Valuation of Startups: Capturing Flexibilities Using Real Options

Master Thesis
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Graz, October 2020

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Kurzfassung

Das Ziel dieser Masterarbeit ist die Bewertung von nicht an der Börse gehandelter Start-ups mit Hilfe des Realoptionsansatzes. Der erste Schritt ist eine Literaturrecherche, um die gängigen Startup-Bewertungsverfahren und deren verwendete Berechnungsmethoden zu erfassen. Anschließend werden verschiedene Arten der Realoptionsbewertung analysiert und deren Eigenschaften diskutiert. Basierend darauf, wird ein passender Ansatz gewählt und dessen Verwendung am Beispiel eines Startups gezeigt. Die Analyse der Realoptionsansätze konzentriert sich auf drei Kriterien: die Datenanforderung, die Plausibilität der verwendeten Annahmen sowie der sowie die Berechnungsmethodik. Basierend auf der Analyse wird festgestellt, dass der "MAD“ Ansatz am besten geeignet ist. Die anschließende Anwendung an einem Beispiel zeigt mehrere Probleme auf: Die fehlende Vergleichbarkeit von Startups führt zu Schwierigkeiten bei der Bestimmung des Diskontzinssatzes. Dieser Zinssatz ist wichtig für die Anwendung der Barwertmethode und muss daher auf eine alternative Art und Weise geschätzt werden. Hierzu wird ein innovativer von Zhang entwickelter Ansatz implementiert. Bei der Anwendung des gewählten Realoptionsansatzes in Verbindung mit diesem Modell treten Probleme auf, welche gelöst werden können. Der Wert eines nicht börsengehandelten Start-Up-Unternehmens wird schlussendlich mit dem identifizierten und optimierten Realoptionsbewertungsansatz erfolgreich berechnet.

Abstract

The goal of this master thesis is to value startups which are not traded on a stock market using the Real Option methodology. A literature review is used to analyze the valuation of startups in general and to identify how the uncertain future development path is captured. Different ways of Real Option Valuation are identified and their characteristics discussed. Based on this insight, one approach is chosen and the operationalization of this approach on a startup example is shown. The selection process indicates that the choice of the Real Option approach is mainly driven by the data availability, the assumptions underlying the different Real Option Valuation approaches and the calculation methodologies. The “MAD” approach is detected to be the most applicable one. The subsequent application on an example shows, that an absolute disconnect from the market requires an optimization of the approach. The discount rate has to be estimated using an alternative way through incorporating Zhangs risk adjustment model. Zhangs model and negative cash flows cause problems in valuing a startup using this approach. With some effort, these problems can be solved. The value of a non-exchange-traded startup is successfully calculated using the identified and optimized Real Option Valuation approach.

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1 Introduction

“Runtastic”, “Shpock” or “Bwin” are well-known names in the corporate world. They started out as startups but made their way into respectable sized businesses in little time. Innovative ideas, a solid business model and excellent management propelled their growth and helped to successfully sail through the vast amounts of uncertainties present in their founding and their development. Several governmental and private initiatives for startup funding try to identify the ventures having a bright future ahead and supply them with adequate capital to realize their growth potential. In order to use scarce capital supporting startups as efficient as possible, it is important to detect and support startups which will turn out as future winners, as early as possible. This puts a lot of stress on the valuation of the startups because the future development should be incorporated most accurately given the vast amount of uncertainties present, when looking ahead.

This requires a valuation approach which integrates the flexibilities regarding the development. One potential approach for fulfilling this requirement is the Real Option Approach. This framework originated from the classical option valuation in the finance domain, which allows for flexibility in decision making and values this flexibility which got carried over to corporate decision making as well as valuation.

While reviewing the literature in the field of Real Option Valuation regarding startups, one problem with the existing research became obvious: The startups valued in the literature were mostly listed on the stock market. This opens a gap in research concerning startups which are in an earlier stage where the assumption of being listed on a stock market is far from being realistic. This research tries to use the Real Option framework for valuing startups whose price for ownership portions cannot be easily detected on any market.

The first part will start with an analysis of startups and the uncertainties present in their development, as well as an identification of “state of the art” valuation methods. Several methods are further analyzed in order to detect ways how future development is estimated and incorporated into valuation of a startup using distinct factors for uncertainty. The second part will focus on real options, their methodology and their value drivers. This part is used to identify one or more approaches usable for valuing startups facing the sheer quantity of valuation literature written about real options. In the third part, the valuation approach chosen will be operationalized on a non-exchange-traded startup example, problems in the application of the approach are analyzed and possible solutions for these issues proposed and implemented.

1.1 Research Question

Based on this goal, two research questions are formulated:

- Which Real Option Approach is most suitable in valuing a non-exchange-traded startup?
- How is the identified Real Option Approach operationalized?

1.2 Research Method

In order to answer the research questions, a methodology has to be specified:

A knowledge basis concerning the research area involved is established first. This is conducted with an analysis of the existing literature concerning startups in general, valuation of startups and valuation using real options.

For using a Real Option Valuation on a startup, the field of real options is searched for specific Real Option Valuation approaches. Based on this, a comparative analysis is conducted and a decision is made based on this comparative insight.

To operationalize the valuation, the insights gained by the literature analysis as well as the selected Real Option approach are incorporated. An example of a startup is used to show the application and to adjust the valuation method to fit the valuation subject.

2 Start Ups, Valuation and Risks

2.1 Definition: Startup

The term Start-up is not precisely defined but several traits are commonly associated with it. It is mainly used in describing two things: In a business sense, a newly founded company and in a cultural sense a mentality with which a company operates (Cook, n.d.).

In a business sense, start-up can be classified as a stage in a revenue lifecycle of a company. This can be seen in Illustration 1 below, a start-up is the stage at the beginning and starts usually with founding. The negative revenue zone is particularly dangerous because around 60% of start-ups don't survive more than 5 years, so many never sufficiently make the turnaround into positive numbers at all (Nobel, 2011).

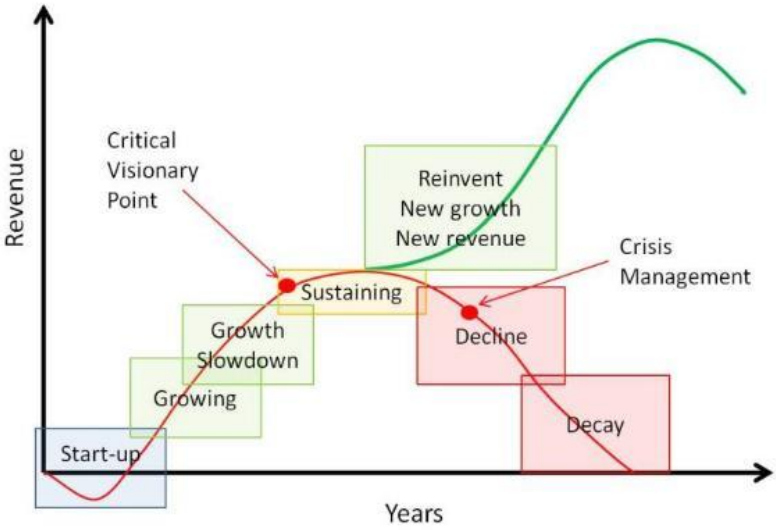

Illustration 1: Business Revenue Lifecycle (Langley, 2017)

An additional definition was given by Robehmed (2013) concerning start-ups which must contain at least three traits to be classified as such:

- Innovative idea
- High growth potential
- International focus

Grbenic (2019) concludes, that there is no common definition of a startup, but quantitative values can be assumed for categorization:

- Organic growth rate of at least 15% per year
- Age of maximum 5 to 6 years up till the IPO
- Risk of failure 35 to 50 % in the first 4 to 6 years
- Located in young industries

The strong positive or negative growth potential leads to sizable uncertainty about the future development of the business. In the Illustration 2 below, different stages of a startup are shown regarding the financing provided in each.

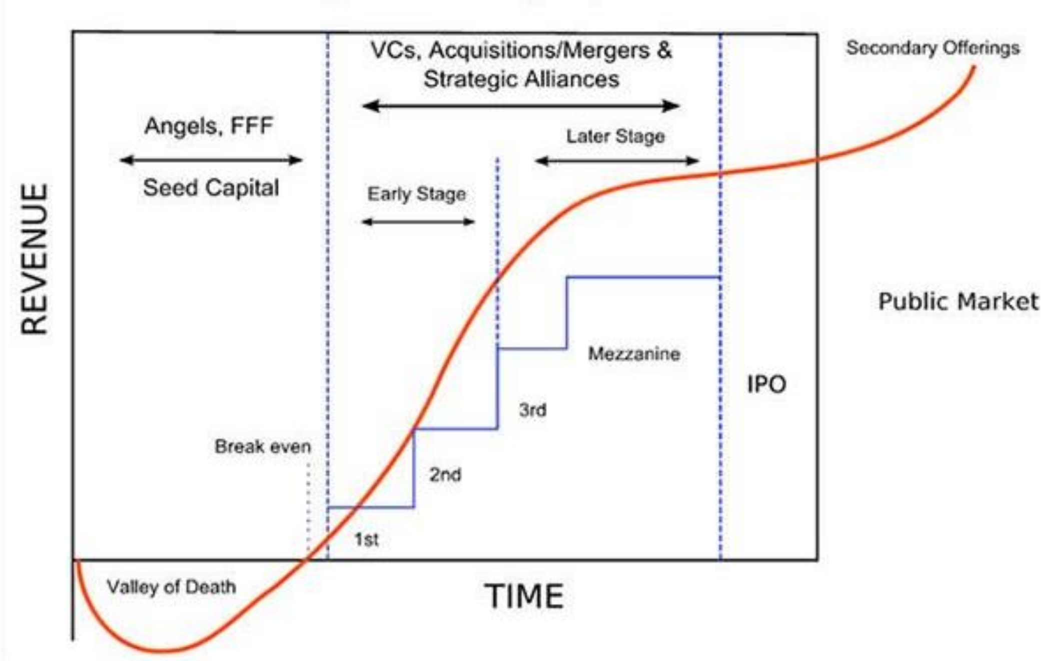

Illustration 2: Startup Financing Cycle (Obedih, 2015)

