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Dynamic Monitoring using Hidden Markov Regression Model for Predicting Remaining Useful Life

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ABSTRACT: This paper presents the remaining useful life (RUL) prediction problem in civil engineering applications using a hidden Markov regression model (HMRM), as a promising approach for model-based degradation. Unlike self-transition hidden Markov models for mass-produced components, where prior lifetime signals are available to estimate state information, the proposed HMRM formulates the conditional probability of RUL in terms of the estimated regressor parameters, after temporally fitting the damage model. The discrete property of state in HMRM makes it possible to handle heterogeneities in the degradation process. The HMRM can also synthesise multiple signals by adopting a decision-level fusion. An adaptive closed-form solution for RUL prediction is presented. The performance of HMRM is demonstrated on synthetic measurements and compared with a Bayesian extended Kalman filter (EKF) updating technique.

KEY WORDS: Hidden Markov chain; model-based degradation; remaining useful life prediction; damage model.

1 GENERAL GUIDELINES

The prediction of Remaining Useful Life (RUL) holds significant importance in both condition-based monitoring (CBM) and the formulation of maintenance strategies for structural components. RUL is defined as the time a structure has before reaching its design threshold, when it can no longer perform under its design function. In CBM, damage is characterised as a change in structural components due to the interaction between internal degradation and the working environment which adversely affects its current and future performance. The progression of damage is heterogeneous, and such heterogeneity can be due to *unit-to-unit variability* [1] of the material, *changing operational conditions* [2], or *periodic loading* [3].

Various damage models have been developed, ranging from empirical laws (such as Paris' laws) to continuum damage mechanics (CDM) methods [4], which requires stochastic-based approaches (such as Markov chain, Weiner processes) for predicting RUL [5]. However, these approaches suffer from parameter correlation. This study focuses on simple power and exponential laws suited in a hidden Markov chain, which, despite their simplicity, provide correlated, good curve fitting by: a linearised and automated regression segmentation and an adaptive parameter updates via a recursive Markov chain to improving accuracy. According to Si, et al. [5][6] hidden Markov models (HMMs) are suited for RUL prediction based on degradation state processes. HMM is composed of two state processes; an unobservable (hidden) Markov chain which accounts for the actual state of degradation such as fatigue at the grain level of a metallic component, and an observable process that interprets the monitoring information, for instance, crack width in a reinforced concrete beam.

Within a degradation process is multiple discrete-hidden states. Past studies [7][8] [9][10] focused on the transition probability matrix and state duration definition. They consider that these states can switch into each other under a predefined transition probability, which is specific to the lifetime datasets. Modelling degradation by this approach is limited to the Markov property [11] which means that: a) the state at any given time step only depends on the previous state and not on any earlier states, b) and that shorter state durations are more likely than longer ones, i.e., state duration follows a geometric distribution. This can be a limitation for modelling real-world degradation processes. To remedy this weakness, hidden semi-Markov models (HSMM) have been proposed to explicitly model state duration distributions rather than assuming a geometric one [12][13]. Unfortunately, this improvement of modelling degradation in components with predefined transition probability and state duration does not extend to modelling the degradation processes on bespoke components that obeys damage laws.

To the best of the author's knowledge, this study frontier a premise of adopting a sequence of state in model-based degradation process. By this approach, the degradation process experiences a state switch at stages of damage triggered by the weighted state posterior distribution [14]. In addition, the proposed approach can handle multiple information of measured signals (i.e., multiple HMRM) earlier introduced in [13]. This improves the state identification since it captures the interacting factors of multiple signals, and reduces measurement noise uncertainty, leading to a more confident RUL prediction. The extended Kalman filter (EKF) is introduced in this study to recursively observe data in real-time, to compare with our inspection approach. As with HMM belonging to a finite-discrete set of states, the EKF is an optimal non-linear filter for finite-dimensional stochastic systems, for model-based analysis, where the states are continuous and described as parameters of the damage model [15].

Within the framework of modelling degradation presented in the paper, two case studies are considered: a fatigue crack growth process, and a multi-sensor beam degradation measurement. A decision-level fusion technique using fisher's weighted discriminant ratio [13] is considered to aggregate the multiple estimated parameters and predict the RUL.

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2 HIDDEN MARKOV REGRESSION MODEL (HMRM)

This section provides the framework that captures the hidden state degradation processes.

