INFORMATION, COMMUNICATION & COMPUTING

Fields of Expertise TU Graz

Source: istockphoto.com

Kay Uwe Römer,
Information, Communication & Computing
Source: Lunghammer – TU Graz

The second TU Graz Science Day will take place on September 21, 2022. This year the focus is on Digitalization, so the FoE ICC in particular will be playing a leading role. The opening keynote will be given by Lothar Thiele, who is not only a leading researcher, but as the "delegate for digital transformation" of ETH Zurich and one of the initiators of the ETH AI Center, is a leading expert on digitalization. In the afternoon, FoE members will organize sessions with invited presentations of colleagues on a topic related to digitalization. Also, an exhibition is planned, to which FoE members can contribute a poster or demonstrator. It is well worth the effort as we expect around 200 visitors from industry, academia, funding bodies, and politics who are keen to see our research and to get in touch. The day after, on September 22, 2022, the final event of the LEAD project Dependable Internet of Things will be held – the first LEAD project funded by TU Graz whose six-year funding period is now coming to a close. The event will showcase the results obtained during six years of intense research and collaboration. You are all cordially invited to contribute to and participate in these two collocated events; please make a note of these dates in your calendars.

In this edition of TU Graz *research*, Cesar Ceballos, postdoc at the Institute of Geometry, shares his *research* on combinatorics and discrete geometry with us. Enjoy reading!

Cesar Ceballos

Combinatorial and Geometric Structures

It is widely recognised that many of the most exciting advances in mathematics rely on building connections between various areas of research, with ideas from one field enriching those of another. Ceballos's research builds on the fruitful interaction between combinatorics and discrete geometry, and focuses on problems motivated by their connections to other fields.

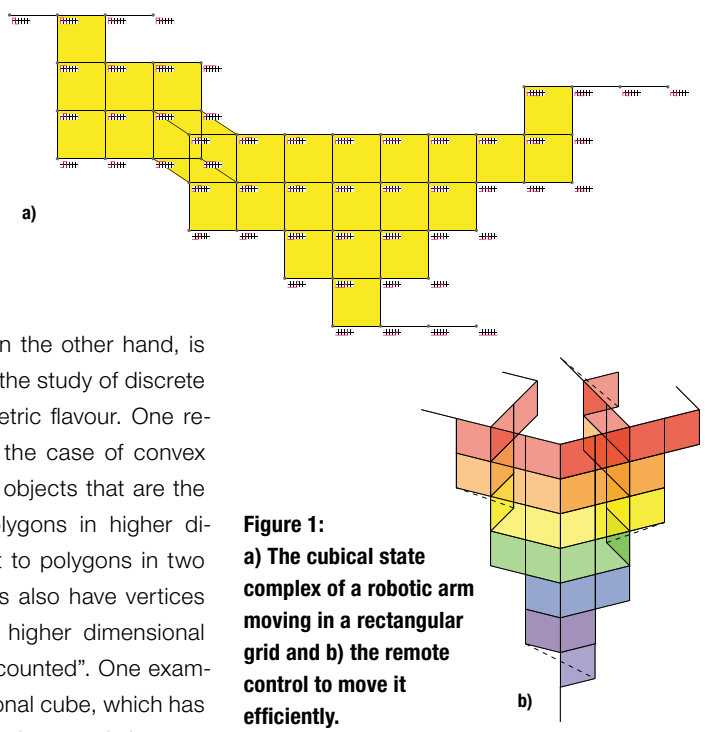


Figure 1:
a) The cubical state complex of a robotic arm moving in a rectangular grid and b) the remote control to move it efficiently.
Source: Federico Ardila and Cesar Ceballos

Combinatorics is an active area in mathematics that deals with the study of discrete structures. Rather than "continuous", discrete structures are built from objects that can be "counted". This simple but important feature is key for applications to real-world problems, where discrete models can be implemented. Examples include scheduling problems, railway planning, networks, cell phone communications, Google Maps, developing of computer algorithms, and many more.

Discrete Geometry, on the other hand, is a field that deals with the study of discrete structures of a geometric flavour. One remarkable example is the case of convex polytopes, geometric objects that are the analog of convex polygons in higher dimensions. In contrast to polygons in two dimensions, polytopes also have vertices and edges, but also higher dimensional faces which can be "counted". One example is a three dimensional cube, which has eight vertices, twelve edges, and six >

squares (two dimensional faces). The theory of convex polytopes is a rich theory on its own and has remarkable applications, such as optimisation problems in linear programming and connections to toric geometry, mirror symmetry and string theory.

As mentioned above, discrete structures are useful for applications to real-world problems, and they also appear naturally in scientific contexts such as computer science, pure mathematics and physics. When dealing with a discrete structure, more than counting the objects involved, one is interested in understanding the full structure behind them. One example of Ceballos's previous work is a geometric model for a theoretical robotic arm moving in a two dimensional grid. The main goal was to provide an algorithm to move the robot from any given position to another in the most efficient possible way. In order to achieve this, a full understanding of the structure of the "map" of all possible states of the robot was given, using techniques from CAT(0) cubical complexes in geometric group theory. Other two examples are permutations and associations of an n-element set, which are connected to each other via certain simple transformations. For associations, these transformations play a fundamental role for efficient searching and sorting algorithms and the representation of data in computer science. The resulting structures are well known polytopes in Discrete Geometry called the permutahedron and the associahedron. Counting faces of these polytopes can be unexpectedly used, for example, to invert formal power series under multiplication and composition.