In the “valley of Death”, the highest risk phase right at the beginning, seed capital is provided in order to set up the start-up. Typically, Angels, a term referring to an investor who provides capital, know-how and network in combination with the founder itself, family and friends (FFF) come into play providing initial capital as visible in Illustration 2. Usually once the startup manages to show some metrics or track record, new types of investors are interested in providing capital, namely Venture Capitalists but also competitors or companies looking for expansion using Merger & Acquisitions or strategic alliances. There can also be several rounds of financing in each stage depending on the capital needs of the start-up. The next milestone of a start-up is the Initial public offering of participation, where parts of the company get sold into the public market in the form of shares on a stock exchange.

A startup begins with no operating history and an innovative, but new, idea. The more a company establishes itself in terms of surpassing the break-even revenue point and further up expanding its operations through establishing a customer base and growing it, the more a company gains credibility and trust from the capital suppliers. For a startup, less risk perceived by capital providers, equals to lower cost sources of capital.

2.2 Uncertainties in Startup Valuation

Defined by Allen, Brealey and Myers (2011), the economic value of an investment is the present value of all future cash flows. In other words: The value of an investment is the sum of all future benefits adjusted for the time value of money. They further elaborate, that valuing through estimating the benefits in the future is a complex topic due to the multitude of factors that have an impact. Following Miloud, Aspelund and Cabrol (2012), valuation goes far beyond pure financial considerations of balance sheets, income statements and financial forecasts. Warren Buffet (1990) writes in his annual letter to shareholders about valuation: "*Clearly, investors must always keep their guard up and use accounting numbers as a beginning, not an end…*" (p.1).

In valuing a newly created start-up, even to start with accounting numbers is tough if not impossible depending on the stage and the short or non-existing financial history. Miloud, Aspelund and Cabrol (2012) conclude that the traditional mainstream valuation approaches like Discounted Cash Flows, earnings multiple method and asset-based valuation only work for companies with financial information of their past existing, which is often not the case at startups. Therefore, depending on the information availability in the different stages of a startup, different valuation approaches are usable. In the seed and early stage of a startup, practitioners and academics developed several techniques for valuing a company using little available information.

The following chapter should draw attention to uncertainties in a startup development which are incorporated into different startup valuation methodologies.

2.2.1 Startup Valuation Approaches

Existing startup valuation approaches were identified in a literature analysis. The purpose is to get a thorough understanding about what uncertainty factors have an impact on the value of a startup and to identify the key impact factors. The literature research identified the following valuation approaches:

- Berkus approach
- First Chicago Method
- Scorecard Method
- Risk Factor Summation Method
- Venture Capital Method
- Valuation by stage
- Schwartz/Moon
- Future Valuation Multiple Method
- Market Multiple Approach
- Comparable Transactions Method
- Discounted Cash Flow Method
- Cost-To-Duplicate Approach
- Liquidation Value
- Book Value

These methods can be divided into two groups based on their origin and usage in valuation: First, the traditional way of valuation according to the Corporate Finance Institute (n.d.) are the Cost-, Income- and Market method and second, non-traditional approaches developed and used valuing startups. Several of the above outlined approaches, although slightly different, use the same basic valuation methods.

Another perspective leads to a categorization into qualitative and quantitative valuation approaches. “Quantitative” means an assessment using measurement and scientific tools, which are based on measurable or countable data reaching to the same result in repeating the method (Collins, n.d.). In contrast, a qualitative approach uses subjective estimation. Traditional valuation approaches are mainly quantitative, in case of limited data availability as often in valuing startups, the use of the traditional approaches is limited (Hand 2005). He emphasizes that a quantitative valuation approach of the financial data is not the most accurate way of valuing a startup, due to the lack of historical data. This leads to a usage of qualitative valuation approach in the earlier stages of a startup where the traditional approaches are of limited value.

Qualitative valuation approaches are useful because they have an explicit way of describing the uncertainty factors which impact the value of a startup. This is important because the identified major uncertainty factors assist in the estimation of the future development of the startup given in the operationalization in chapter 5.

The traditional quantitative approaches identified are well explained in literature and not further discussed here. Non-traditional valuation methods are sorted out if they do not include explicit uncertainty factors in their estimation because otherwise, major impact factors could not be captured.

This shortens the list of the identified startup valuation approaches to the following:

- Berkus approach
- Scorecard method
- Risk Factor Summation Method

For its importance and due to the close connection to the Real Option methodology, the discounted cash flow (DCF) methodology is added to the analyzed and explained methods below.

The Berkus Approach

One qualitative valuation method is called the Berkus approach which was invented by Dave Berkus (2012). At this method, startup value consists of five impact factors, as shown in Table 1 below:

If Exists	**Add to Company Value up to:**
1. Sound Idea (basic value, Product Risk)	$1/2 Million
2. Prototype (reducing Technology Risk)	$1/2 Million
3. Quality Management Team (reducing execution risk)	$1/2 Million
4. Strategic relationships (reducing market risk and competitive risk)	$1/2 Million
5. Product Rollout or Sales (reducing financial or production risk)	$1/2 Million

Table 1: Berkus Valuation Scorecard (Berkus, 2012)

In each category, value is given to the company up to the maximum amount in the right column. The sum-total reachable for a startup equals to $2,5 million. Then each point is subjectively assessed by the analyst according to experience and trust, so basically subjectively judged. This approach is aimed at startups which project to reach at least $20 million in revenue within 5 years. Depending on the location of the startup, the maximum value reachable in each category can be adjusted depending on the geographical environment of the start-up. These adjustment factor are the regional differences detected in the HALO report 2019 by the Angel Resource Institute (2020): Different locations of the startup lead to a different valuation. Berkus (2016) commented on the flexibilities in the methodology:

"*For example, in Silicon Valley, a "big data" startup might competitively call for a $1.5 million maximum value per element, while the same startup in Nebraska might find $500,000 appropriate"* (p.1)

This can be attributed to the stronger demand and competition for deals between investors identified by Payne (2006). This approach is limited to startups before revenues are generated and can lead to widely different valuation outcomes but has the benefit of reaching a result without the need of quantitative data.

Scorecard or Benchmark Method

This method uses slightly different qualitative factors for its valuation. For each of the comparison factors listed below in Table 2 , there is a corresponding possible range of achievement emphasized by Payne (2019).

COMPARISON FACTOR	RANGE	TARGET COMPANY	FACTOR
Strength of Entrepreneur and Team	30% max	125%	0.375
Size of the Opportunity	25% max	150%	0.375
Product/Technology	15% max	100%	0.150
Competitive Environment	10% max	75%	0.075
Marketing/Sales/Partnerships	10% max	80%	0.080
Need for Additional Investment	5% max	100%	0.050
Other factors (great early customer feedback)	5% max	100%	0.050
Sum of Factors			1.075

Table 2: Scorecard Method (Payne 2019)

Each of these factors is subjectively assessed and compared to an “average” startup. The analyst compares it to other startups and subjectively judges based on experience, what the “average” start-up scores for each factor. Table 2 incorporates this comparison in the column “Target Company”, where the startup is graded with a percent factor. The product of “Range” and “Target company” equals the individual factor and the sum of the factors is to be multiplied with the average valuation for the average company in the region:

$$Factor = Range * Target\ Company$$

$$Startup\ Price = Factor * Average\ Price$$

Problematic with this approach is that the subjective judgment leads to differences in valuation results, Payne (2019) adds that it helps to compare companies and to decide whether the startup should be valued rather near the high end or low end of a reasonable range in valuation.

Risk Factor Summation Method

A more detailed method regarding the number of qualitative factors, is the risk factor summation method (Payne, 2011). This method separates the total perceived risk of a startup into different components and assesses each component with a factor between -2, which equals very negative up to 2 for very positive grading. The individual factors are stated below:

- Management
- Stage of the business
- Legislation/Political risk
- Manufacturing risk
- Sales and marketing risk
- Funding/capital raising risk
- Competition risk
- Technology risk
- Litigation risk
- International risk
- Reputation risk
- Potential lucrative exit

The value of one-point equals to \$250 000. The total sum of points times value per point is then added or deducted, depending on the result, from the "average" start-up value in the region:

$$Startup\ value = \text{"standard" startup} + Sum\ of\ points * 250\ 000\ \$$$

This more detailed approach requires the identification of a "standard" startup similar to the "Scorecard" method. The more factors present lead to a better insight about the potential problems a startup might face.

Discounted Cash Flow (DCF)

Dittmann, Maug and Kemper (2002) state that the DCF is the valuation method most commonly used by investors. Reverte, Hernandez and Ramirez (2016) emphasize that for startups valuation methods based on DCF are recommended by prior studies. Further this method is most dominant in the capital budgeting process for companies in several countries. (Bennouna, Meredith and Marchant, 2010).