2.1 HMRM Parameter estimation

The unobserved degradation processes u_t represent a sequence of K hidden states of data points, formulated at index time t in a particular state k as:

$$u_t = U_t \cdot \exp(\sigma_k^2 \varepsilon_t), \quad (t = 1, \ldots, T) \tag{1}$$

where U_t represent the damage model that characterises the degradation process up till failure time T. To model the hidden Markov chain, we take the log-transform of equation 1 and put in matrix form as:

$$y_t = \log u_t = \mathbf{x}_t \boldsymbol{\beta}_k + \sigma_k^2 \varepsilon_t \tag{2}$$

y_t becomes degradation processes that follows a Gaussian distribution of mean $\mathbf{x}_t \boldsymbol{\beta}_k$ and variance σ_k^2, the parameter vector $\theta = (\boldsymbol{\pi}, \mathbf{A}, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K, \sigma_1^2, \ldots, \sigma_K^2)$ defines the model of the degradation process. ε_t is a random variable that follows the standard Gaussian distribution (zero mean and unit variance) representing an additive measurement noise with standard deviation σ_k. $\mathbf{x}_t$ is the covariate vector at index time t that translates scatter due to environmental/operational conditions and $\boldsymbol{\beta}_k$ is a $\mathbb{R}^{2\times1}$ vector containing the regression coefficients. The hidden sequence $\mathbf{k} = (1, \ldots, K)$ is assumed to be a homogeneous Markov chain (as shown in Figure 1) of the first order and parameterised by an initial state distribution $\boldsymbol{\pi}$ and a transition matrix $\mathbf{A}$.

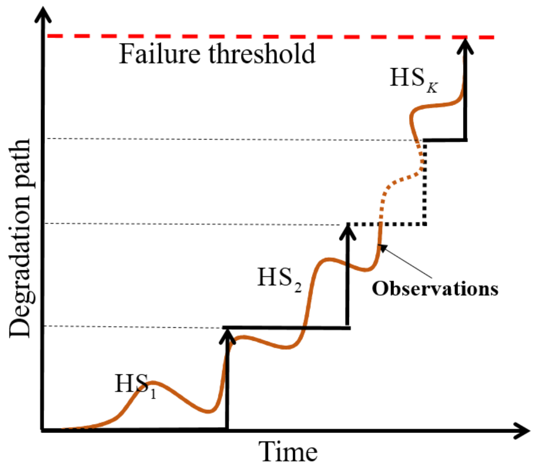

Figure 1. Schematic representation of the actual degradation process over a sequence of hidden states (HS), from 1 to K.

As shown in Figure 1, the hidden state discretises the observation that is continuously measured, in a sequence. This sequence of discretization is irreversible since degradation process is progressive, except when a maintenance action is implemented. Figure 2 describes the probability of stay in a hidden state is defined by the transition probability $A_{k,k}$ in the transition matrix $\mathbf{A}$. Hence, $A_{k,k}$ determines whether the degradation process is either staying or moving to the next degradation stage.

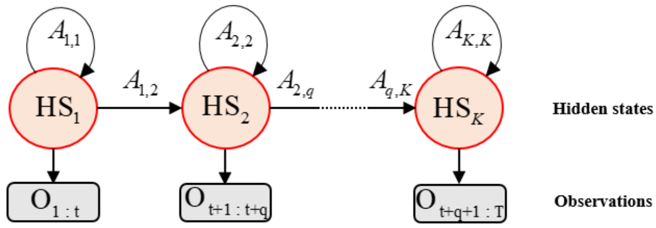

Figure 2. Illustration of the possible sequence of transition path between hidden states.

The vector θ can be estimated using the limiting properties of consistency, and asymptotic normality of the so-called maximum likelihood method. Its efficiency is subject to a considerable number of data points, so the sample size is suitable for taking advantage of the limiting properties of the likelihood estimator. The log-likelihood is presented in the form:

$$\begin{aligned} \mathrm{L}(\theta^{(m)}) &= \log\ p(\mathbf{y}_1, \ldots, \mathbf{y}_t; \theta^{(m)}) \\ &= \log \sum_{\mathbf{k}} p(\tau_1; \boldsymbol{\pi}) \times \prod_{t=2}^{T} p(\tau_t | \ \tau_{t-1}; \mathbf{A}) \prod_{t=1}^{T} \mathrm{N}\ (\mathbf{y}_t; \mathbf{x}_t \boldsymbol{\beta}_k, \sigma_k^2) \end{aligned} \tag{3}$$

