ONE-DIMENSIONAL CRITICAL VELOCITY FORMULATION – AN ASSESSMENT OF THE DEFICIENCY OF THE CURRENT MODELS, AND THE INTRODUCTION OF A NEW CONCEPT

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ABSTRACT

A review of the most classic set of critical velocity (V_c) equations in the tunnel ventilation industry is presented. Assumptions, and limitations in the derivation of V_c are explained. A new methodology is presented, which takes into account the fundamental laws of physics described in the conservation of mass, momentum, energy, and combustion. Simple, but robust enough to capture the different velocities, temperatures and mixing of gases to calculate the minimum velocity that is required to control the backlayer of smoke in the event of a tunnel fire. The results presented are in agreement with expectations – lower fire heat release rate, lower Vc; higher fire heat release rate, higher Vc. The new methodology results do not asymptote to 3.5 m/s, as the classic Vc set of equations point, but progressively increases Vc as the fire heat release rate increases. Geometrical aspects of the tunnel are taken into account to provide a proper set of results.

Keywords: critical velocity, tunnel ventilation, fire modelling

1. INTRODUCTION

The classic critical velocity (V_c) was developed in the mid 1970's as part of the SES Handbook [1]. Danziger and Kennedy [2] presented its application for the first time in 1982; and Kennedy, Gonzalez, and Sanchez [3] presented the derivation of these equations in 1996 at ASHRAE for the first time. The following year, Kennedy [4] edited the paper outlining the derivation [3] and added information about the Memorial Tunnel Fire Test Program. After being published in 1996, these equations were implemented as an Annex into NFPA 502 for information. Up until 2014, NFPA 502 used the formulation presented in the 1996 publication by Kennedy et al. [3]. In 2017, NFPA 502 Annex included a revision to the equations varying the Froude Number (*Fr)* as a function of the Fire Heat Release Rate (FHRR). Y.Z. Li, and H. Ingason [5] presented a discussion where they found some problems with the critical Fr_c. In 2020, NFPA 502 issued a revised set of equations that were very controversial, which led NFPA 502 to pull out all calculation methods for V_c. Stacey and Beyer [6] presented an argument that researchers have not yet captured the physics and geometry. In 2022, NFPA 502 issued an Annex that did not endorse any method, but called out the 2014, 2017 versions, and CFD, and described the subject open to research.

Shi, De Los Rios and Lopez [7] presented an argument that modifying *Fr* as a function of the FHRR would increase the ventilation capacity by 50%, and they called it "a penalty". But the fact is that physics is not politics, and we should focus on the physics that would give us the right answer first. We need the facts before we assess the risk.

2. CLASSIC CRITICAL VELOCITY

Kennedy et al. [3] presented the derivation of the SES V_c equations in 1996. A summary of the derivation is presented as a review here to lay the foundation for discussion.

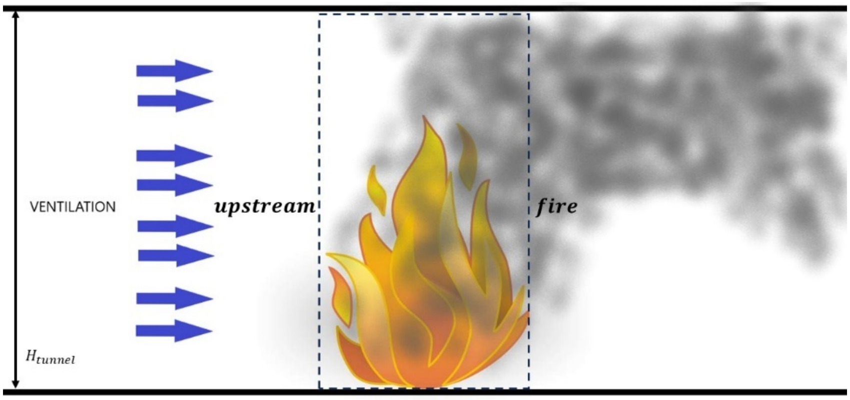

Figure 1: Classic V_c control volume.