The DCF approach uses future cash flows (CF) which are generated by the startup. Each year's cash flow is discounted at an appropriate discount rate back to one specific date. At the end of the planning horizon, a terminal value is estimated to capture the value of the company behind the planning horizon. The simple decision process which is inherent in the DCF method is based on a Net Present Value analysis (NPV): If the NPV of cash inflows minus outflows discounted back to the present, is a positive value, the benefits outweigh the costs and the investment should be undertaken. If the NPV value is negative, the investment should not be made because the capital costs surpass the benefits achievable with the investment.

The size of the cash flows and the terminal value are estimated using different forecasting techniques. Traditionally, historical data is used, adapted and extrapolated into the future. An approach which is problematic for startups because of strong non-linear growth and partly negative cash flows.

Mun (2005a) states, that due to the uncertainties of point estimates of cash flows, stochastic simulation like Monte Carlo (MCS) can be used to widen the estimation bandwidth. For incorporation of risk, the discount rate is traditionally calculated as a weighted average cost of capital (WACC). This rate consists of costs for debt and equity capital and the specific composition of the two components in the company. The cost of equity capital is usually calculated using the capital asset-pricing model (CAPM), multifactor asset-pricing model (MAPT)), the arbitrage pricing theory (APT) or a subjective rate. CAPM developed by Sharpe (1964) derives the risk out of comparison of the company's return volatility to the return volatility of the market. In the MAPT, several factors are identified which influence the return, then correlation between return and specified factors is established. These variables are usually based on macroeconomic, fundamental or statistical indicators. APT is a multifactor model based on estimating returns using a linear relationship between the returns of the company and macroeconomic variables (Triana, 2010). Problematic at these approaches as emphasizes by Mun (2005a) is that all the correlations are computed out of historical data with the underlying assumption, that the market correlations of the past remain the same also in the future which must not be the case. This historical approach is again, as in the estimation of cash flows above, especially difficult to use at startups due to lack of history.

Smith and Bliss (2011) explain, that in startup financing it is common to adjust the discount rate for perceived risks instead of working on a common estimation of the future development where the founders and the investors can agree on. Desaché (2014) argues, that the discount factor can be calculated from comparable listed companies with similar growth rates but very difficult in the case of a startup company. Subjective internal rates of return (IRR) are used, a concept similar to the discount rate, in order to account for risks regarding the DCF approach in startups (Laitinen, 2019). The hurdle rates are above 50% per year regarding on the stage of the startup (Bhgat, 2014).

2.3 Summary of Valuation Approaches

In the above outlined qualitative approaches for valuing startups, the uncertainties regarding future startup development are subjectively estimated and startup value is composed of several distinct value drivers in the analyzed approaches. For startups, before generating revenues, the major problem is the basis of estimating a future development without basing it on an operating history. This severe limitation of available data is problematic in applying quantitative valuation approaches. One solution used by practitioners in the startup area is the qualitative valuation as outlined earlier in this chapter. The lack of historical information about the startup is compensated through experience and trust in the proposed business idea, environment and management.

Differences in subjective estimates can lead to vast differences in calculated value. Through using a variety of valuation tools, it is believed to minimize the number of failed investments significantly (Payne, 2011).

Academic research on these qualitative non-financial valuation methods is rare, although there is lot of information about their application in the startup environment, concluded by an extensive online search.

As revenue for startups is already available for valuation, the qualitative methods are no longer the dominant valuation tools emphasized by Payne (2019). Qualitative information value is fading as the startups mature and quantitative financial information is gaining in value which is supported by Hand (2005).

A further analysis of the value drivers of the qualitative valuation approaches is conducted in order to detect and understand the essence of value inherent in a startup and to assist with these identified value drivers in the estimation of the future startup development in the operationalization section 5. Therefore, in the following chapter 2.4 and 2.5 the value drivers of the qualitative startup valuation approaches are explained and further analyzed.

2.4 Risk and Uncertainty

There are several definitions in literature concerning risk. One definition which qualitatively approaches the topic, and which connects risk and uncertainty, was developed by Kaplan and Garrick (1981):

Risk equals uncertainty plus a potential danger rooted in this uncertainty.

Uncertainty is primarily the non-knowledge about the outcome, which is neutral, but the term risk introduces a general focus on the downside of an uncertain event. One trait of risk is, that it is relative to the observer and the same situation would be judged vastly different by different observers. Garrick and Kaplan (1981) further conclude that it is an accepted conception, that all possible risks cannot be listed, therefore the term "perceived risk" is often used.

Regarding the financial dimension of risk, the American securities and exchange commission (SEC, 2011) defines risk as: "…*the degree of uncertainty about the rate of return on an asset and the potential harm that could arise when financial returns are not what the investor expected*" (p.28). A relationship between return and uncertainty is established by them. Depending on the risk preference of the investor, different levels of risks are accepted. One assumption, which underlies the CAPM is that variance of returns equals risks, the more volatile a stock in the stock market, the higher its risk.

The Nobel laureate Robert Engle (2003, p.405) concludes that the central paradigm of finance is, that optimal behavior takes risks that are worthwhile: "…*we must take risks to achieve rewards but not all risks are equally rewarded. Both the risks and the rewards are in the future, so it is in the expectation of loss that is balanced against the expectation of reward*."

The modern portfolio theory connected risk with the variation of the value of an asset which lead to a widespread belief that volatility is a good estimate for risk. The 2008 financial crisis has shown the major flaws of this theory (Triana, 2010). Assets which profited heavily of the building of the financial bubble were of low volatility and were therefore perceived as low risk assets, instead of some government treasuries which are usually considered safe, with were then perceived risky. In the financial crisis 2008 several assets with low volatility suddenly lost all their value Triana (2010, p.2): *"…low volatility can act as a camouflage for toxic assets*".

The next two terms in the risk context are "probability" and "frequency". "Probability" is a "state of knowledge, a degree of belief, a state of confidence" defined by Kaplan and Garrick (1981, p.7) which is not a subjective value, rather a general state of knowledge where two individuals with the same knowledge should assign the same probabilities to the same situation. In contrast, "frequency" is a well-defined objective value defined after testing. These two values are connected through the known frequency of a similar situation, a judgement can be made with a degree of confidence, which is called probability. The term "probability" is used to discuss issues where there is no frequency information available but a state of confidence in decision making is needed. They add that risk is a relative concept because no risk is acceptable, viewed in isolation. Decision theory states that options, costs, benefits and the individual risks of each option must be considered.

Daly (2008) states that volatility is defined as the changeability of the variable under consideration. In finance volatility is used to describe the dispersion of a value, price or model.

It is also is also emphasized by Florea (1999) that although, “Risk” and “Uncertainty” are often used synonymously, there is a difference between these two which supports the definition by Kaplan and Garrick (1981).

Even though it is not completely correct, from here on “Risk” and “uncertainty” are used synonymously. The reason for this is that several academics use them this way which would lead to confusion on the following pages.

Damodaran (2005a) breaks the risks a firm is facing down into a range of factors. The extremes are “Firm-specific” risks and “Market” risks below in Illustration 3.

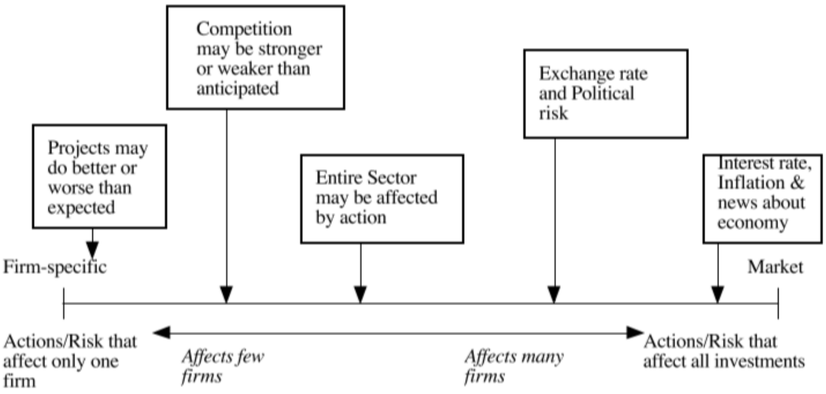

Illustration 3: Risk Range (Damodaran, 2005b)

To understand the major sources of the risks influencing startups, the startup valuation approaches outlined in chapter 2.2.1 are further analyzed. A categorization of the specific factors into market risks and private risks is conducted in order to understand the origin of the value.

2.4.1 Market Risks

The following risk factors can be attributed to public risks, which are commonly perceived as risks that affect all companies in an economic sector. Of the qualitative valuation approaches outlined in 2.2.1, only the Risk factor summation method explicitly incorporates public risks:

- Legislation/Political risk

The quantitative approaches have different ways of dealing implicitly with public risks, they use often inseparable ways of dealing with both types of risks.

2.4.2 Private Risks

Comparing the identified private risk factors to public risk factors used in the qualitative startup valuation approaches shows the importance of private risks as dominating source impacting value. The quantity of private risk factors incorporated far exceeds the number of public risk factors.