The Baum-Welch algorithm is referred to as the expectation-maximisation (EM) algorithm [11][16], which performs a recursive iteration over E-M steps while updating estimated parameters that govern the regression model. The regressor mean is:

$$\begin{aligned} \hat{\boldsymbol{\beta}}_k^{(m+1)} &= \left[\sum_{t=1}^{T} \mathbf{x}_t' \gamma_{tk}^{(m)} \mathbf{x}_t \right]^{-1} \sum_{t=1}^{T} \mathbf{x}_t' \gamma_{tk}^{(m)} \mathbf{y}_t \\ &= \left[\mathbf{X}' \mathbf{W}_k^{(m)} \mathbf{X} \right]^{-1} \mathbf{X}' \mathbf{W}_k^{(m)} \mathbf{Y} \end{aligned} \tag{4}$$

where

$$\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_T]' \tag{5}$$

is the $\mathbb{R}^{T\times2}$ regression matrix and $\mathbf{W}_k^{(m)}$ is a $\mathbb{R}^{T\times T}$ diagonal matrix of weights whose diagonal elements represent the posterior probabilities $(\gamma_{1k}^{(m)}, \ldots, \gamma_{Tk}^{(m)})$. On the other hand, the covariance matrices are updated as a weighted variant of the estimation of a Gaussian density with the polynomial mean $\mathbf{x}_t \hat{\boldsymbol{\beta}}_k^{(m+1)}$:

$$\begin{aligned} \hat{\sigma}_k^{2(m+1)} &= \frac{1}{\sum_{t=1}^{T} \gamma_{tk}^{(m)}} \sum_{t=1}^{T} \gamma_{tk}^{(m)} \left(\mathbf{y}_t - \mathbf{x}_t \hat{\boldsymbol{\beta}}_k^{(m+1)} \right)' \left(\mathbf{y}_t - \mathbf{x}_t \hat{\boldsymbol{\beta}}_k^{(m+1)} \right) \\ &= \frac{1}{\sum_{t=1}^{T} \gamma_{tk}^{(m)}} \left(\mathbf{Y} - \mathbf{X} \hat{\boldsymbol{\beta}}_k^{(m+1)} \right)' \mathbf{W}_k^{(m)} \left(\mathbf{Y} - \mathbf{X} \hat{\boldsymbol{\beta}}_k^{(m+1)} \right) \end{aligned} \tag{6}$$

2.2 Multiple HMRM (M-HMRM)

Regarding the case of multiple signals, the model can be expanded as a set of q time-series:

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$$\mathbf{y}_j = \begin{cases} \mathbf{x}_t \boldsymbol{\beta}_{1k} + \sigma^2_{1k}\epsilon_t \\ \mathbf{x}_t \boldsymbol{\beta}_{2k} + \sigma^2_{2k}\epsilon_t \\ \vdots \\ \mathbf{x}_t \boldsymbol{\beta}_{qk} + \sigma^2_{qk}\epsilon_t \end{cases}, \quad \forall j = 1,\ldots,q, \tag{7}$$

where the latent degradation state k simultaneously governs all of the univariate time series components. Having observed the multivariate degradation process with the M-HMRM in equation (7) and obtaining the estimated posterior distribution, the weighted discriminant type fusion technique [13] is used to re-define the regression parameters to a univariate process, and consequently the sampled distribution. The core concept of discriminant function analysis is to determine whether groups differ in terms of the mean of a variable, and then use that variable to predict which group the sample distribution might belong to. When dealing with a single variable, the final test to assess whether the underlying assumption of homogeneity of variance (i.e. homoscedasticity) distinguishes between groups is the F-test. This test is calculated by comparing the variance between groups to the pooled (average) variance within groups. If the variance between groups is significantly higher, it indicates significant differences in the prediction. The weighting process is influenced by the respective F-values. The F-value indicates how statistically significant a variable is in distinguishing between groups, reflecting its unique contribution to predicting group membership or fusion.

Here, the estimated covariance $(\hat{\sigma}^2_k)$ of the M-HMRM is a $\mathbb{R}^{q\times q}$ matrix. Since F-value is a measure of the extent to which a variable makes a unique contribution to the prediction of group membership, one can easily obtain the F-value of the different sensors from $(\hat{\sigma}^2_k)$ to be:

$$F_{jk} = \frac{\text{variance between groups}}{\text{variance within groups}} \tag{8}$$

The final fusion becomes a weighted linear combination, as follows:

$$\left\{\hat{\boldsymbol{\beta}}_k, \hat{\sigma}^2_k\right\} = \sum_{j=1}^{q} \frac{F_{jk}}{\sum_{j}^{q} F_{jk}} \left\{\hat{\boldsymbol{\beta}}_{jk}, \hat{\sigma}^2_{jk}\right\} \tag{9}$$

and the estimated bivariate coefficients are then integrated into the failure function. Integrating these coefficients into the degradation process facilitates the consideration of time-varying dynamics of systems, making it a preferred approach among researchers.