The starting point was based on a Fr a scale modeling approach, defined as:

$$Fr = \frac{gravitational\,force}{intertial\,force} \tag{1}$$

Density, ρ, was introduced to address buoyancy effects

$$Fr = \frac{gH(\rho_{upstream} - \rho_{fire})}{\rho_{upstream} V^2} = \frac{gH}{V^2}\left(1 - \frac{\rho_{fire}}{\rho_{upstream}}\right) \tag{2}$$

which was converted into temperature relationships derived from the gas laws

$$Fr = \frac{gH}{V^2}\left(1 - \frac{T_{upstream}}{T_{fire}}\right) \tag{3}$$

Based on the control volume shown in Figure 1, using only the Convective FHRR ($\dot{Q}_c$), the temperature rise is calculated to be

$$\dot{m}C_p T_{upstream} + \dot{Q}_c = \dot{m}C_p T_{fire} \tag{4}$$

Assuming constant properties - ρ, mass flow rate ($\dot{m}$), and specific heat (C_p), the approaching temperature is formulated as a function of the $\dot{Q}_c$

$$T_{upstream} = \frac{\dot{m}C_p T_{fire} - \dot{Q}_c}{\dot{m}C_p} \tag{5}$$

which is then introduced into Equation 3 and a Fr as a function of $\dot{Q}_c$ and T_{fire} is formulated.

$$\mathrm{Fr} = \frac{gH\dot{Q}_c}{\dot{m}C_p T_{fire} V^2} \tag{6}$$

Assuming ρ constant, the mass flow rate is converted in terms of air velocity and tunnel area (A), yielding,

$$Fr = \frac{gH\dot{Q}_c}{\rho C_p A T_{fire} V^3} \tag{7}$$

For all practical purposes, the critical Fr (Fr_c) is now correlated with V_c

$$Fr_c = \frac{gH\dot{Q}_c}{\rho C_p A T_{fire} V_c^3} \tag{8}$$

Introducing a constant, K_{grade}, to address the slope effect on the fire, the V_c is formulated as

$$V_c = K_{grade}\left(\frac{gH\dot{Q}_c}{\rho C_p A T_{fire} Fr_c}\right)^{1/3} \tag{9}$$

Kennedy et al. [3], recommended the conservative value for Fr_c be 4.5. This leads to

$$V_c = 0.61K_{grade}\left(\frac{gH\dot{Q}_c}{\rho C_p A T_{fire}}\right)^{1/3} \tag{10}$$

To calculate T_{fire}, still assuming constant properties, the following is carried out.

$$\rho A C_p V_c T_{upstream} + \dot{Q}_c = \rho A C_p V_c T_{fire} \tag{11}$$

$$T_{fire} = \frac{\dot{Q}_c}{\rho A C_p V_c} + T_{upstream} \tag{12}$$

Equations 10 and 12 constitute the classic V_c equations used since 1975.

3. FROUDE NUMBER DERIVATION

Based on a traditional fluid mechanics theory definition, as shown in Szirtes [8],

$$Fr = \sqrt{\frac{inertial\ force}{gravitational\ force}} \tag{13}$$

Under this definition,

$Fr < 1$ is subcritical and waves move upstream,
$Fr = 1$ is critical and waves moves with the bulk fluid,
$Fr > 1$ is super critical and waves move downstream.

First, let us review what "inertial force" (ma) is. It is the force that a mass has to maintain its motion or rest position. Second, let us review what "gravitational force" (mg) is. Quite often confused with buoyancy force, gravitational force is the force that pulls all masses downwardly towards the center of the earth, while buoyancy force is the force that pushes any mass upwardly (against the gravitational vector direction) and keeps this mass afloat fluids.

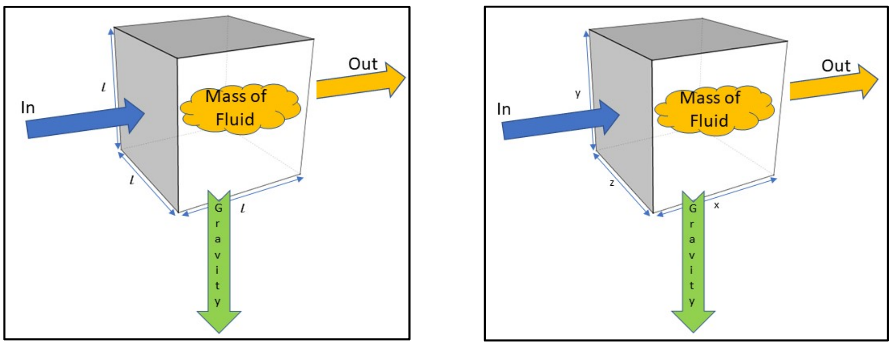

Figure 2: (a) left schematic assumes all dimensional length are l; (b) right schematic assumes dimensions following the coordinate system x,y,z.

