}_k^{(i)}$ is the normalized weight, which can be written as,

$$\hat{\omega}_k^{(i)} = \frac{\omega_k^{(i)}}{\sum_{j=1}^{N} \omega_k^{(j)}} \tag{6}$$

For the weight $\omega_k^{(i)}$ of each particle, it can be recursively expressed as,

$$\begin{aligned} \omega_k^{(i)} = \omega\left(x_{0:k}^{(i)}\right) &\propto \frac{p(x_{0:k} \mid y_{1:k})}{q(x_{0:k} \mid y_{1:k})} \\ &= \omega_{k-1}^{(i)} \frac{p\left(y_k \mid x_k^{(i)}\right) p\left(x_k^{(i)} \mid x_{k-1}^{(i)}\right)}{q\left(x_k^{(i)} \mid x_{k-1}^{(i)}, y_k\right)} \end{aligned} \tag{7}$$

Accordingly, the implementation steps of the PF algorithm are as follows,

(1) By sampling from the known prior probability distribution $p(x_0)$, the initial sample $\left\{x_{0:k}^{(i)}, \omega_k^{(i)}\right\}_{i=1}^{N}$ is obtained, where $\{\omega_k^{(i)}\}_{i=1}^{N} = 1/N$.

(2) A new particle set $\left\{x_k^{(i)}\right\}_{i=1}^{N}$ is obtained by sampling from the importance density function $q(x_k \mid x_{k-1}^i, y_k)$.

(3) Calculating the weight of each particle according to Equation (7).

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(4) Normalized weights, $\left\{ \hat{\omega}_{\mathrm{k}}^{(\mathrm{i})} = \omega_k^{(i)} / \sum_{j=1}^{N} \omega_k^{(j)} \right\}_{i=1}^{N}$.

(5) k=k+1, and return to step (1) to continue the iteration.

2.4 Resampling

Because the variance of the particle weight increases with time, the particle degradation in the basic PF is inevitable. After multiple iterations, most of the particle weights are so small that they can be ignored, while the weights of individual particles are too concentrated. To improve this situation, the resampling method is used to discard the particles with small weights, copy the particles with large weights and make them have equal weights, so as to reduce the phenomenon of particle degradation. The system resampling method is adopted to avoid particle degradation in this paper.

2.5 EKF importance sampling density

At time k, according to the new observation, the EKF algorithm is used to calculate the particle mean estimation $\bar{\boldsymbol{x}}_k^i$ and variance estimation $\hat{\boldsymbol{P}}_k^{(i)}$, and then the particles are extracted from the approximate Gaussian distribution $N(\bar{\boldsymbol{x}}_k^{(i)}, \hat{\boldsymbol{P}}_k^{(i)})$. This method of using EKF to generate importance density function is called EKPF. For a state vector $\boldsymbol{X}(t)$, the specific algorithm is as follows,

(1) The initial particle samples are obtained by sampling from the known prior probability distribution.

(2) The initial particles are updated by EKF,

$$\tilde{x}_{k+1|k}^{(i)} = f\left(\hat{x}_{k|k}^{(i)}, 0\right) \tag{8}$$

$$P_{k+1|k}^{(i)} = \boldsymbol{\Phi}_{k+1|k}^{(i)} \boldsymbol{P}_{k|k}^{(i)} \left(\boldsymbol{\Phi}_{k+1|k}^{(i)}\right)^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{K}_{k+1}^{(i)} = \boldsymbol{P}_{k+1|k}^{(i)} \left(\boldsymbol{H}_{k+1}^{(i)}\right)^{\mathrm{T}} \left(\boldsymbol{H}_{k+1}^{(i)} \boldsymbol{P}_{k+1|k}^{(i)} \left(\boldsymbol{H}_{k+1}^{(i)}\right)^{\mathrm{T}} + \boldsymbol{R}_{k+1}\right)^{-1} \tag{10}$$

$$\hat{\boldsymbol{x}}_{k+1|k+1}^{(i)} = \tilde{\boldsymbol{x}}_{k+1|k}^{(i)} \boldsymbol{K}_{k+1}^{(i)} \left[\boldsymbol{y}_{k+1} - \boldsymbol{h}\left(\boldsymbol{x}_{k+1|k}^{(i)}, 0\right)\right] \tag{11}$$

$$\boldsymbol{P}_{k+1|k+1}^{(i)} = \left(\boldsymbol{I} - \boldsymbol{K}_{k+1}^{(i)} \boldsymbol{H}_{k+1}^{(i)}\right) \boldsymbol{P}_{k+1|k}^{(i)} \tag{12}$$

