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# Manifold-Based Diffusion Models for Generating Synthetic Motor Imagery EEG

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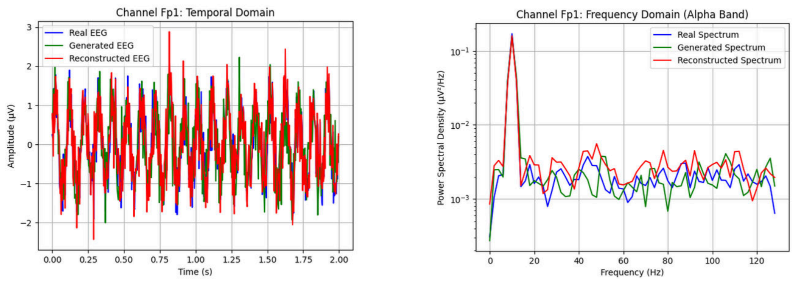
*Introduction:* Generating realistic synthetic EEG can alleviate dataset size, privacy, and session variability concerns in motor imagery (MI) brain–computer interfaces. Conventional diffusion-based models risk off-manifold artifacts, ignoring the manifold structure of EEG in space and time[1]. We employ *Symmetric Positive Definite* (SPD) matrices derived from EEG covariance to capture electrode correlations. Constraining diffusion on this SPD manifold yields higher-fidelity reconstructions and generated signals for MI tasks[2].




(a) Time-domain comparison of Real (blue), Generated (green), and Reconstructed (red) EEG signals for Channel Fp1.

(b) Spectral comparison of Real (blue), Generated (green), and Reconstructed (red) EEG signals, emphasizing alpha-band retention.

Figure 1: Combined time-domain and spectral comparisons of real, generated, and reconstructed EEG signals for Channel Fp1.

*Material, Methods, and Results:* We used the BCI Competition IV dataset 2a (BCIC-IV-2a), which includes four-class motor imagery EEG from nine subjects, with 288 trials per session recorded over 22 electrodes at 250 Hz. Preprocessing steps involved noise reduction, down-sampling to 128 Hz, and segmenting 3.5-second epochs, producing $22 \times 22$ covariance matrices on the SPD manifold. A forward stochastic differential equation (SDE) was defined,

$$d\mathbf{z} = f(\mathbf{z}, t)\, dt + g(\mathbf{z}, t)\, d\mathbf{w},$$

where $\mathbf{z}$ represents latent SPD coordinates, and $f(\cdot), g(\cdot)$ regulate manifold-constrained noise injection. The reverse-time SDE was approximated by a neural network using mean squared error and cosine similarity, incorporating *log* and *exp* mapping operations alongside a multi-scale diffusion strategy. Synthetic SPD outputs were mapped back to time-domain EEG through Cholesky factorization. Figures 1a and 1b demonstrate that the generated signals preserve amplitude fluctuations and alpha-band peaks (10–12 Hz), improving classification accuracy and reducing Fréchet distance compared to Euclidean baselines.

*Discussion and Significance:* Constraining diffusion to the SPD manifold minimizes off-manifold artifacts and preserves critical temporal and spectral features of the MI EEG. The overlap between real and synthetic signals demonstrates the retention of inter-electrode correlations and alpha oscillations, making this approach valuable for data augmentation in small-sample or privacy-restricted scenarios. Future work could refine Riemannian metrics, extend to multi-class MI tasks, and validate on larger clinical datasets, solidifying manifold-based diffusion as a reliable method for generating realistic EEG data.

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