Let us examine a small fluid control volume of dimensions x, y, and z, and a horizontal velocity u that travels from *in* to *out*, to evaluate the terms for Fr (Equation 13). If we assume a uniform length scale for all dimensions, l, as shown in Figure 2(a), we lose directionality, as shown in Equation 14, and l can take either x, y, or z.

$$Fr = \sqrt{\frac{ma}{mg}} = \sqrt{\frac{\rho l^3 a}{\rho l^3 g}} = \sqrt{\frac{\rho l^3 \frac{V}{t}}{\rho l^3 g}} = \sqrt{\frac{\rho l^3 V \frac{V}{l}}{\rho l^3 g}} = \sqrt{\frac{\rho V^2}{\rho l g}} = \sqrt{\frac{V^2}{lg}} = \frac{V}{\sqrt{lg}} \quad (14)$$

If we consider the planes where the forces act upon - y, and z for direction of motion, and x, and z for direction of the gravitational pull, following Figure 2(b), the correct Fr is thus derived. The fluid is moving along the, as in tunnel ventilation. The inertial force acts perpendicular to the gravitational force. This leads Equation 15 to define the Fr with the length scale being the x direction, parallel to the motion. William Froude derived Fr to quantify the resistance of floating objects when navigating at a given speed. Under his application, the length scale was the length of the ship, with water and ship travelling in parallel in the *x-axis,* thus the length scale is in the length of the ship. Furthermore, the formulation presented in Equation 16 becomes the correct form.

$$Fr = \sqrt{\frac{ma}{mg}} = \sqrt{\frac{\rho xyza}{\rho xyzg}} = \sqrt{\frac{\rho xyz \frac{V}{t}}{\rho xyzg}} = \sqrt{\frac{\rho xyzV \frac{V}{x}}{\rho xyzg}} = \sqrt{\frac{\rho yzV^2}{\rho xyzg}} = \sqrt{\frac{V^2}{xg}} = \frac{V}{\sqrt{xg}} \quad (15)$$

$$Fr = \frac{V}{\sqrt{xg}} \quad (16)$$

4. FIRE HEAT RELEASE RATE, REACTANTS, AND PRODUCTS

Figure 3 outlines the split of the masses, velocities, and temperatures in simplicity: upstream, into the fire and out to the fire chamber, the annular bypass, and the downstream mixture chamber. The total fire heat release rate ($\dot{Q}_t$) is a result of the combustion of the reactants. Sanchez [9] described in detail the non-stoichiometric effect fires have. There are three regions of the combustion process: lean, stoichiometric, and rich. This is assessed via the equivalence ratio, ϕ, which is defined to be:

$$\phi = \frac{AFR_{stoich}}{AFR_{actual}} \quad (17)$$

$\phi<1$ *lean* (lighter blue color, no CO, no soot, no radiation) (18)
$\phi=1$ *stoichiometric* (dark blue color, no CO, no soot, no radiation) (19)
$\phi>1$ *rich* (orange color, CO, soot, radiation) (20)

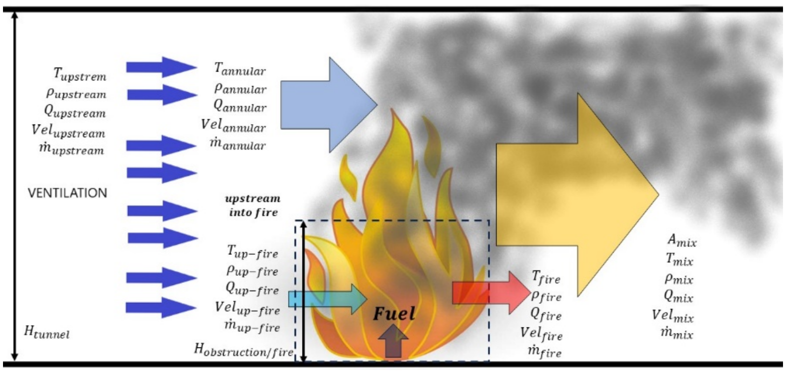

Figure 3: Full V_c Fire Control Volume

Sanchez identified the effect ϕ has on $\dot{Q}_t$ and its properties. For tunnel fire applications, the fires are classified as rich because they generate a flame orange in color, generate soot, CO, and radiation. Fresh air comes down the tunnel from the inlet (upstream), the upstream airflow will split into airflow to sustain the fire and the rest will be bypass airflow to push the fire plume. Using Equation 21, the products of combustion are calculated. For this one-dimensional analysis, it assumed that all products of combustion are fully mixed and all together represent the smoke from the fire. In a similar way, when the annular airflow mixes with the fire smoke, it is assumed that the mixture is fully mixed and this one body of mass will be the smoke plume the tunnel ventilation system is trying to control.