where $\boldsymbol{\Phi}_{k+1|k}^{(i)}$ is the state transition matrix, $\boldsymbol{H}_{k+1}^{(i)}$ is the observation coefficient matrix, there are,

$$\boldsymbol{\Phi}_{k+1|k}^{(i)} = \frac{\partial f_{\mathrm{k}}\left(\boldsymbol{X}_k^{(i)}, \boldsymbol{y}_k\right)}{\partial \boldsymbol{x}_k^{(i)}} \mid \boldsymbol{x}_k^{(i)} = \hat{\boldsymbol{x}}_{k|k}^{(i)} \tag{13}$$

$$\boldsymbol{H}_{k+1}^{(i)} = \frac{\partial h_{\mathrm{k}}\left(\boldsymbol{x}_{k+1|k}^{(i)}, 0\right)}{\partial \boldsymbol{x}_{\mathrm{k}}^{(i)}} \mid \boldsymbol{x}_{\mathrm{k}}^{(i)} = \tilde{\boldsymbol{x}}_{k+1|k}^{(i)} \tag{14}$$

Considering the cause of filtering divergence and the infinite growth of Kalman filter memory, the data error at the previous moment will cause the error covariance matrix $\boldsymbol{P}$ and the gain matrix $\boldsymbol{K}$ to lose the ability to correct the state estimation with the iteration, resulting in filtering divergence. To increase the weight of new data and relatively weaken the influence of old data, a fading factor is introduced to reduce the negative impact of old data on filtering estimation. The formula (9) is modified as follows,

$$\boldsymbol{P}_{k+1|k}^{(i)} = \boldsymbol{\Phi}_{k+1|k}^{(i)} \boldsymbol{P}_{k|k}^{(i)} \left(\boldsymbol{\Phi}_{k+1|k}^{(i)}\right)^{\mathrm{T}} S \tag{15}$$

Among them, the forgetting factor $\lambda = 1/S$, the literature suggests $0.95 < \lambda < 1.0$, then the weighted weight of the fading memory is $1.0 < S < 1.05$.

(3) Complete the sequential importance sampling with reference to Section 2.3.

(4) Complete system resampling with reference to Section 2.4.

3 PARAMETER-FREE RESTORING FORCE IDENTIFICATION METHOD BASED ON EKPF AND CHEBYSHEV POLYNOMIAL

3.1 The equivalent linearization theory

For a multi-degree-of-freedom nonlinear dynamic system, the equation of motion can be written as,

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{f}_{non}(t) = \boldsymbol{f}(t) \tag{16}$$

In the formula, $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$ are the mass, stiffness and damping matrices of the system respectively. $\ddot{\boldsymbol{x}}(t)$, $\boldsymbol{x}(t)$ and $\dot{\boldsymbol{x}}(t)$ are the acceleration, displacement and velocity vectors respectively. $\boldsymbol{f}_{non}(t)$ is the nonlinear restoring force vector provided by the nonlinear element, and $\boldsymbol{f}(t)$ is the excitation vector the system.

The dynamic equation of the equivalent linear system is,

$$\boldsymbol{M}_E \ddot{\boldsymbol{x}}(t) + \boldsymbol{C}_E \dot{\boldsymbol{x}}(t) + \boldsymbol{K}_E \boldsymbol{x}(t) = \boldsymbol{f}(t) \tag{17}$$

where $\boldsymbol{M}_E$, $\boldsymbol{C}_E$ and $\boldsymbol{K}_E$ represent the equivalent linear mass, equivalent linear damping and equivalent linear stiffness, respectively. Since the structural mass does not change during the nonlinear development process, $\boldsymbol{M}_E$ can be regarded as the identification value of the mass, that is, and $\boldsymbol{M}$ is numerically equal $\boldsymbol{M}_E$. The nonlinear restoring force of the structure in the equation (17) will be reflected in the parameters $\boldsymbol{C}_E$ and $\boldsymbol{K}_E$ of the equivalent linear system, which is,

$$\boldsymbol{R}_{non}(t) = \boldsymbol{C}_E \dot{\boldsymbol{x}}(t) + \boldsymbol{K}_E \boldsymbol{x}(t) \tag{18}$$

3.2 Model-free nonlinear restoring force representation based on Chebyshev polynomial

The Chebyshev polynomial is one of the most important function sets in mathematics. Any continuous function can be represented by a set of orthogonal function sequences on [-1,1], the expression is as shown in the literature[16].