3 REMAINING USEFUL LIFE PREDICTION

Having established the single and multiple signal HMRM degradation process, in this section the PDF of the RUL of their underlying damage models are formulated using a derived closed-form expression. As stated in the previous sections, the degradation is modelled by a random process $\{\hat{u}_k(\tau); \tau \le 0\}$, assumed Gaussian for simplicity. Under the concept of first hitting time (FHT), the conditional RUL $(\hat{\ell}_k)$ of the system on the observation $\hat{u}_k(\tau)$ at degradation rate τ of state k is defined as the time from the initial state of performance degradation until the failure threshold (λ) is reached for the first time, as:

$$\hat{\ell}_k = \inf\left\{\tau : \hat{u}_k(\tau + \hat{\ell}) \ge \lambda \,|\, \hat{u}_k(\tau) < \lambda\right\} \tag{10}$$

A simple system architecture of the approach is presented in Figure 3. The HMRM parameters are obtained by sampling the observed data over the underlying state-based damage model

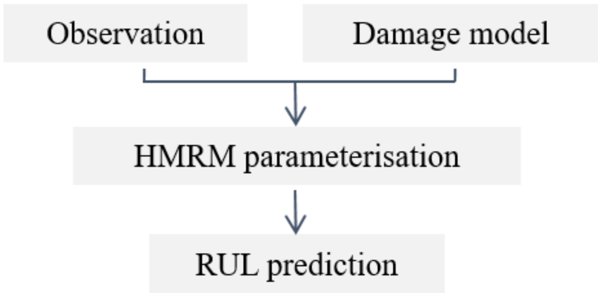

Figure 3. Remaining useful life flowchart.

3.1 Power model

Consider that multiplicative measurement error in equation 1 can be approximated as $\exp(\hat{\sigma}^2_k \epsilon_\tau) \approx 1 + \hat{\sigma}^2_k \epsilon_\tau$ and the variability in sampling the observation is very small, i.e., $\left(\hat{c}_k \tau^{\hat{b}_k} \hat{\sigma}^2_k \sim \hat{\sigma}^2_k\right)$, then the failure function becomes:

$$\hat{u}_k(\tau) = \hat{U}_k(\tau) + \hat{\sigma}^2_k \epsilon_\tau = \hat{c}_k\, \tau^{\hat{b}_k} + \hat{\sigma}^2_k \epsilon_\tau \tag{11}$$

τ controls switching between one degradation point $u(\tau)$ and another across the K hidden states. From the log-linear transform, $\mathbf{x}(\tau) = [1 \ \log\tau]$ and $\hat{\boldsymbol{\beta}}_k = [\log\hat{c}_k \ \hat{b}_k]'$. Since the FHT of the nonlinear degenerate model at the current rate τ satisfies the inverse Gaussian distribution [17][18]. The degradation path upon hitting λ can be expressed as the PDF of the system RUL as:

$$f_T\left(\hat{\ell}_k | \ \hat{\theta}, \lambda\right) = \frac{\lambda - \hat{u}_k}{\sqrt{2\pi\hat{\ell}^3_k\left(\hat{\sigma}^2_k \hat{\ell}_k + \hat{\rho}_k\right)}} \times \exp\left[-\frac{\left(\lambda - \hat{u}_k - \hat{c}_k \hat{\ell}_k^{\hat{b}_k}\right)^2}{2\hat{\ell}_k(\hat{\sigma}^2_k \hat{\ell}_k + \hat{\rho}_k)}\right] \tag{12}$$

Where $\hat{\rho}_k$ equal to one [19].

3.2 Exponential model

Under the measurement error condition of log-normal distribution as assumed in the power model, the exponential failure function is presented as:

$$\hat{u}_k(\tau) = \hat{c}_k\, e^{\hat{b}_k \tau} + \hat{\sigma}^2_k \epsilon_\tau \tag{13}$$

From the log-linear transform, $\mathbf{x}(\tau) = [1 \ \tau]$ and $\hat{\boldsymbol{\beta}}_k = [\log\hat{c}_k \ \hat{b}_k]'$. The degradation path upon hitting λ can also be expressed in terms of the PDF of the system RUL:

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$$f_T\left(\hat{\ell}_k \mid \hat{\theta}, \lambda\right) = \frac{\mathrm{In}\ \lambda - \mathrm{In}\ \hat{u}_k}{\sqrt{2\pi\hat{\ell}_k^3\left(\hat{\sigma}_k^2\hat{\ell}_k + \hat{\rho}_k\right)}} \times \exp\left[-\frac{\left(\mathrm{In}\ \lambda - \mathrm{In}\ \hat{u}_k - \hat{b}_k\hat{\ell}_k\right)^2}{2\hat{\ell}_k(\hat{\sigma}_k^2\hat{\ell}_k + \hat{\rho}_k)}\right] \tag{14}$$

3.3 Performance error

The threshold λ is chosen based on expertise judgement. However, since this degradation dataset lacks historical data, as typical for bespoke components, the value of the last data point will be used as the threshold. In the second case study (for multiple sensor information), the mean of the signals' last data points is considered as the threshold.