The Berkus approach, Scorecard method and the Risk factor summation method provide the following risk factors which are categorized into blocks due to their familiarity:

Risks concerning the success of the Product/Service:
Product risk (idea)
Technology risk (prototype)
Product/Technology
Technology
Litigation
Development stage
Manufacturing
Opportunity

Risks concerning Management, Organization:
Management
Execution risk(management)
Marketing
Sales/marketing
Reputation

Risks concerning Funding of the startup:
Financial or production risk (sales or rollout)
Need for capital
Funding
Potential exit

Risks concerning the competitive environment:
Competition
International
Competitive risk or market risk(relationships)

2.5 Major Uncertainty Factors Impacting Startup Value

The above derived risk categories can be summarized:

- Product and Technology
- Management and Organization
- Funding
- Competition

Private risks as being the majority of risk factors in the qualitative valuation approaches can be assumed to be the major risk drivers regarding startup value. The quantitative approaches also use value drivers which are in part subjectively estimated, but risk factors cannot be derived explicitly like in the qualitative approaches. Just in comparison, regarding the value composition of "mature" exchange traded companies, Damodaran (2005c) emphasizes that 75-80% of risks are due to firm specific factors. This number is assumed to be higher in startups based on the quantity of private risk factors in the qualitative approaches in comparison to the public ones.

Subjective Estimation of Risk Impacts

The risk factors which have an impact on startup value, were identified and categorized above. In order to be able to estimate the quantity of impact, the identified risk factors have on startup value, subjective estimation is used in the qualitative approaches. In section 5, the future development of a startup is estimated subjectively based on qualitative valuation methods. In order to support this estimation process, a method was searched for, which connects the qualitative estimates with a quantitative outcome.

A technique was found which uses a transparent mathematical approach in order to rank different influence factors: "Analytical hierarchy process" (AHP), a process to support a multi-objective selection problem, which is present when estimating the impact of several risks on future development. It can be used for qualitative and quantitative risk analysis as stated by Emblemsvåg and Kjølstad (2006). With this analytical process, a hierarchy of risk importance can be created.

AHP Method

Miller (1956) proposed that parent risks are used which are allowed to consist of 9 sub-risks at a time. As parent risks, the major risk factors identified in chapter 2.5 are used. The sub risks are identified individually at each startup. A scale for measurement is needed in order to do a pair-wise comparison shown below in Table 1:

Intensity of importance	**Definition**	**Explanation**
1	Equal importance	Two items contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one over another
5	Strong importance	Experience and judgment strongly favor one over another
7	Very strong importance	An activity is strongly favored, and its dominance is demonstrated in practice
9	Absolute importance	The importance of one over another affirmed on the highest possible order
2,4,6,8	Intermediate values	Used to represent compromise between the priorities listed above
Reciprocals of above numbers		If item i has one of the above non-zero numbers assigned to it when compared with item j, the j has the reciprocal value when compared with i

Table 1: Scales of Measurement (Saaty, 1990)

The scale in Table 1 above is used to compare the risks of the four identified risk categories with each other in a matrix in order to quantify the relative importance to each other.

In chapter 5, this process is implemented in operationalization. With the definition of a startup and the identification and categorization of uncertainty factors impacting startups, the focus now can be drawn to the Real Option Valuation.

3 Real Options

3.1 Definition: Option

Lütolf, Pirnes and Indaimo (2011) conclude, that in finance, an option is defined as having a right but not an obligation to buy or sell an underlying financial asset at a fixed price over a specified time period.

There are roughly two types of options: call and put options. A call options is a right for a holder to buy an underlying asset for a fixed price, a put option is a right to sell an underlying asset at a fixed price. The counterparty of the transaction is the seller of the option which has the obligation to conduct the transaction in case the holder exercises his right. Further, there are two differences in the right of the option regarding the timing of exercising the option. An American option can be exercised from initiation of the contract till the specified date, a European option is only exercisable at the specified date.

During the life of the option, if the underlying asset is changing in price, also the option changes its value. The option is generally classified to be “in-the-money”, “at-the-money” or “out-of-the-money”. In the following Illustration 4, there is an example for a call option on WTI crude oil with an exercise price of 95$. The blue line signals the underlying asset price. If the underlying price is below the strike price the option is “out of the money” signaled with the red area. In this case rational option holders would not exercise their right because paying 95$ to purchase an asset which has a lower price would not make economic sense. In this case the option is assumed worthless and valued at 0$. If the option is “at-the-money” the asset price is at 0$ but same for the option value. For a rational investor the economical decision whether to buy or wait and not exercising the right at this point is indifferent. When the option is “in-the-money”, the exercise is valuable because the option holder can acquire an asset for costs lower than its actual price which gives the option a positive value. A put option works vice versa, if the price drops below the strike price the put option is “in the money”.

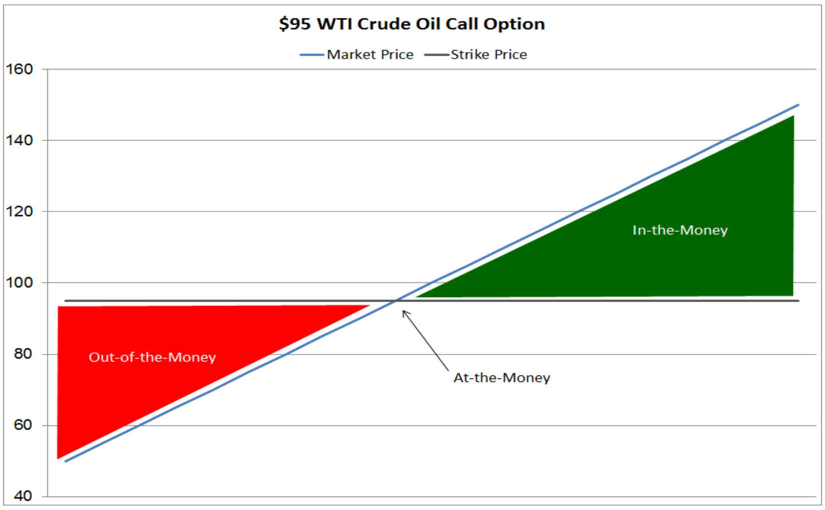

Illustration 4: Call Option (Mercatus Energy Advisors, n.d.)

3.2 Difference Between Options and Real Options

Myers (1977) was the first to introduce the “Real options” concept. Amram and Kulatilaka (1999) used option pricing theory and financial market rules to evaluate assets and support management decisions under uncertainty.

A real option is the application of the financial option pricing theory on “real” assets. One definition of “real asset” by Chen (2020, p.1):
“*Real assets are physical assets that have an intrinsic worth due to their substance and properties. Real assets include precious metals, commodities, real estate, land, equipment, and natural resources*.”

According to Mun (2005a), Real Options Analysis can be used to value a variety of assets, not strictly limited to physical ones but also intangibles. Due to the basic framework he further elaborates that it is useable in several settings like valuation, research and development decision making, production planning, capacity planning for infrastructure and risk management.

Kang (2009) defines a real option as an option to invest in the “Real economy”, his examples are investing in goods and services instead of financial contracts. For a real option an initial transaction in which the right for later use is given from the option offeror to the holder must take place but this can be conducted in several ways. Then the option right can be exercised with paying the exercise price. Trigeorgis (1993) divides real options into seven categories according to the type of managerial flexibility and discusses this categorization on an investment example:

- Option to defer
    - Hold or lease till price justifies full investment
- Option to alter operating scale
    - To expand, contract, shutdown and restart
- Option to abandon
    - If market does not improve, project can be abandoned
- Staged investment option
    - Series of outlays, option to abandon and minimize loss
- Option to switch
    - Change output mix or input mix
- Option to grow
    - Early investment e.g. acquisition, projects etc. to open-up future opportunities
- Multiple interacting options
    - Collection of various options, both upward potential enhancing and downward protection

The options listed above are not exhaustive due to the wide applicability of real options and provide just basic types. In case of multiple uncertainties present, Copeland and Antikarov (2001a) call this option a rainbow option. They further recognize that combinations of several options are called compound options and the most used option type in practice is the rainbow compound option.

Triantis and Borison (2001) state, that under practitioners there are three different interpretations of real options:

- Way of thinking
- Analytical tool
- Organizational process

Luehrmann (1998) concludes, that business strategy is in financial terms like a sequence of major decisions, a series of holding or exercising real options. The corporate strategy sets the framework but leaves room for learning from ongoing developments and acting based on this gain of information.

Amram and Kulatilaka (2000, p.17) define real options as "*The subset of strategic options in which the exercise decision is largely triggered by market priced risk*", as shown in Illustration 4. Oil price fluctuation can be a market priced risk, because the risk is captured in the value of oil future contracts.

Copeland and Antikarov (2001a) state three main differences between predominant financial options and real options:

- In a financial option the issuer cannot control the underlying asset.
- Financial options usually have stock, an index or a bond as underlying asset.
- In real options the management can influence the risk to a certain degree through influencing competitors' actions.

3.3 Value drivers of Real Options

As explained in 3.2, the fact that a real option is a close proxy of an option applied on a real asset, leads to a similar structure of the value drivers. Copeland and Antikarov (2001a) list six value drivers in addition to the type (call or put) which together define a real option:

Value drivers:

- Value of the underlying asset
- Exercise price
- Time to expiration
- Uncertainty in the underlying asset (Volatility)
- Risk-free rate of interest
- Dividends

The value drivers are briefly explained below:

Value of The Underlying Asset

An underlying asset was introduced in the example shown in section 3.1: The oil price underlying an oil option. Generally, it is the asset on which the option is based on. For financial options, the value is derivable from financial markets. In case of a real option, the underlying assets is either traded in the market, then the value is publicly available, or a similar asset is available then the value of a “twin security”, an ideally perfect correlating comparable asset value, can be derived. If the two solutions are not possible, Copeland and Antikarov (2001a) emphasize that the NPV without flexibility is the next best unbiased estimator of market value of the asset, they call this assumption “Market Asset Disclaimer” or “MAD” approach.