Therefore, the restoring force of the nonlinear element between the two degrees of freedom of the structure can be expressed by a set of relative velocity and relative displacement between the stories,

$$\boldsymbol{R}_{i,i-1}^{\mathrm{non}}(t) \approx \sum_{a=0}^{A} \sum_{b=0}^{B} c_{i,i-1,a,b}^{\mathrm{non}} C_a\left(v_{i,i-1}'\right) C_b\left(s_{i,i-1}'\right) \tag{19}$$

where $\boldsymbol{R}_{i,i-1}^{\mathrm{non}}(t)$ denotes the NRF of the nonlinear member between the i-th and (i-1)-th degrees of freedom of the system, and $c_{i,i-1,a,b}^{\mathrm{non}}$ denotes the coefficient of the Chebyshev polynomial. $C_a\left(v_{i,i-1}'\right)$ and $C_b\left(s_{i,i-1}'\right)$ are Chebyshev polynomials. A and B are integers, and their values are related to the degree of nonlinearity of the structure. In this paper,

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A+B=4. $v'_{i,i-1}$ and $s'_{i,i-1}$ denote the relative velocity and relative displacement between the *i*-th and *i*-1th degrees of the normalized system, respectively.

$$v'_{i,i-1} = \frac{v_{i,i-1} - \min(v_{i,i-1})}{\max(v_{i,i-1}) - \min(v_{i,i-1})} \tag{19}$$

$$s'_{i,i-1} = \frac{s_{i,i-1} - \min(s_{i,i-1})}{\max(s_{i,i-1}) - \min(s_{i,i-1})} \tag{20}$$

Among them, $v_{i,i-1}$ and $s_{i,i-1}$ are the relative velocity and relative displacement between the layers of the structure before normalization.

From Eqs. (17) and (18), the motion equation of the *i*-th DOF of the structure can be discretized as,

$$m_i \ddot{x}_i + \sum_{a=0}^{A}\sum_{b=0}^{B} c^{\text{non}}_{i,i-1,a,b} v^a_{i,i-1} s^b_{i,i-1} + \sum_{a=0}^{A}\sum_{b=0}^{B} c^{\text{non}}_{i,i-1,a,b} v^a_{i,i+1} s^b_{i,i+1} = f_i(t) \tag{21}$$

Therefore, the nonlinear member is introduced into the structure. After the complete structural parameters and dynamic response are obtained by the EKPF method, the Legendre polynomial coefficients are identified by the least square method, and the total nonlinear restoring force $\boldsymbol{R}_{non}(t)$ of the structure can be calculated. Finally, according to Eqs. (16) to (18), the damping force provided by the nonlinear member can be inversely derived.

4 NUMERICAL VERIFICATION

4.1 Example 1

To verify the effectiveness of the proposed method, numerical simulations are carried out to validate the four-degree-of-freedom shear-type frame with MR damper as an example. The nonlinear behavior of the structure is simulated by introducing the MR damper in the four-story concentrated mass shear-type frame shown in Figure 1. The mass of each layer of the structure m_i=150kg, the inter-story stiffness k_i=2.0×10^{5}N/m, and the damping coefficient c_i=160N·s/m, where *i*=1, 2, 3, 4. A horizontal external excitation f(t) with an action time of 2s is applied to the third story, and the time profile is shown in Figure 2. The structural response is obtained by the fourth-order Runge-Kutta method with a time step of 0.001s.

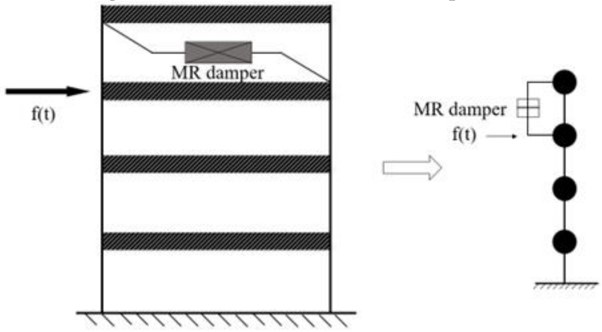

Figure 1. Nonlinear model equipped with MR dampers

Define the structure state vector as,

$$\boldsymbol{X}(t) = [x_1, x_2, x_3, x_4, \dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, k_{E1}, k_{E2}, k_{E3}, k_{E4}, c_{E1}, c_{E2}, c_{E3}, c_{E4}, m_{E1}, m_{E2}, m_{E3}, m_{E4}]^{\text{T}} \tag{22}$$