Since the true internal states of degradation are not available, the number of states for the lifetime of either case is unknown. However, any chosen number of discrete states should be able to capture the degradation effects, encompassing material degradation, loading, environmental conditions and maintenance regimes. For prediction performance accuracy, we compute the root mean square error (RMSE) metric over each state k of the RUL, given as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(\ell_k - \hat{\ell}_k\right)^2} \tag{15}$$

where ℓ_k and $\hat{\ell}_k$ represents the actual and predicted RUL respectively.

4 FATGIUE CRACK GROWTH (FCG)

A FCG degradation process in a structural component has been simulated based on the Paris law of fracture mechanics described in [20], and the synthetic data is shown in Figure 4. The synthetic data will be used as the available data for the both models discussed in the previous section.

4.1 Results

Since RUL prediction solely depends on the available data, signals were continuously updated at every 25 cycles until failure. The advantage of this is to capture the heterogeneity of the degradation process. Typical for damage propagation in FCG, 2 and 3 state has been considered [21]. The parameter estimation procedure in section 2 and the PDF of RUL in section 3 is repeated recursively per inspection points.

The estimated PDF of RUL in terms of recursion in 2-state HMRM by power model is shown in Figure 5. At early stages of prediction (i.e., 25, 50 cycles), the prediction is dominated by uncertainty (high variance) due to limited degradation process, which does not clearly indicate the propagation trends. As failure approaches, the degradation process becomes more predictable and the variance shrinks. In addition, states switches is seen to adjust continuously as more data is introduced. However, the model seems to suffer a positive bias in predicting the RUL, as these models are conservative and obviously may not match the growth dynamics of Paris' law.

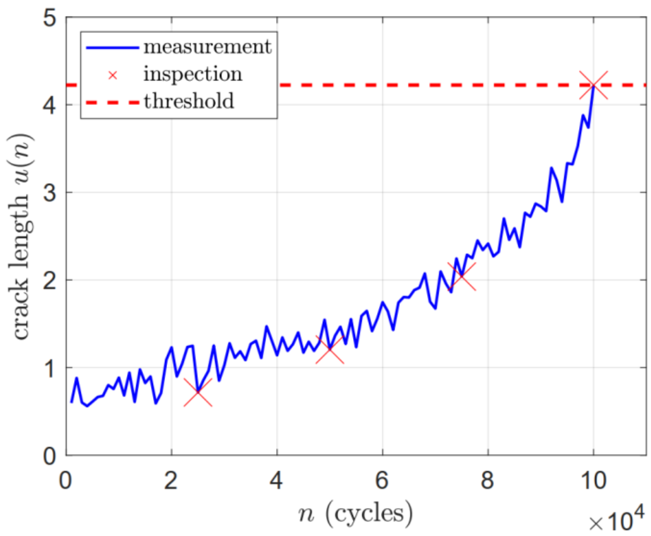

Figure 4. Fatigue crack length measurements $u(n)$ versus magnitude of cycles n.

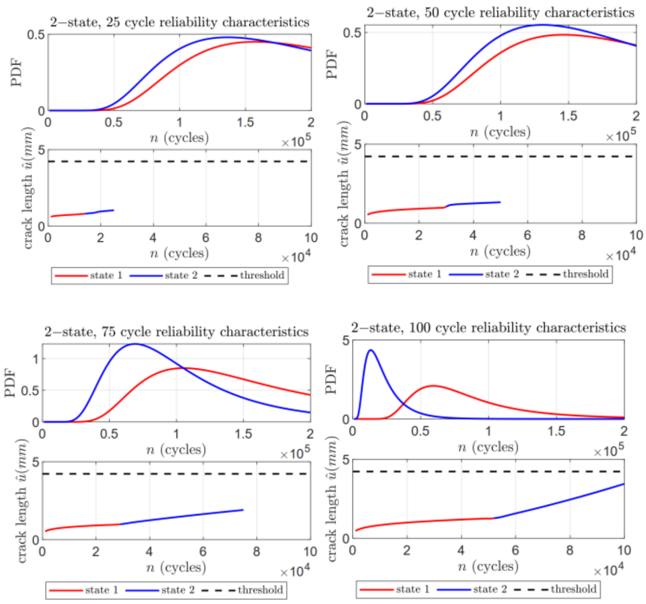

Figure 5. 2−state power HMRM reliability characteristics showing the predicted degenerate slope and the accompanying PDFs of RUL for real-time monitored information from the populated 25 initial cycles, updated by a populated 50, 75, and 100 cycle of monitored information.