Exercise Price

Referring to the example in section 3.1, the amount that has to be paid at exercising a call is named exercise price. At put options, for example: The option to sell a good at a fixed price, the exercise price is “in the money” if the value of the asset drops under the exercise price and the holder therefore exercises the option. In financial options, the exercise price is also called strike price and is defined at initiation of the contract according to Ganti (2020). In real options, the exercise price can also be fixed at the beginning, like for example in expansion options with a price guarantee for a machine or an offer on the acquisition of land. Usually the real options exercise price is harder to determine and in case of non-knowledge, the exercise price should be equal to the present value of the future investments, as concluded by Aarle (2013a).

Time to Expiration

The time to expiration is the time from acquiring the option till the date where the option expires. At financial options, this is a clearly defined period. At real options it depends on the type of option and the way the option is acquired. In case of an option to expand, the option can expire when competition closes the gap in the market and the expansion would lose its economic benefits. An option to defer an investment can be limited by the availability of the asset in the future. Generally said, time to expiration is tough to specify at a real option but the longer the timespan till expiration, the higher the value of an option because there is more time for a positive development to happen.

Uncertainty in the Underlying Asset or Volatility

The higher the uncertainty, the bigger the possible upside and downside in the development of the underlying asset. The estimation of the future variation of the underlying asset value is not trivial. At startups, this variation is driven by the identified risk factors in chapter 2.5. Ways to derive the volatility “ex ante”:

- Industry variance
- Historical company variance
- Subjective estimation

Risk-free-rate of Interest

Risk free rate as discount rate for risk-free alternative opportunities is derived from generally accepted risk-free assets like government bonds. An increase in the risk-free-rate generally increases the value of the option because the costs for exercising the option, which remains at the option holder up to the day of exercise, can earn interest up to this date.

Dividends

The value lost due to payouts of financial assets and outflows of capital of real assets diminishes the value of the option.

3.4 Valuation with Real Options

Copeland and Antikarov (2001b) state that the academic literature about real options contain the “..*most outrageously obscure mathematics anywhere in finance.* “(p.29) from a practitioner’s view. This is seen by Mun (2005a) as one major reason why this method is not widely spread in finance.

One advantage of real options is flexibility in defining an option, which makes it a wide applicable tool in valuation and corporate decision making. Real Options Valuation can be roughly divided into four steps:

- Compute the basic present value (PV) without flexibility or growth
- Model uncertainties into a lattice and understand how the PV changes over time in order to quantify the change of the underlying.
- Include managerial flexibility into the lattice to graphically show the decision options
- Calculate the real option value and add it to the basic PV

3.4.1 NPV Compared to Real Option Valuation

As described in chapter 2.2 , the DCF method with the subsequent calculated NPV is the most widespread technique for valuation and decision-making present. The bold claim of Copeland and Antikarov (2001c,vi): “*In ten years, real options will replace NPV as the central paradigm for investment decisions.*” is not validated yet if one regards the current research: Espinoza and Morris (2013) state that despite the shortcomings of the NPV assumptions, NPV and the strongly related IRR method remain the most popular methods for investors. Due to the close connection between NPV and Real Option Analysis, it is useful to start with the NPV valuation and to introduce uncertainties to move up to the ROA framework and the different developed approaches in this field.

As illustrated by Mun (2005a), the NPV is a special case of real options, a real option where the uncertainty equals 0.

In chapter 2.2.1, the basic assumption of the DCF method with the subsequent calculation of the NPV was introduced. Three variables in the valuation process were introduced: the expected cash flows, the discount rate and the terminal value.

Copeland and Antikarov (2001a) criticize, that the uncertainties of the cash flows are not explicitly included in the NPV approach and that the simple NPV systematically undervalues every project. There are uncertainties about the future development of the cash flows, but alternative developments are neglected. There is one most probable case calculated and the investment decision is broken down into one NPV which states the increase in project or company value, that can be expected with the calculated scenario. Then the manager or investor must make a go or no-go decision. The NPV approach is constrained to make an often "premature" yes or no decision, where often after more information is revealed the early made commitment based on an NPV decision rule can turn out worse.

The Real-Option approach is the application of a finance idea to optimize the structuring of corporate investment valuation and decision making that helps capturing more of the upside potential and limiting the downside.

The decision rule differences between NPV and ROA:

$$NPV\ rule: MAX(at\ t = 0)[0, E_0V_T - X]$$

E_0…Probability of future at time 0
V_T…Value at T
X…..Exercise Price

It states that at time=0 a decision is made for the project if $(E_0V_T - X) > 0$. Only information known at t=0 is incorporated because the decision is made at t=0. Therefore, the decision is made for the maximum of expectations E_0V_T.

The ROA decision rule looks at the problem from another perspective:

$$ROA\ rule: E_0\ MAX(at\ t = T)[0, V_T - X]$$

Here the decision is made for the expectation of maximums. The project is undertaken at a future time T only if $V_T > X$. Future information can be incorporated because the decision happens in the future. The decision point is shifted from t=0 up to t=T compared to NPV, if the probability for a beneficial development is possible from the viewpoint at t=0 which is expressed in the probability E_0.

If the uncertainty is 0, so that the probability E_0=1, the two rules are the same.

Illustration 5 below shows the risk-return distribution difference between the NPV rule and the ROA rule. Due to the flexibility of not executing a project, which generates a loss under the ROA rule, the expected return is improved and the distribution therefore is shifted to the right.

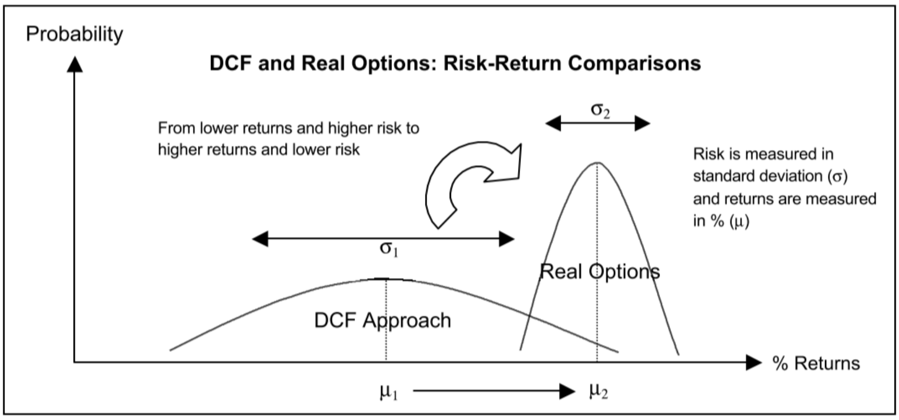

Illustration 5: DCF and Real Options (Mun, 2005b)

To detect additional value, the NPV framework neglects, the basic idea is discussed in the following Illustration 6:

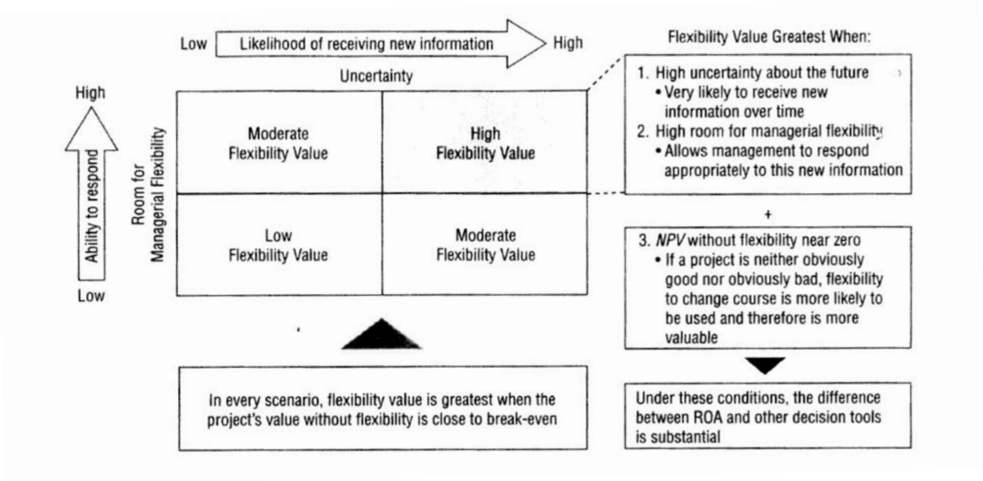

Illustration 6: Flexibility Value (Aarle, 2013b)

High flexibility value is not captured in the NPV model due to the deterministic assumptions, Real Options Valuation is therefore useful. This managerial flexibility to respond appropriately to new information is a trait of a startup due to its lean decision structure and generally high strategical, as well as operating flexibilities.