The MR damper introduced in the structure is a Bingham model with a damping force that satisfies the relation,

$$F^{\text{Bh}}_{\text{non}} = f^{\text{Bh}}_{\text{c}} \cdot \text{sgn}(v, v_{i,i-1}) + C^{\text{Bh}}_0 \cdot v_{i,i-1} + f^{\text{Bh}}_0 \tag{23}$$

where $F^{\text{Bh}}_{\text{non}}$ is the damping force provided by the Bingham model, $f^{\text{Bh}}_{\text{c}} = 20\text{N}$, $C^{\text{Bh}}_0 = 600\text{N}\cdot\text{s/m}$ and $f^{\text{Bh}}_0 = 0$ are model coefficients. The damping force calculated from Eq. (23) is accurately calculated by the mathematical relation equation and thus can be used as the theoretical damping force of the structure to evaluate the identified value.

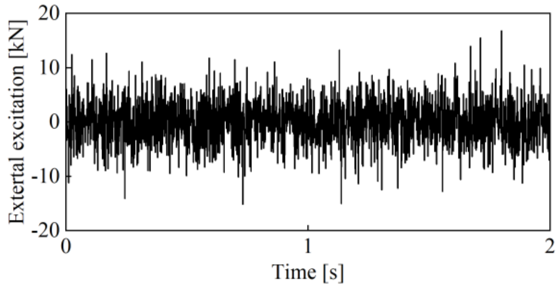

Figure 2. External excitation force time history

The setting range of the initial parameters of the structure is estimated based on the real parameters of the structure, and the number of particles is set to 20000, the initial value of the stiffness of each floor is 130kN/m~210kN/m, the initial value of the damping coefficient is 0kN·s·m^{-1}~1000kN·s·m^{-1}, and the initial value of the mass is 100kg~160kg. In fact, due to the use of EKF as the importance function for sampling, the identified range of particles after EKF update can exceed the setting range of initial parameters. Assuming that the acceleration of the second story of the structure cannot be measured, only the accelerations of the first, third and fourth stories of the structure are observed, and 5% non-Gaussian noise is added to the observations.

Taking the mean value of the last step in each iteration as the parameter identification result, the structural stiffness, damping, and mass are identified using EKPF, the convergence process of the equivalent linear parameter is shown in Figure 3. Figure 3(a) gives the convergence process of the identified equivalent stiffness of each story of the structure. It can be seen that from Figure 3(b) the equivalent damping identification value of the first to the third stories tends to the true value, but the identified value of the fourth story identifies with the other stories in the completely opposite direction, which can be judged that the structure has undergone a nonlinear behavior in the fourth story, which is also consistent with the actual installation of the damper. The convergence result of mass identification is shown in Figure 3(c), which can accurately converge to the actual value, which also shows that the nonlinear behavior of the structure does not affect the mass of the structure after it occurs. Table 1 and Table 2 give the identified results of the mass and the equivalent stiffness and equivalent damping, respectively. It can be seen from Table 1 that the identified result of the structural mass is better, and the error of each story is less than 1 %. The equivalent linear parameters have also achieved good identification results.

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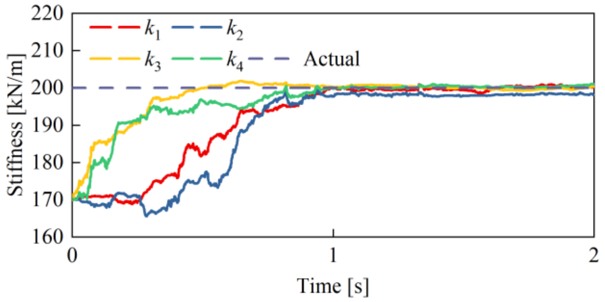
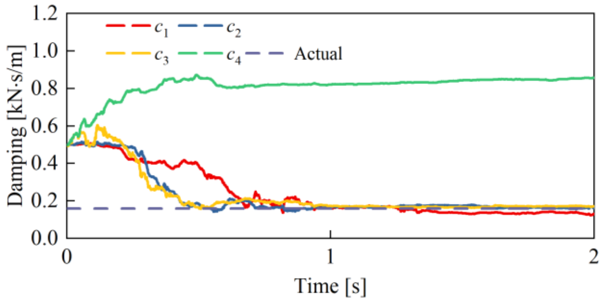
Table1. Identified mass results