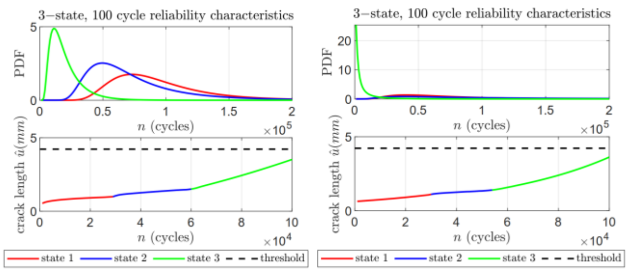

Figure 6. Reliability characteristics showing predicted degenerate slope and the accompanying PDFs of RUL for 100 cycles of monitored information in 3−state power (left) and exponential (right) HMRMs.

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Figure 6 shows the complete (100 cycles of) predicted degenerate slope and the accompanying PDFs of RUL for 3-state HMRMs. It compares the prediction by power and exponential model. From the PDF plots, it shows that prediction from power model suffers more uncertainty than of the exponential model, especially at the late stage of damage propagation. This uncertainty is associated with the regressor parameter $\hat{b}_k$, which is gradual in the power model than in exponential model.

Figure 7 presents the RUL prediction based on the PDF of RUL plots. From the plot, it is observed that 2-state hidden states satisfy the heterogeneity in the datasets, and an additional state does not contribute to the dynamical distribution in the dataset. This is reasonable since FCG simulation is quasi-static, and rarely models the micro-cracks at grain levels. As time passes, it is shown that the predicted RUL converges towards the actual RUL in the absence of a prior degradation. The RUL distribution of the 2- state exponential model are observed to converge best than the power model. This is evident as the model follows damage accumulation scenario, which is a typical exponential. An EKF model based on the power model is also presented, for comparison. The EKF-power model converges to the true RUL better than 2-state power model. This is because, for an inspection routine with HMRM, the variance of estimating the regressor parameters over a sequence of observed data is higher than that of an EKF monitoring process. Along the true RUL is a 95% confidence interval.

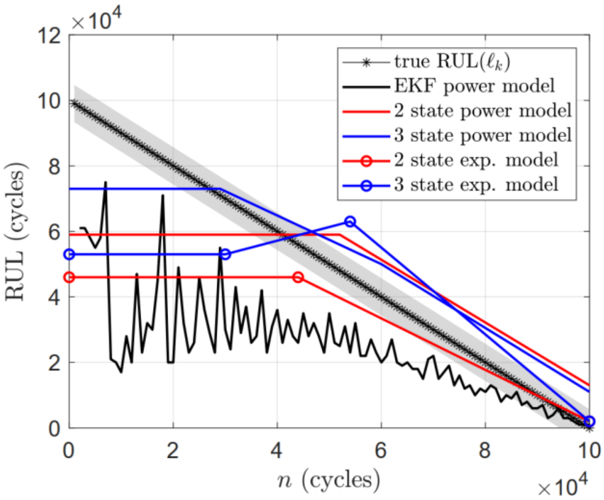

Figure 7. RUL prediction versus magnitude of cycles.

Figure 8 shows the corresponding RMSE using equation (15). Since error is cumulative, the model's performance is conditioned on state's discretisation and its ability to effectively capture the heterogeneity due to any dynamic effects by the switch operation mechanism of HMRM. The cumulative RMSE for 2−3 states of power model are 2.51×10^4 and 1.50×10^4 cycles; of exponential model are 3.12×10^4 and 2.60×10^4 cycles. The RMSE of EKF is 2.86×10^4 cycles.

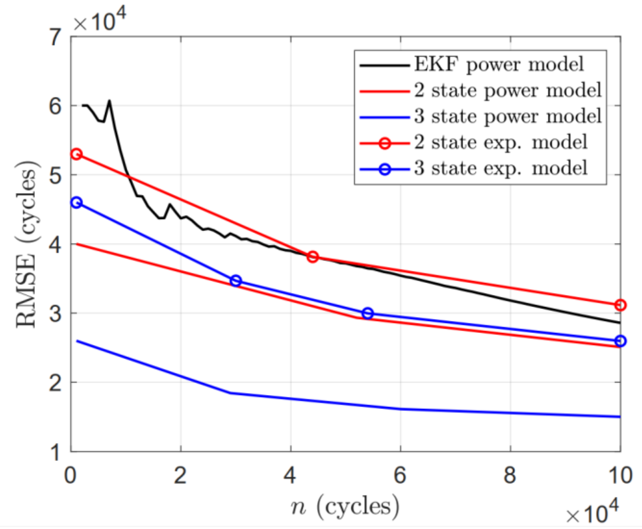

Figure 8. RMSE performance accuracy.