Different Models for Valuing Real Options

Important for valuing real options are the following concepts:

- ➔ Black Sholes
- ➔ Binomial lattices
- ➔ Replicating portfolio approach
- ➔ Risk neutral probability approach
- ➔ Binomial decision tree
- ➔ Compound rainbow options

Black Sholes

The Black and Sholes formula is widely used for valuing options on the financial markets (Bodie, Kane and Marcus, 2008). There are seven assumptions incorporated into the Black and Sholes formula which limit the application of the option pricing formula (Hull, 2015):

- Exercise of the option only at maturity
    - European option pricing
- Only one source of uncertainty present
    - No rainbow options
- Contingency on one single underlying risky asset
    - No compound options
- No dividends from underlying asset
- Current market price and stochastic process known
- Constant volatility through time
- Exercise price known and constant

The Black Sholes formula for a Call and a Put are listed below:

The formula for a Call option:

$$C_o = S_0 * N(d_1) - X * e^{-r_f * T} * N(d_2)$$

The formula for a Put option:

$$P_o = X * e^{-r_f * T} * N(-d_2) - S_0 * N(-d_1)$$

S_0…Price of underlying
$N(d_1)$…Cumulative normal probability of unit normal variable d_1
$N(d_2)$…Cumulative normal probability of unit normal variable d_2
X… Exercise price

T…Time to maturity
r_f…The risk-free-rate

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + r_f * T}{\sigma * \sqrt{T}} + \frac{1}{2} * \sigma * \sqrt{T}$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

Triantis and Borison (2001) note that the Black and Sholes is useful in providing a quick estimate of the Real Option Value but is not suited for most real-life applications due to their restricting assumptions. Further, the Black and Sholes framework expects a lognormal distribution of the price of the underlying, which equals a Geometrical Brownian Motion evolution of the underlying. This is further explained in the Binomial lattice chapter below. The underlying asset must be traded at all times without costs, which is the assumption of a continuous frictionless market. Further the law of one price must be abided which as well prevents arbitrage opportunities.

Binomial Lattices

Below in Illustration 7, a basic one step binomial model is illustrated. S_0 is the price of an asset at time 0, p is the probability of an up move and $1-p$ the probability of a down move u stands for the rising factor and d for falling factor of the asset value.

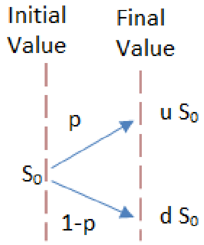

Illustration 7: Binomial Tree (Invest Excel n.d. a)

A time step leads to new branches in the lattice. The lattice in Illustration 7 above has one time step and in Illustration 8 below there are three time steps. There are two types of lattices, recombining and non-recombining. In Illustration 8, a recombining lattice is displayed a non-recombining needs vastly more computational effort and yield the same results.

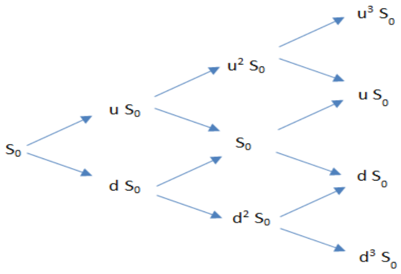

Illustration 8: Multistep Binomial Model (Invest Excel n.d. b)

Non recombining lattices are needed when there are two or more uncertainties present or when the volatility changes over time. The accuracy of the framework is acceptable, compared to the widely accepted Black and Sholes framework which equals the same value as the Binomial approach if using 1000 timesteps Mun (2005a). This is because the Black and Sholes differential equation approach is a continuous calculation in comparison to the discrete binomial model. When using enough timesteps the discrete model becomes approximately continuous.

The principal idea behind the binomial model is that the underlying value development is modeled with a discrete lattice where the underlying value either moves up or down in each time step.

If the volatility is set to zero, the lattice would be equal to a straight line. The cash flows would be deterministic due to zero uncertainty and the value could be calculated with a simple NPV using the DCF method, because the value of flexibility would be zero. Displayed in Illustration 9 below are different binomial lattice frameworks for different volatility values:

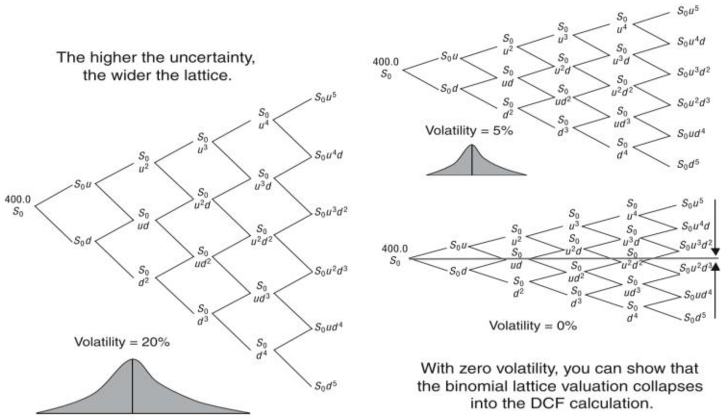

Illustration 9: Volatility Impact on Lattice (Mun, 2005c)

The lattice evolution is set to mimic the Geometric Brownian motion (GBM), the widely accepted standard assumption necessary for pricing options. The GBM consists of a deterministic and a stochastic term:

$$\frac{\delta S}{S} = e^{\mu(\delta t)} e^{\sigma \varepsilon \sqrt{\delta t}}$$

$\frac{\delta S}{S}$…the change rate of the underlying price

$e^{\mu(\delta t)}$...the deterministic part where μ is the average growth rate, δt the time between steps

$e^{\sigma \varepsilon \sqrt{\delta t}}$…the stochastic term is the main focus for option valuation because the deterministic term can be incorporated into valuation with a sole NPV.

The volatility component is symbolized by σ in the term, ε stands for the simulation random variable. The stochastic price development therefore is only dependent on the volatility and the timesteps, shown in Illustration 10 below as the volatile line above and below of the straight deterministic line.

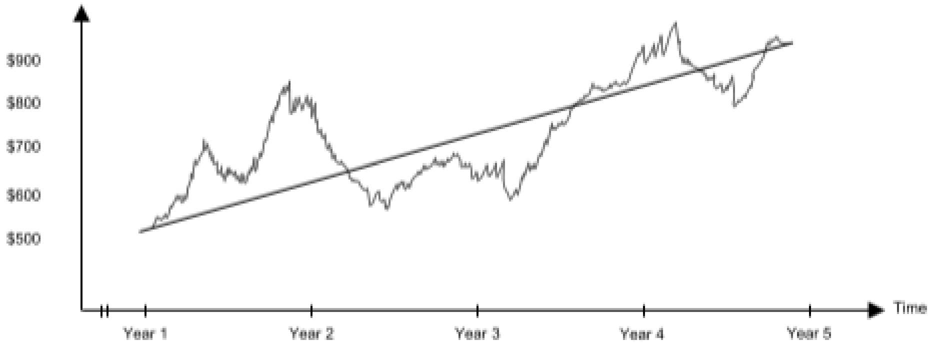

Illustration 10: Geometric Brownian Motion (Mun, 2005d)

Based on the stochastic term, the binomial lattice can be calculated. The size of the up movement equals $u = e^{\sigma\sqrt{\delta t}}$ and down movement $d = e^{-\sigma\sqrt{\delta t}}$.

The assumption of a stochastic evolution of underlying asset price is based on Samuelson (1973) study: Returns on securities fluctuate randomly around the mean. This means that multiple uncertainties can be incorporated into one stochastic process with one volatility measure. When this is the case, the binomial lattice can be used.

Valuing binomial lattices is possible either with risk neutral probabilities, or the replicating portfolio approach which are explained below:

Replicating Portfolio Approach

The law of one price in option valuation dictates that for a real option to be priced correctly in relation to the market, the same potential benefit must have the same price as well as the same risk. In practice, a combination of stocks and bonds with similar payoffs in the market must have the same price as the startup. This leads to defining payoffs, the underlying is expected to generate and searching for a portfolio of marketed securities which has exactly the same benefits and risks as the asset. Based on this, the price of the portfolio must be the same as the price of our real option.

Replicating portfolio:

$$Up\ state: m(€30) + B\left(1 + r_f\right) = €60$$

$$Down\ state: m(€15) + B\left(1 + r_f\right) = €0$$

m…number of stocks
B…number of bonds

Assuming a risk-free rate r_f of 3% and the price of the stock in the market is 20€/share, solving the 2 equations for 2 unknowns leads to m=4 and B= -58,24. The option payoffs can be replicated with buying 4 shares and borrowing 58,24€ which equals a value of the replicating portfolio of:

$$Value\ of\ portfolio = 4 * shareprice - quantity\ borrowed = 4 * 20 - 58{,}24 = 21{,}76$$

The difference between the value of the portfolio which has the same payoffs as the option and the NPV of fixed precommitment equals the value of the flexibility.

$$Value\ of\ portfolio - NPV = value\ of flexibility$$

Problematic with this approach is to find twin securities in the open market which can be used as a replicating portfolio. Correlated payouts in the up and down state between the to-be-valued asset and the traded portfolio are needed.

Risk Neutral Probability Method

Leskisenoja (2015) emphasizes that this method is a mathematical convenience for discounting at the risk-free rate in a binomial lattice. Through adjustment of the probabilities of the cash flows from “objective” ones into “risk neutral” ones, the cash flows are turned into certainty equivalent ones which qualify for a risk-free discounting.

The idea behind the risk neutral probability approach is to “hedge” the portfolio in a way that it becomes risk-free (Mun, 2005a). This is done by the right combination of long and short positions between the option and the underlying asset.