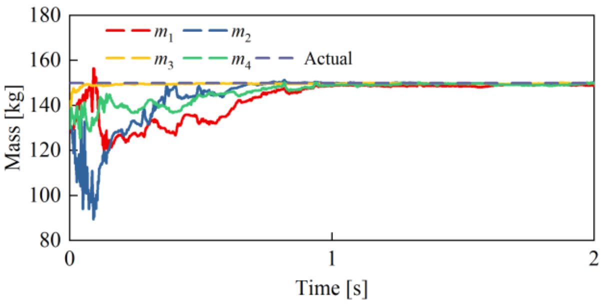
Mass	Identified [kg]	Actual [kg]	Error [%]
m_1	148.85	150	0.77
m_2	150.26		0.17
m_3	149.85		0.10
m_4	149.71		0.19



(a) Equivalent linear stiffness



(b) Equivalent linear damping



(c) Equivalent linear mass

Figure 3. Structural parameter convergence process

Table2. Identified equivalent stiffness and damping results

Parameter	Identified	Actual
k_{E_1} (kN/m)	200	200
k_{E_2} (kN/m)	198	200
k_{E_3} (kN/m)	200	200
k_{E_4} (kN/m)	201	200
c_{E_1} (kN·s·m^{-1})	0.13	0.16
c_{E_2} (kN·s·m^{-1})	0.16	0.16
c_{E_3} (kN·s·m^{-1})	0.17	0.16
c_{E_4} (kN·s·m^{-1})	**0.86**	0.16

Figures 4 and 5 show the comparison between the identified results and the actual values of the displacement and velocity response of each story of the structure, and Figure 6 shows the comparison between the identified values and the true values of the unobserved acceleration of the 2nd story of the structure. The identified results of displacement, velocity and acceleration of the second story are in good agreement with the true values.

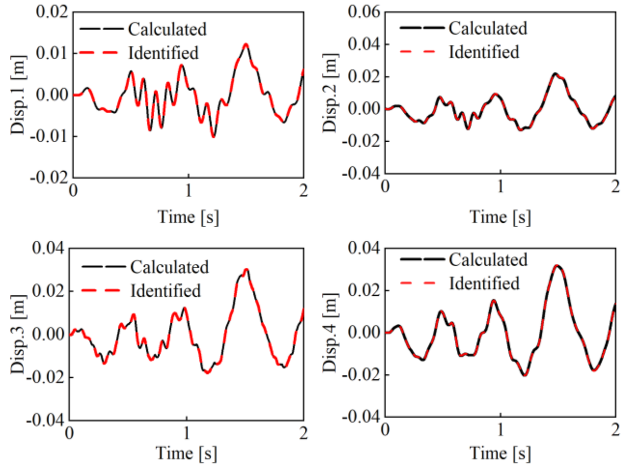


Figure 4. Identified structural displacement responses

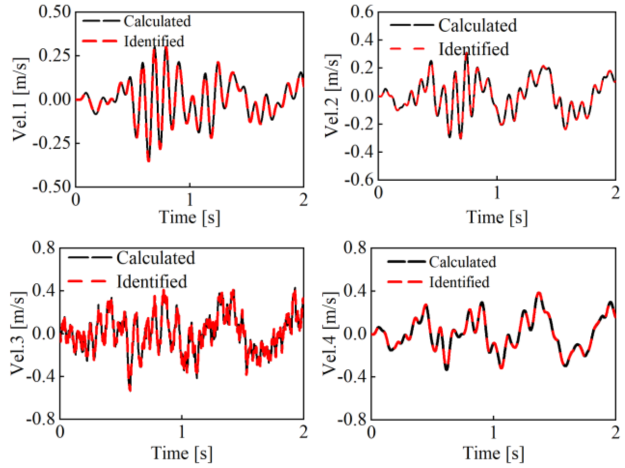


Figure 5. Identified structural velocity responses

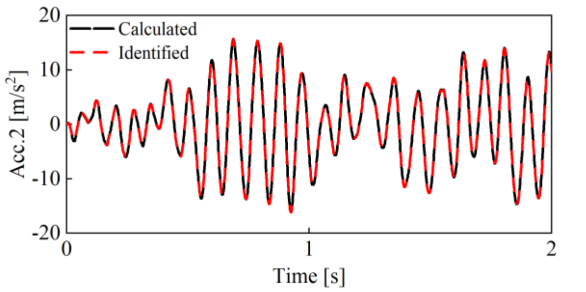


Figure 6. Identified structural acceleration responses on second floor

Based on the identified structural parameters, displacement and velocity responses, the NRF of the 4th floor of the structure can be further obtained using the least squares algorithm in a nonparametric manner, and the results of the comparison between the identified and the true values of the NRF of the 4th floor of are shown in Figure 7.