Over the years, the permutahedron and the associahedron have inspired a vast amount of research due to their significant appearance in many different contexts, and have led to numerous connections to diverse fields in mathematics, computer science and physics. These objects are particular cases of a much wider family of discrete structures for which many open problems and connections to other areas arise. Ceballos is the Austrian coordinator of the ANR-FWF project PAGCAP, an international cooperation project between Austria and France which concentrates on a selection of open problems that go beyond the study of the permutahedron and the associahedron. Some problems to be approached within this project include the development of new methods for specific geometric construction models, the study of the space of all possible geometric realizations of certain combinatorial structures, as well as the foundations of free probability and its connections to combinatorial Hopf algebras. Other team members involved in this project are Franz Lehner from the Institute of Discrete Mathematics and Oswin Aichholzer from the Institute of Software Technology at TU Graz.

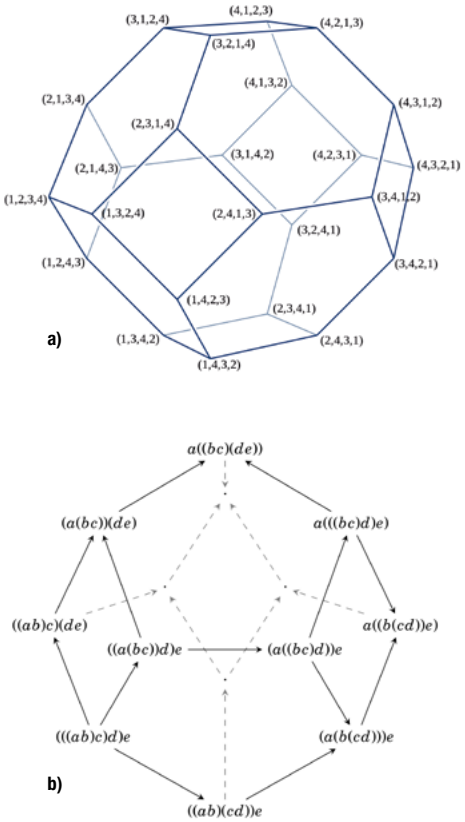


Figure 2: The three dimensional a) permutahedron and b) associahedron.

Source: a) Wikipedia, David Eppstein; b) Wikipedia, Niles Johnson

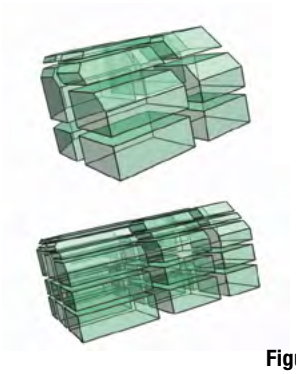
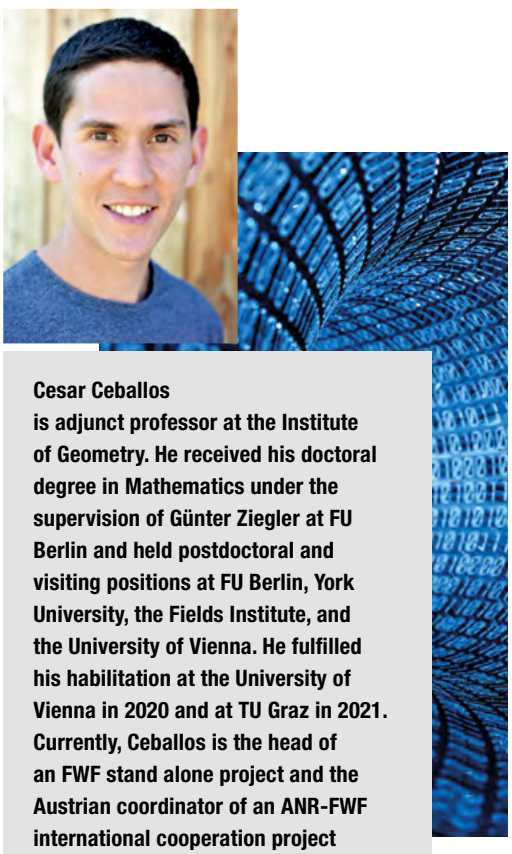


Figure 3: Geometric constructions of v-Tamari lattices using tools from tropical geometry.

Source: Camilo Sarmiento

Ceballos's second FWF project also deals with the study of convex polytopes, as well as their connections to diagonal harmonics in representation theory and applications of Hopf algebras.



Cesar Ceballos is adjunct professor at the Institute of Geometry. He received his doctoral degree in Mathematics under the supervision of Günter Ziegler at FU Berlin and held postdoctoral and visiting positions at FU Berlin, York University, the Fields Institute, and the University of Vienna. He fulfilled his habilitation at the University of Vienna in 2020 and at TU Graz in 2021. Currently, Ceballos is the head of an FWF stand alone project and the Austrian coordinator of an ANR-FWF international cooperation project between Austria and France. The latter involves three institutes from two faculties at TU Graz and five different universities in France. He wishes to obtain a permanent professorship at TU Graz and continue developing a top international research group in combinatorics and discrete geometry.

Source: Stephanie Duus-Glöckner