$$1\ kg\ fuel + AFR\ kg\ air = (1 + AFR) kg\ products \tag{21}$$

Prescribing the design $\dot{Q}_t$, the fuel consumption rate is calculated.

$$\dot{m}_{fuel} = \frac{\dot{Q}_t}{\chi\ \Delta H_c} \tag{22}$$

χ is defined to be the fuel burning efficiency for the fire process. It shall be noted that $\dot{Q}_t$ is composed of two components – convection and radiation.

$$\dot{Q}_t = \dot{Q}_c + \dot{Q}_r \tag{23}$$

For the time being, we shall prescribe the percentage radiation fraction. Only the convective term will be used to derive the critical velocity. The upstream mass flow rate is calculated by defining the upstream velocity and ρ, while the annular velocity is derived from the upstream mass flow rate minus the mass flow used to sustain the fire.

$$\dot{m}_{upstream} = (\rho VA)_{upstream} \tag{24}$$

$$\dot{m}_{up-fire} = (AFR_{actual})\dot{m}_{fuel} \tag{25}$$

$$\dot{m}_{annular} = (\rho VA)_{annular} = \dot{m}_{upstream} - \dot{m}_{up-fire} \tag{26}$$

$$\dot{m}_{fire} = (1 + AFR_{actual})\dot{m}_{fuel} \tag{27}$$

$$\dot{m}_{mix} = (\rho VA)_{mix} = \dot{m}_{annular} + \dot{m}_{fire} \tag{28}$$

$T_{annular} = T_{upstream}$ are known. T_{flame} is the actual temperature of the flame and calculated based on ΔH_c, $\dot{Q}_t$, and kinetics (not discussed in this paper, but it is a needed parameter). T_{flame} has to be converted into the effective convective temperature, T_{fire}, that mixes with the annular mass flow by multiplying it by the ratio of $\dot{Q}_c$ to $\dot{Q}_t$. Therefore, the temperature of the fire gases after mixing the fire plume with the upstream annular mass flow is calculated as follows:

$$T_{mix} = \frac{(\dot{m}c_pT)_{annular} + \left[\left(\dot{m}c_pT_{flame}\left(\dot{Q}_c/\dot{Q}_t\right)\right)_{fire}\right]}{(\dot{m}c_p)_{mix}} \tag{29}$$

It should be noted that *Cp* has to be the corresponding values at the local temperature of the various mass flows. As a result of the conservation of mass and energy, ρ is computed as follows:

$$\rho_{upstream} = \frac{101325}{RT_{upstream}} \tag{30}$$

$$\rho_{annular} = \frac{101325}{RT_{annular}} \tag{31}$$

$$\rho_{fire} = \frac{101325}{RT_{fire}} \tag{32}$$

$$\rho_{mix} = \frac{101325}{RT_{mix}} \tag{33}$$

And consequently, the velocities are calculated as follows:

$$V_{upstream} = \frac{\dot{m}_{upstream}}{\rho_{upstream} A_{upstream}} \quad (34)$$

$$V_{annular} = \frac{\dot{m}_{annular}}{\rho_{annular} A_{annular}} \quad (35)$$

$$V_{fire} = \frac{\dot{m}_{fire}}{\rho_{fire} A_{fire}} \quad (36)$$

$$V_{mix} = \frac{\dot{m}_{mix}}{\rho_{mix} A_{mix}} \quad (37)$$

These lay out all the parameters derived for the fire scenario. It should be noted that for fires, *V,* ρ*,* and *Cp* are not constant due to the conservation equations of mass and energy. They vary as a function of local temperature.

5. CLASSIC CRITICAL VELOCITY DEFICIENCIES

The following are deficiencies in classic critical velocity derivation:

1. The definition of *Fr* used by Kennedy et al. [3] is not consistent with the standard fluid dynamics definition [8]. As it can be seen, Equation 1 is the inverse of Equation 13, and already squared. Furthermore, following Equation 13 convention, and taking the square root of 4.5, the value for Fr_c would have been 2.1213, which then inverting this value to concur with Equation 13, it would yield Fr_c =0.4714, which indicates that the value is a subcritical value that would allow the wave (backlayer) move upstream. The value desired should be >1.0.
2. The tunnel height is used as the length scale (Equation 2). But Equation 15 shows that *Fr* depends on the length scale parallel the motion of the fluid, *x*.
3. *Fr* is not a function of the buoyancy force; ρ does not appear in its standard definition (Equation 16). Equation 2 introduced density ratios to address buoyancy effects, but as shown on Equation 16, *Fr* is a function of the gravitational force (*mg*); not the buoyancy force (ρg). Moreover, *Fr* is not a function of any fluid density ratio. Kennedy et al. added an extra term K_{grade} to address the effect the slope has on the buoyancy forces. *Fr* does not depend on slope. In a similar way, it should be pointed out that *Fr* is not a function of any $\dot{Q}_c$.
4. Equation 4 is not correct. ρ and C_p vary with temperature. It does not account for the mass of the fire, which leads to miscalculate the velocity.

Fr should not be used to derive the critical velocity as Kennedy et al. [3] so derived it. Li and Ingason [5] identified problems with the critical value of Fr_c=4.5. The above description presents evidence why Li and Ingason were having problems validating Fr_c=4.5.

In 2017, NFPA 502 presented values for *Fr* based on $\dot{Q}_c$. But $\dot{Q}_c$ is not part of the *Fr* fundamentals. The Fr_c values are presented in the Table 1 below. These values are misleading. They are all subcritical.

In 2020, NFPA 502 presented a curve fit trying to match a plot from the Memorial Fire Test. Such correction was not based on any physics, and thus the controversy was created.

Table 1: NFPA 502 -2017 Annex D – Range of K1 values for various $\dot{Q}_c$

$\dot{Q}_c$ (MW)	K1	Fr_{cSES}	$\sqrt{Fr_{cSES}}$	$Fr_{\text{fluid mech}}$	Flow Regime
>100	0.606	4.493	2.120	0.472	Subcritical
90	0.620	4.196	2.048	0.488	Subcritical
70	0.640	3.815	1.953	0.512	Subcritical
50	0.680	3.180	1.783	0.561	Subcritical
30	0.740	2.468	1.571	0.637	Subcritical
<10	0.870	1.519	1.232	0.811	Subcritical

6. NEW CRITICAL VELOCITY FORMULATION

A new critical velocity methodology is presented in this paper founded on the conservation of mass, momentum, energy, and combustion in a simple one-dimension. Equations above (17 through 37) show how to calculate the various mass flow rates, velocities, temperatures. Equation 39 represents the conservation of momentum. Many researchers and engineers have focused on the buoyancy component. Ingason and Li [10] has expressed that the slope effects are overestimated using NFPA 502. That is correct. Based on Equation 39, that component is very small. The component that is missing is the throttling effect the temperature rise has on the flow. Adding this factor into Equation 39 makes all the difference. Equation 39, in conjunction with Equations 17 through 37, represent the final critical velocity equation.

$$(\rho A V^2)_{annular} = (\Delta P_{throttle} A)_{annular} + \sin(\alpha)\ F_{buoyancy} \quad (38)$$

$$(\rho A V^2)_{annular} = \left(1 - \frac{T_{annular}}{T_{mix}}\right)\left(\frac{g\rho H A}{2}\right)_{annular} + \sin(\alpha)\ g\ m_{mix}\left(\frac{\rho_{annular} - \rho_{mix}}{\rho_{mix}}\right) \quad (39)$$

Where m represents mass, g the gravitational constant, and $\sin(\alpha)$ the sine function of the angle of the slope.

7. SAMPLE CALCULATIONS

Calculations were performed with the following assumptions: $\dot{Q}_t$.= 20 MW, $\dot{Q}_c = 0.7\dot{Q}_t$, χ=0.7, ΔH_c=20MJ/kg, ϕ= 1.25, AFR_{stoich}= 12.9, $T_{annular} = T_{upstream} = 25C$, and $T_{fire} = 1153\ C$ (This value has been calculated based on ΔH_c, $\dot{Q}_t$, and kinetics; not discussed in this paper, but submitted as an input parameter). The area of the fire obstruction = 3.38 m^2 (Height=1.5m, Width=2.25m). The tunnel area is as the Height and Width shown in the figures.

Figure 4 plots the V_c for various $\dot{Q}_t$ (10, 20, 30, 50, 75, and 100MW) as a function of various slopes (-10%, -5%, 0%, +5%). The plot shows that V_c increases both as $\dot{Q}_t$ increases, and as the slope increases downward; not like the classic V_c equation, which leads to a limit of 3.5 m/s for all slopes and $\dot{Q}_t$. The increase of $\dot{Q}_t$ as the upstream tunnel ventilation rate increases is in agreement with Li et al. [11], which states that "the fire growth rate increases with the ventilation velocity". But I would like to restate the statement – to sustain a higher $\dot{Q}_t$, the upstream airflow rate must be increased; otherwise, there will not be enough air and $\dot{Q}_t$ cannot increase more than the upstream airflow rate can support.