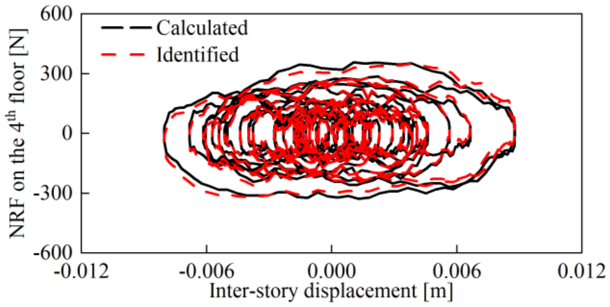


Figure 7. Identified MR damper force

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The identified results of the unknown acceleration and NRF are quantified by the root mean square error (RMSE)[23], which are calculated to be 0.12m/s^2 and 12.59N for the acceleration of on the 2nd floor and NRF, respectively.

4.2 Example 2

Considering the actual situation that the structure may have multiple damages under strong dynamic loading, in order to verify the generality of the proposed algorithm, MR dampers are introduced in the first and fourth stories of the structure, and horizontal external excitation is applied to the second floor, and the nonlinear structural model and the external excitation are shown in Figure 8 and Figure 9, respectively.

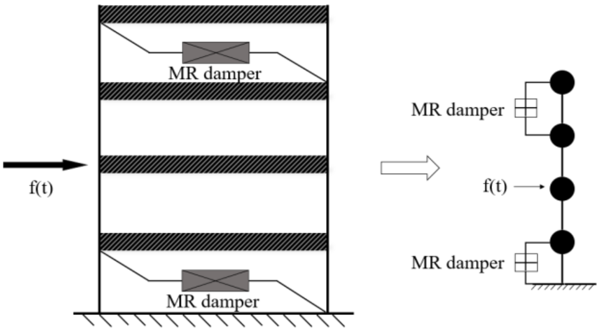


Figure 8. Four-story shear frame model with MR dampers

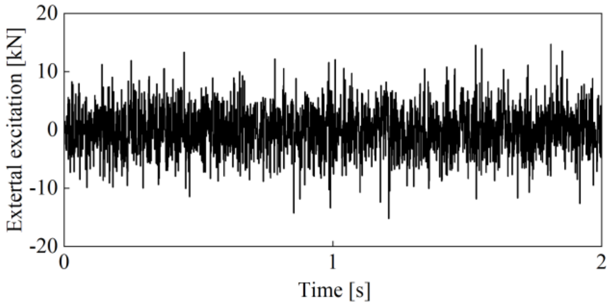


Figure 9. External excitation force time history

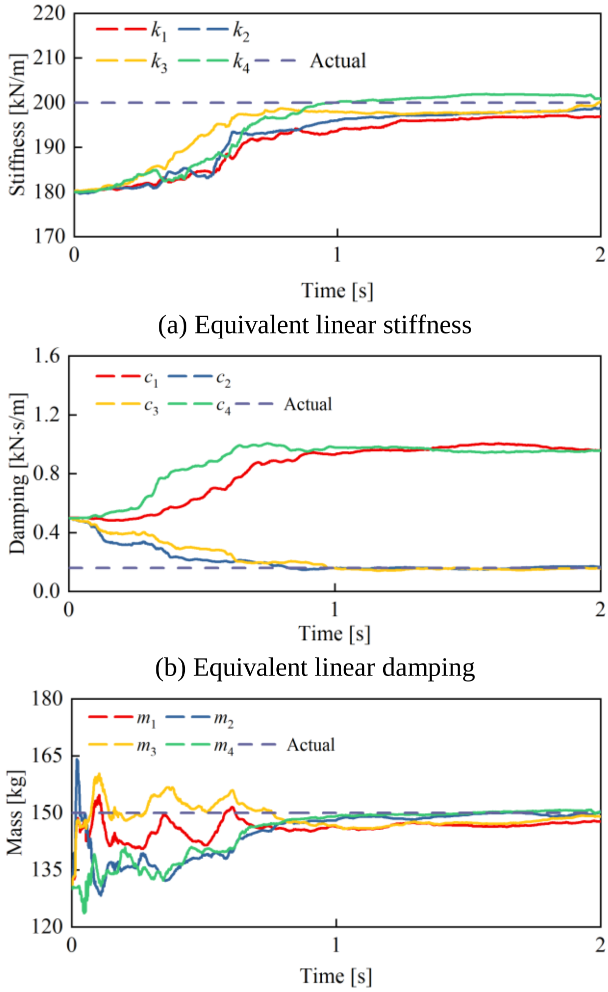
Distinguishing from the Bingham model in Example 1, the two MR dampers in this example use the modified Dahl model with the expression,