The equation of risk neutral probability in a binomial lattice is:

$$p = \frac{e^{(r_f)\delta t} - d}{u - d}$$

$p\ldots$risk neutral probability of an up move
$r_f\ldots$risk free rate
$u\ldots$ size of up move
$d\ldots$ size of down move
δt...time step between nodes

Risk Adjusted Decision Tree

In order to incorporate flexibility value into the NPV, decision tree analysis is a method for doing this. According to Copeland and Antikarov (2001a) , the classical weighting of the decision branches and discounting back with a constant WACC rate throughout the tree, as done in a classical decision tree, is wrong due to the difference in risk perception between the branches when time is factored in. They show that this violates the law of one price. In order to for the tree to be correct, they suggest a risk adjusted rate of return (RAR) and the discount rate must be calculated for each decision node.

3.4.2 Company Valuation Using Real Options

As first introduced in chapter 2.2.1, the DCF method doesn't catch managements flexibility appropriately and this flexibility is quantifiable with the Real Options Analysis. Putten and Macmillen (2005a) recommend this as an essential complement to capture the value of uncertainty.

Their way for evaluation a project which is applicable on any type of asset:

$$TPV = NPV + AOV + ABV$$

TPV… Total project value
NPV…Net present value
AOV…Adjusted option value
ABV…Abandonment value

Below in Illustration 11 project A is shown at two points in time. The total project value is the same at both times, only the uncertainty changes which enhances the NPV but decreases the option value.

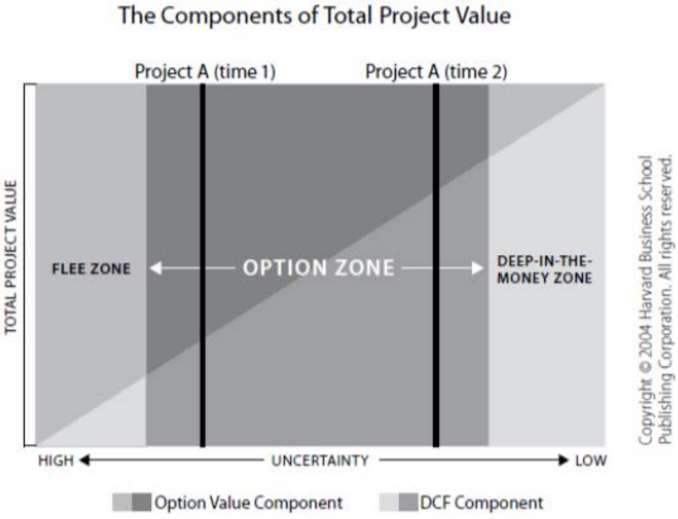

Illustration 11: Option Zone (Putten and MacMillan, 2005b)

When valuing startups, this venture can be seen similar to a project with small initial setup value, a comparable big follow up investment for expansion or market rollout. This perspective is important when defining the real option. Due to the high uncertainty in the development of a startup, qualitatively expressed, the major value can be attributed to the option value component left of the center in Illustration 11.

3.5 Real Option Valuation Approaches and their Assumptions

In an extensive literature research, several distinct methodologies dominating the field of real options were identified. The most influential academics contributing to the field:

- Trigeorgis
- Luehrman
- Copeland and Antikarov
- Amram and Kulatilaka
- Dixit and Pindyck
- Smith and Nau
- Borison

Several valuation techniques are proposed or further researched by the above listed academics and are explained in short in this chapter. In the next chapter 4, one of these approaches is identified suitable for startup valuation.

Classic

Amram and Kulatilaka (1999) publicized this approach for valuing real options the "classic" way using the Black and Sholes equation. To get the input values needed in the Black and Sholes framework, a replicating portfolio is traditionally searched for in the market, which mimics the returns of the asset one wants to value. Then, the asset in question is sized using the replicating portfolio model, the price and volatility are derived from the market.

Problematic at this approach is finding the replicating portfolio as a composition of stocks and bonds which have the same return. The replicating portfolio technique explained earlier is used.

Subjective

Luehrman (1998) emphasizes the valuation of real options using subjectively estimated price and volatility data of the underlying asset and then applying the standard Black and Sholes equation. Basically, using the "Classic" approach without the replicating portfolio technique.

Marketed Asset Disclaimer (MAD)

Copeland et Antikarov (2001d, p.94) argue that "*What is better correlated with the project, than the project itself?*". They propose that the present value of the projects cash flows without flexibility is the best estimate of the market value of the project and call this assumption the Marketed Asset Disclaimer (MAD). The MAD approach uses subjective data for the expected future cash flows analog to the NPV approach. The discount rate is derived similar to the DCF framework using the WACC and the CAPM model which needs market data for finding an appropriate rate. The underlying asset is assumed to behave like a GBM, which makes the binomial lattice approach applicable. The value of the underlying asset is derived with a NPV calculation and then volatility is computed using Monte Carlo Simulation based on subjective estimated projections for the development of the underlying. To calculate the value and visualize the development, a binomial lattice is used.

The disconnect from the market is an issue, the valuation is internally consistent but can be mispriced to the market as emphasized by Borison (2005). Also, the issues in estimating future cash flows without flexibility and in estimating the variability of cash flows due to flexibility are problematic. One other issue is the calculation of the WACC which implicitly relies on a replicating assumption.

Revised Classic

The revised classic approach allows for more flexibility and requires identifying the dominating risks before application. This valuation technique was developed and is explained thoroughly by Dixit and Pindyck (1994). It leads to a decision about the major risk sources: If the value is mostly influenced by market risks which allow a replication approach, then the classic approach should be chosen. If replication is not possible because dominantly private risks influence the value, decision analysis should be used.

Limiting at this approach is the black and white categorization of influencing risks, in case the decision is made, the other ones are neglected.

Integrated

The integrated approach, emphasized by Smith and Nau (1995), recognizes that most asset values are influenced by both, private and public, risks. Therefore, the risks which influence the value are categorized first for the public risks, a replicating portfolio is identified and for private risks, subjective estimates are used. These two risks are subsequently incorporated into a decision tree with risk neutral probabilities for public risks and subjective probabilities for private risks. The final values of the tree are discounted back to derive the option value and the optimal strategy.

This approach is useful because it doesn´t limit itself to be an either-or approach, in case a replicating portfolio is available, it is incorporated. Issues with finding a replicating portfolio are present as well as problems with subjective estimation of private risks. The integrated approach has its origin in decision analysis using a decision tree and real options. One advantage of this framework is, that incorporating several flexibilities into one option is possible and decision advise can be drawn from the model because the way value is maximized, can be shown. The risk adjustment of the subjectively estimated probabilities is the same problem as adjusting the discount rate in a decision tree.

4 The “right” Real Option Approach

Startup as a Real Option

The Real Options framework allows for a wide variety of flexibilities and ways of valuing startups. Due to the simple, project like structure, the whole startup can be seen as an option. The major investment is the exercise price, which is the capital the startup needs for growth and further expansion. In case of not exercising the option, the startup is terminated and only has an abandonment value. The information enhancement on decision day compared to the time the option is set up, is based on a gain of information about the startup development. The value drivers of an option can be interpreted as the following, when applied on a startup:

- ➔ Value of underlying: Value of the startup.
- ➔ Exercise prices: Price for expansion, capital increase, product/service rollout.
- ➔ Volatility: Change in value of the startup.
- ➔ Risk free rate: Market rate of risk-free bonds.
- ➔ Dividend: Can be neglected at startup because the capital generated is assumed to be reinvested.

4.1 Comparison of Different Approaches

Based on reviewing the different Real Option approaches and their assumptions introduced in chapter 3.5, three dimensions appear essential in choosing an applicable framework:

- Data availability
- Underlying assumptions
- Calculation methodology

All three dimensions can limit the usage of the identified frameworks. If assumptions underlying an approach are not acceptable and would be breached in application, it would be problematic to accept the result. The data availability needed in applying the frameworks has to be sufficient as well in order to compute a result. Generally, concerning data availability as identified in chapter 2, a more qualitative approach in the early stages is needed and a possible quantitative consideration in the later stages should be possible to incorporate all information available.

The major proposed analytical approaches for applying real options from chapter 3.5 are more thoroughly analyzed regarding these three dimensions in order to derive a suitable solution for real option application on a startup.

In Table 2 below, a comparative analysis is conducted focusing on the two objectives:

Valuation approach	**Classical**	**Subjective**	**MAD**	**Revised classic**	**Integrated**
Value of the underlying source	Replicating portfolio	DCF using subjective estimation	DCF using subjective estimation	Replicating portfolio or subjective estimated	Replicating portfolio or subjectively estimated
Assumptions	Black and Sholes assumptions GBM	Black and Sholes assumptions which are relaxed for incorporating subjective data GBM	No complete market assumption for replicating portfolio Twin security is same beta company GBM	After complete market not detected: dynamic programming Twin security is comparable listed company (same Beta)	Market assumed partially complete Twin security is comparable listed company (same Beta)
Data	Market data	Subjectively estimated data Market data for risk adjusted discount rate	Subjectively estimated data Market data for Risk adjusted Discount rate	Market data or after not finding correlation in the market subjectively estimated data Market data for risk adjusted discount rate	Market data for market risks Subjectively estimated data for firm specific risks

Table 2: Possible Real Option Approaches for Startups

4.1.1 Data Availability

Concerning the data availability, there are two important points to consider:

- Historical data of a startup
- Price data of a startup or a correlating asset detectable on the market for replicating purposes

The historical data can be used as a basis for estimating the future development, as discussed at the quantitative valuation approaches in chapter 2.3. This idea is also incorporated into the Real Option Valuation approaches of Table 2. Projections of the future development based on historical quantitative numbers is done in the DCF method which is used in the “Subjective” and the “MAD” approach. In case historical numbers are not available for projecting a development, the needed data can be subjectively estimated based on qualitative factors. This is done at the “Subjective”, “MAD”, “Revised Classic” and “Integrated” approach.