Figure 5 plots the variation of V_c at various tunnel H/W aspect ratios. The results show that V_c increases as the tunnel becomes narrower, and as the tunnel height increases.

Figure 6 plots the variation of V_c as a function of the radiation factor. The results are as expected. As the radiation fraction increases, the convective fraction decreases, and the V_c decreases as well. Between 0% to 40% radiation factor, the V_c decreases by 0.5 m/s.

8. CONCLUSION

This paper has identified that the classic V_c set of equations derived by Kennedy et al. [3] in 1996, and used throughout the world since, have some fundamental limitations due to the way Fr was defined, the value for Fr_c, and the implementation of constant properties. The new methodology presented in this paper has demonstrated how to account for all the physics involved in the determination of the critical velocity. The new methodology is consistent with expectations – low $\dot{Q}_t$, low V_c is required; high $\dot{Q}_t$, high V_c is required. Although still a one-dimensional approximation, the effect of properties as a function of temperature helps identify that the velocities downstream are greater because of lower ρ, and higher temperature. This paper brings the tunnel ventilation industry to a closer understanding on how to determine ventilation requirements based on a one-dimensional V_c approximation.

9. ACKNOWLEDGEMENTS

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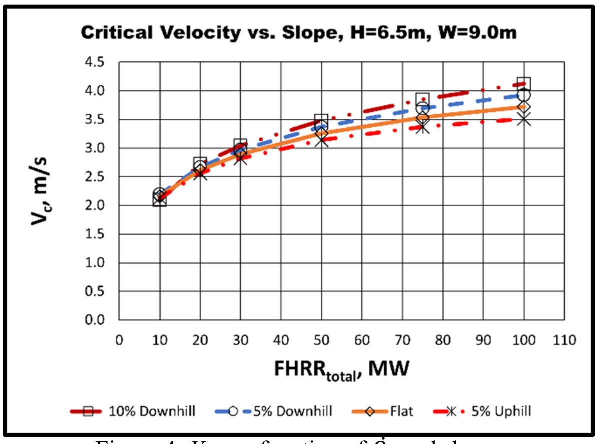

Figure 4: V_c as a function of $\dot{Q}_t$ and slope.

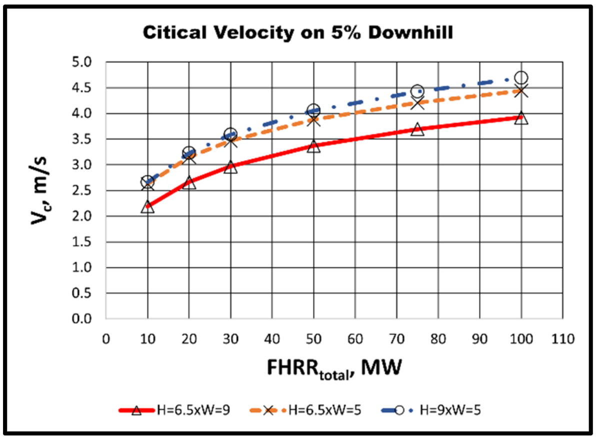

Figure 5: V_c as a function of $\dot{Q}_t$ and various tunnel H/W aspect ratios.

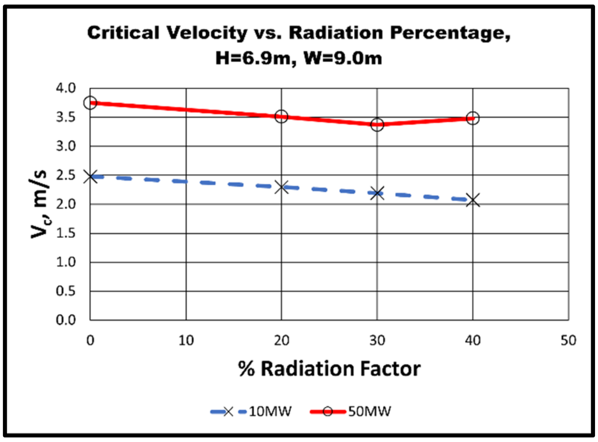

Figure 6: V_c as a function of % radiation factor.