$$F_{non}^{\mathrm{Dh}} = K_0^{\mathrm{Dh}} s_{i,i-1} + C_0^{\mathrm{Dh}} v_{i,i-1} + F_0^{\mathrm{Dh}} Z + f_0^{\mathrm{Dh}} \tag{24}$$

$$\dot{Z} = \sigma v_{i,i-1}\left(1 - Z sgn(v_{i,i-1})\right) \tag{25}$$

where F_{non}^{Dh} is the damping force provided by the Dahl model, $K_0^{\mathrm{Dh}} = 30\mathrm{N/m}$, $C_0^{\mathrm{Dh}} = 600\mathrm{N \cdot s/m}$, $F_0^{\mathrm{Dh}} = 35\mathrm{N}$, $\sigma = 500\mathrm{s/m}$ and $f_0^{\mathrm{Dh}} = 0$, $f_0^{\mathrm{Dh}} = 0$, C_0^{Dh}, F_0^{Dh} and σ are model coefficients, and Z is the dimensionless hysteresis.

The settings of the initial parameters and the structural true values in Example 2 are the same as those in Example 1, and the identified process of the equivalent linear stiffness, damping and mass is given below. From Figure 10 the structural stiffness, damping, and mass parameters of the other floors converge to the actual value of the structure, except for the equivalent damping values of the first and fourth floors in Figure 10(b), which deviate from the actual value, and according to which it can be shown that the structure undergoes nonlinear behavior in the first and fourth floors. The identified results of the mass, the equivalent linear stiffness and damping are shown in Tables 3 and 4, respectively. The identified results of the parameters have small errors.



(a) Equivalent linear stiffness

(b) Equivalent linear damping

(c) Equivalent linear mass

Figure 10. Structural parameter convergence process

Table 3. Identified mass results

Mass	Identified [kg]	Actual [kg]	Error [%]
m_1	147.70	150	1.53
m_2	150.48		0.32
m_3	149.01		0.66
m_4	150.13		0.09

Table 4. Identified equivalent stiffness and damping results

Parameter	Identified	Actual
k_{E_1}(kN/m)	197	200
k_{E_2}(kN/m)	199	200
k_{E_3}(kN/m)	200	200
k_{E_4}(kN/m)	201	200
c_{E_1}(kN·s·m^{-1})	**0.96**	0.16
c_{E_2}(kN·s·m^{-1})	0.16	0.16
c_{E_3}(kN·s·m^{-1})	0.15	0.16
c_{E_4}(kN·s·m^{-1})	**0.95**	0.16

Figures 11 and 12 show the comparison of the identified results of displacement and velocity response of the nonlinear structure respectively, it can be found that the dynamic response of the structure in all floors are identified with good results.

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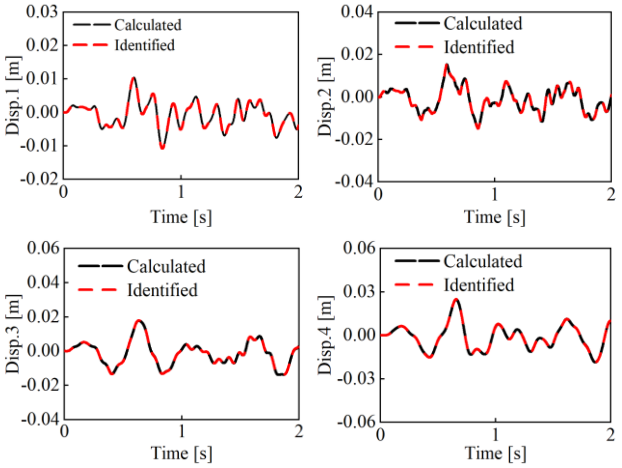