5 STRUCTURAL BEAM DEGRADATION

The second case study presents the corrosion degradation problem of a structural beam (see Figure 9). Sensors are deployed at intervals across the beam length to monitor the degradation process. For this study, synthetic measurements of signals from [20] were also considered which obey the power model in equation 11, with assumed log-normal state distribution and normal multiplicative measurement error as detailed in the mentioned reference.

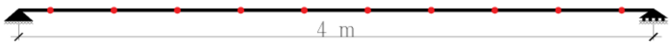

Figure 9. Multi-sensor condition monitoring scenario of a structural beam under progressive spatial varying damage accumulation, specific to the red "dotted" location.

Figure 10 illustrates the degradation distribution of the 10 sensor locations which were placed at 400mm sensor-sensor, numbered from left to right. Each degradation data point approximates the degradation level per year.

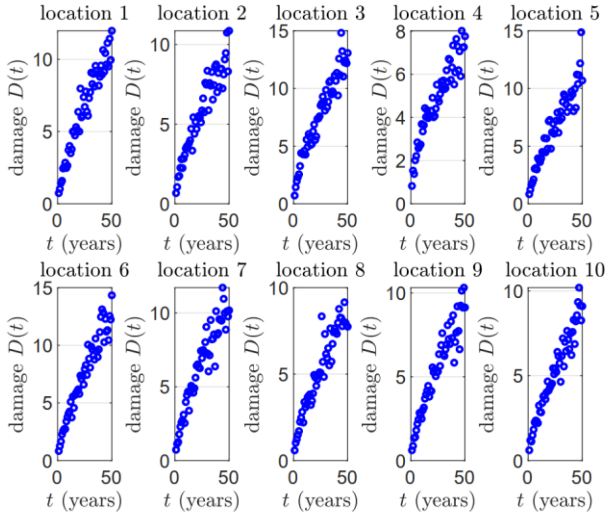

Figure 10. Synthetic sensor-based monitoring datasets of a structural beam subjected to damage accumulation. Observed dataset over the 10 sensor locations.

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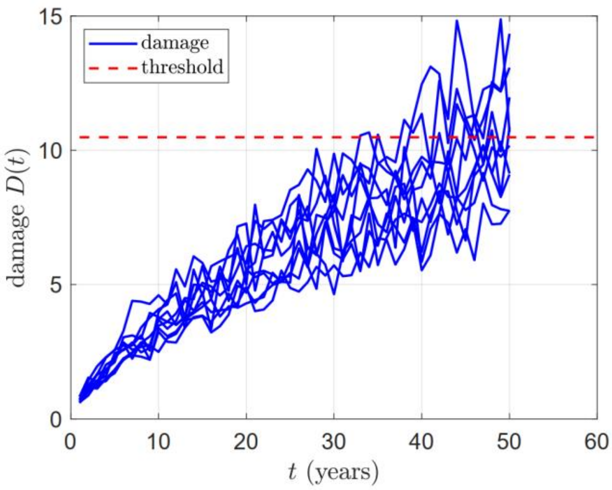

Figure 11. Combined distribution of the datasets.

As in the previous example, the sensor signals are inspected in 25 years' intervals until failure, when available, which was observed to be a minimum data points for a 2-state M-HMRM regression parameter estimation problem, according to the model. The same procedure of sequential updating of degraded sensor measurements applies to track the RUL change with time.

In Figure 11, the observed degradation in equation 7 is a $t \times 10$ matrix for t −year cycle, which is used to estimate the degradation parameters by adopting the decision-level fusion in equation 9. A $t \times 1$ predicted degenerate slope is obtained and the accompanying PDFs of RUL for 2 and 3-state M-HMRM are also obtained. The sensitivity of the concave degenerate slope is captured in the PDF distribution, in the hidden states, respectively. Whereas, a mean distribution of the j^{th} sensors is the assumed observation for the EKF.

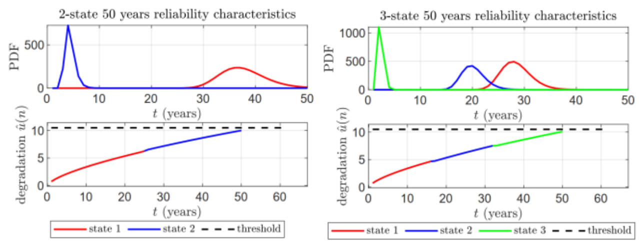

Figure 11. Multi-Sensory monitoring of a structural beam based on 2-state (left) and 3-state(right) segmentation process.