Figure 11. Identified structural displacement responses

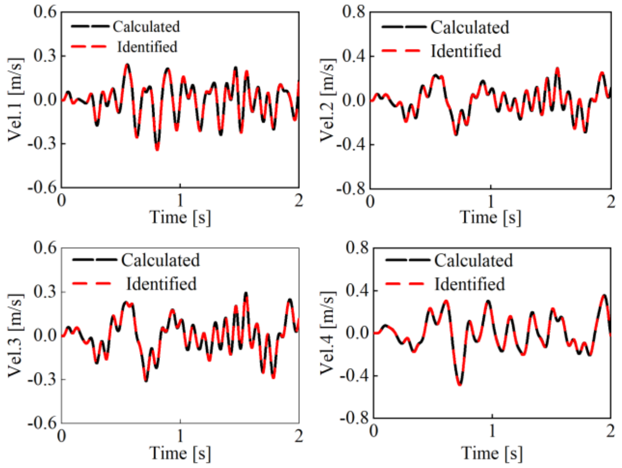
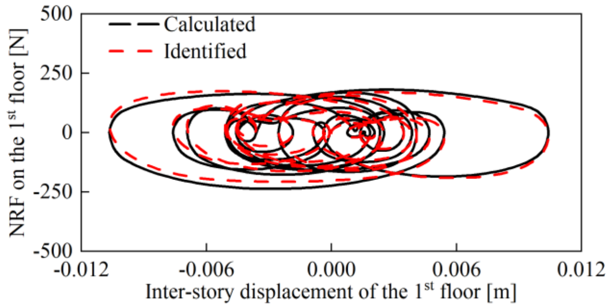
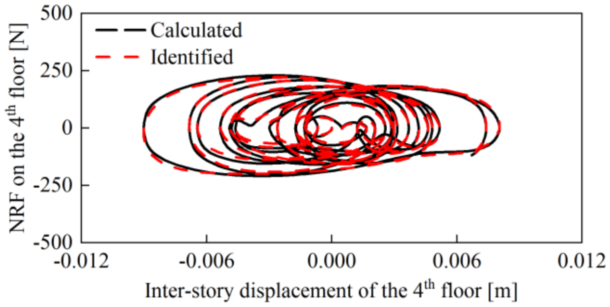


Figure 12. Identified structural velocity responses

The same as in Example 1, based on the parameters and responses identified by the structure, the NRFs between the first and fourth floors can be identified, as shown in Figure 13. The RMSE of the identified acceleration value of the second floor is 0.30m/s^2, the RMSE of the identified value of NRF of the first floor is 14.43 N, the RMSE of the identified value of NRF of the fourth floor is 11.46N. It can be seen in Figure 13 that the identified values of the first and the fourth floors of the damping force are in good agreement with the real values, which indicates that the proposed algorithm is not only able to effectively identify the nonlinear behavior of the structure at a single unknown location under strong dynamic loading, but also applies to the case of a nonlinear system where multiple nonlinear locations are unknown.



(a) Identified result of MR damping force on the 1st floor



(b) Identified result of MR damping force on the 4th floor

Figure. 13 Identified MR damper force

5 CONCLUSIONS

In this paper, based on the EKPF algorithm and equivalent linear theory, the structural parameters and nonlinear locations are identified under limited acceleration observations, and a model-free identification of structural NRF is proposed based on Chebyshev polynomial.

Numerical simulations of a four-degree-of-freedom concentrated mass nonlinear shear frame model were performed. In two examples, different numbers and models of MR dampers (Bingham model vs. Dahl model) are sequentially introduced to the structure to simulate different nonlinear behaviors for different numbers and locations. Considering the effect of measurement noise and changing the location of horizontal external excitation application, the structural stiffness, damping, mass parameters, and the inter-story NRF at the arrangement of MR dampers are identified with limited observations of the acceleration response, and the validity and applicability of the proposed methodology are verified by comparing the identified values with the theoretical values.

The method proposed in this paper is general in that it does not need to utilize a parametric model of the NRF of the structure in the identified process. The identified NRF under strong dynamic loads such as earthquakes is an intuitive description of the hysteretic performance of the structure, and through the restoring force characteristics at different moments, it can reflect the occurrence and development of the damage of the structure or sub-structure at different moments in the process of dynamic loading, and can be used for the quantitative description of the energy dissipation of the structural components in the process of the loading. The method proposed in this paper is of great significance for damage localization, quantitative assessment and post-disaster structural performance evaluation of structures subjected to dynamic loads such as earthquakes.

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