Figure 12 describes the RUL prediction at different times and its corresponding prediction performance accuracy. The prediction performance is influenced by the number of states used per observed cycle. The result shows that the 3-state predicted RUL converges better towards the actual RUL, consequently posing a better confidence of RUL prediction. The RUL for The RMSE for the prediction performance accuracy of both models is also presented. Figure 13 shows the cumulative RMSE for 2 and 3 states to be 10.47 and 10.94 years which is twice more accurate, in absolute terms, than 20.09 years of EKF.

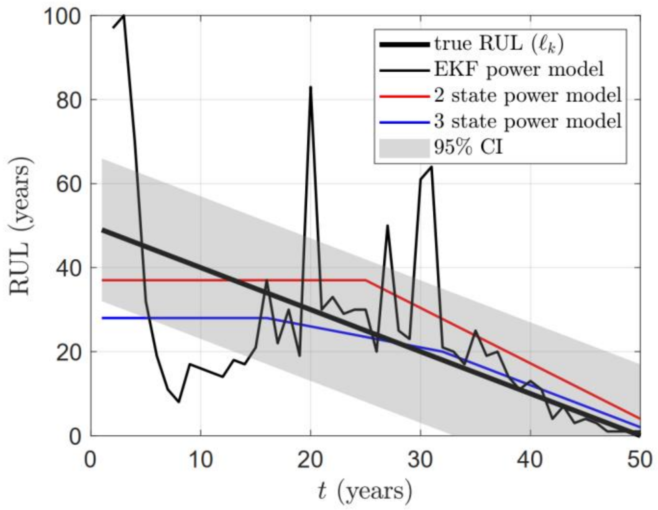

Figure 12. RUL prediction versus duration of degradation.

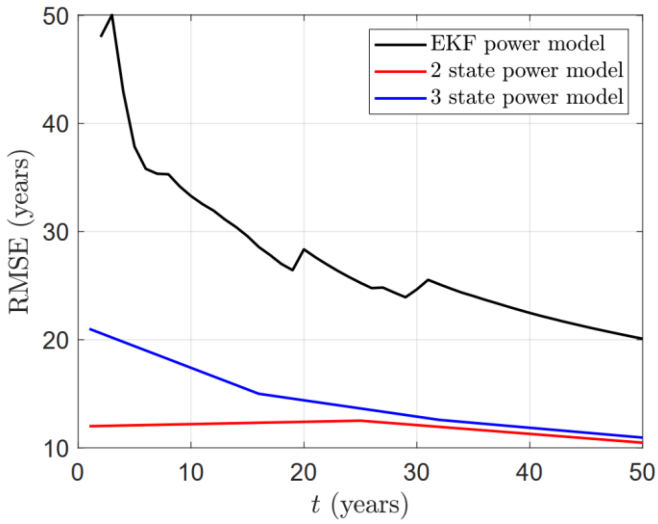

Figure 13. RMSE performance accuracy.

LIMITATION IN PRACTICAL APPLICATION

The models discussed in this paper are generalized empirical power models to approximate self-accelerating crack growth behaviour and corrosion damage in a structural beam. While corrosion in reinforced concrete beam obey empirical power laws [22], crack growth may require differential or rate-based power laws, such in Paris or NASGRO equation [23], to capture multiple parameters (or uncertainties) which characterizes real-world systems. Crack growth phenomenon are influenced by multiple factors (or uncertainties) such as, material variability, environmental, and operational effects. Provided that these laws can be approximately linearised and adapted into hidden-state process to quantify these parameters or uncertainties, a closed form solution of it FHT to predict the remaining useful life distribution can be obtained. Although, the solution may not follow closely a similar trend as discussed in this paper, but the principle is the same.

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CONCLUSIONS

A framework to estimate the RUL of bespoke components was presented and illustrated on two examples. A regression-based hidden Markov model was proposed for degradation process prediction and two power failure functions were used to illustrate the state-dependent degradation scenarios for each problem. A case of multiple sensor measurements has also been presented.

Both cases presented address the situation where the system's prior lifetime dataset is unavailable, which makes it difficult to estimate RUL using traditional methods. The observations are modelled as sets of data that become available, and the parameters of the failure function are updated. Finally, the PDFs of RUL can be evaluated based on the re-estimated parameters of failure function, for each state. Compared to extended Kalman filter method, the model demonstrates its ability to estimate the RUL of a structural component in the absence of failure history.

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