The replication of an asset builds on finding a perfect correlating security or a perfect correlating portfolio of securities on the market. The underlying assumption, for this to work, is the “complete market hypothesis”. This hypothesis builds on two conditions according to Buckle and Thompson (2004):

- Negligible transaction costs
- A price for every asset in every state is available

To conclude weather the replication assumption applies or not at startups for deriving the price data, the completeness of the market regarding correlating assets must be assessed.

Generally, a startup has its first access to the public pricing mechanism of a stock market at the IPO stage. For the startups, which have not reached this stage in their development, the stock market pricing is not directly useable. The “Classic” approach, “Revised classic” approach and the “Integrated” approach try to circumnavigate this issue through replication. A replicating asset, or portfolio of assets, which has the same payoffs as the startup is searched for on the market using the no arbitrage assumption incorporating the “law of one price”. This “law” is based on the logic that two assets with the same payoffs have the same price. In Real Option Valuation literature, several examples are given where this approach is successful in valuing companies, for example in situations where the value of the company mainly consists of the value of a publicly traded asset. The way this is calculated: The value of an oil company consists in part of the oil fields it possesses; Oil is traded in the market therefore a value can be derived.

As Copeland and Antikarov (2001a) point out, it is often practiced but problematic to assume that the volatility of one asset, the company possesses, is the same as the volatility of the company.

Additionally, they further question the usability of the replication approach in general. Mogi et. Al (n.d.) further studied the replication hypothesis and concluded, that perfect correlativity and volatility matching are necessary to qualify as a “twin” asset which is assumed to represent the “objective” value. They applied a random selection approach to find a replicating “twin” security on the stock market; this was not successful they concluded. They used a business project with FCFs over 9 years and searched in commercial databases for stocks and indexes which could replicate their project (Mogi et. al, n.d., p.1): *“… finding a perfect twin asset is virtually impossible*” was the final insight of their study. The structure of a startup approximately reflects the project Mogi et. Al. (n.d.) used in their study, therefore the result they reached can be assumed applicable on a startup as well.

Based on these insights it can be concluded, for valuing startups using real options, the market is not complete “enough” to use this valuation approach. This leads to an abandonment of the replication approach for startup valuation using real options.

4.1.2 Underlying Assumptions

The Real Option approaches shown in Table 2 are now being discussed regarding their assumptions.

The “Classic” approach developed in finance and applied on real options uses the Black and Sholes framework and its underlying assumptions. The most prominent point is, that it assumes complete capital markets. As concluded above, this assumption does not hold when tried to use a replicating approach for deriving the value of a startup due to the non-existence of a perfect correlating asset. The GBM assumption of a lognormal distribution of the underlying and the no arbitrage assumption are applicable in theory but cannot be tested due the problems with the replication theory.

The “Subjective” approach is similar to the “Classic” approach also using the standard Black and Scholes assumptions but incorporating a subjective estimation about the future development of the asset instead of the replication approach. For deriving the value of the underlying and its volatility, a DCF analysis is used which needs a connection to the market for calculating its discount rate. The Real Option Value is calculated with the Black and Sholes model. Because the “Subjective” approach is based on the standard Black and Sholes assumptions, it implicitly assumes complete capital markets but derives the price data through subjective estimation, a conflicting view as emphasized by Borison (2005). For this approach to be applicable, the complete market assumption has to be relaxed.

The “MAD” approach does not use a complete market assumption. Due to the DCF usage in the calculation of the underlying value, the discount rate calculation uses a modest form of replication assumption incorporated in the WACC using the CAPM. Otherwise it is completely decoupled from the market. The other inputs in the valuation are subjectively estimated.

The “Revised classic” approach is an either-or valuation approach using replication or subjective estimation. In case of a replication possibility, the Black and Sholes framework is applied with all its restrictive assumptions. If this is not feasible, subjective estimation is used. Subjective judgements are incorporated into the decision tree. The tree is based on the DCF methodology but has the issue of a constant changing discount rate depending on the position in the tree, as emphasized by Zambon and Marzo (2007). If the risk-adjustment of the discount rate is done in every tree branch, the value derived is the same as using a binomial tree or the Black and Sholes equation as shown by Copeland and Antikarov (2001a).

The “Integrated” approach uses both data sources in one framework. A decision tree is built where risks are incorporated using a replicating approach with incorporation of market data and subjective estimates as well. The assumption used is, that the market is “partially complete”. The public risks are incorporated into the tree as risk neutral probabilities.

4.1.3 Calculation Methodology

For a more thorough analysis regarding the applicability of the calculation methodology, the “MAD”, “Integrated” and “Revised Classic” approaches are further discussed here. The “Classic” and the “Subjective” approach use the basic Black and Sholes calculation method which was already discussed.

MAD

Problematic with this approach is the use of the WACC, which by default derives the capital costs using the standard CAPM. It compares the historical return of the startup with the market to derive a correlation. Lack of historical information and the special startup structure limit the application of the WACC using the CAPM to startups, which already have historical data available.

It is possible, in case historical market data is not available, to subjectively estimate the discount rate based on experience and subjective adjustments to comparable assets. The result of this approach can vary significantly and has potential for error due to human judgement. This is due to the high sensitivity of the DCF approach to a change of the discount rate.

In the “MAD” approach, the NPV is calculated using the DCF approach incorporating the free cash flow (FCF). The DCF method is based on inflexible “fixed” future benefits and the value of the inflexibility is calculated as base NPV. The flexibility is incorporated using subjective projections of the “fixed” FCF incorporated into the NPV. The derivation is calculated as volatility using MCS. This single volatility measure includes “public” and “private” risks which are incorporated into the FCF projection.

The uncertain value of the underlying is assumed to follow a Geometric Brownian Motion (GBM), an assumption which is needed for using the binomial lattice tree to value the real option. The binomial lattice is practical because it allows for European and American option to be incorporated.

Problematic from a theoretical perspective with using the MAD approach in valuing a startup are the following points:

- ➔ Traditional CAPM for risk adjustment is problematic for a startup due to the replication assumption
- ➔ Classic WACC does not consider flexibility
- ➔ Estimation about future development is based on subjective judgement

Revised Classic

If an investment can be replicated in the market with a portfolio of securities with similar payoff, the Black and Sholes approach should be used incorporating this data according to this approach. In case of startups, where private risks dominate and replication is not possible, a decision tree approach has to be conducted. The decision tree models decision alternatives in discrete time and incorporates decision flexibilities, discounting them back with the NPV approach using the WACC. Different decision alternatives have to be defined and probabilities as well as values for the outcomes following the paths have to be estimated. In addition to the demanding estimation of distinct scenarios, the discount rate is problematic which cannot be assumed constant throughout the tree. The risk estimated at the beginning of the tree, is not the same as in later stages when new information can have appeared and changes the probabilities of success of the project and therefore the discount rate.

A decision tree is useful in valuing companies which have a simple structure and focus on one product is emphasized by Hubbard (2009): Decision tree is a project management decision and valuation tool. The estimation of the probabilities and the generation of possible value outcomes gets complicated the more complex the company structure.

To properly discount the decision tree in order to derive a value consistent with classical real option pricing, a risk adjusted discount rate depending on the risk profile of the location in the tree has to be implemented as shown by Copeland and Antikarov (2001a). The probabilities of cash flows and the discount rates have to be consistent in order not to count the risk twice.

The inputs that need to be estimated for DTA:

- ➔ Cash flows
- ➔ Distinct scenarios
- ➔ Probabilities of Cash Flows
- ➔ Changing discount rate

Integrated

Any corporate investment can be valued with this approach according to Borison (2005), because for public risks a replicating portfolio is created and for private risks probabilities are estimated subjectively. A risk-adjusted decision tree is used, similar to the one incorporated into the “revised classic” approach. Public and private risks are both identified explicitly and are incorporated in the decision tree for valuation. This methodology makes the approach applicable in “partially complete” markets. Troublesome are the same areas as in the “revised classic” approach, namely the changing discount rate as well as the distinct development path which have to be estimated.

4.2 Characteristics of the Real Option Approaches

In this chapter, a summary of the discussed approaches is given, where the main positive and negative points regarding the applicability of the analyzed approaches on startups are presented. The overall goal is the usability from a theoretical and practical perspective, of a Real Option Valuation on a non-exchange traded startup. These main benefits and problems of each approach discussed earlier are summarized and shown below in Table 3:

Approach	*Classical*	*Subjective*	*MAD*	*Revised*	*Integrated*
Pro	Relatively easy to implement	Relatively easy to implement	Valuation in incomplete markets except discount rate	Incomplete markets possible	Incorporation of market data and subjective data Incomplete markets possible All assets possible
Con	Restrictions of Black and Sholes equation Market data need	Restrictions of Black and Sholes equation Subjective judgment	Discount rate for underlying Subjective judgement	Changing discount rate throughout tree Subjective judgement Black and white categorization Detailed and distinct development path	Changing discount rate throughout tree Subjective judgement Detailed and distinct development paths

Table 3: Comparison of Valuation Characteristics

4.3 Choosing the Most Suitable Approach

The first conclusion that can be drawn is, that of the five above introduced approaches, the “Classical” approach is ruled out for its complete market assumption which is based on a replication hypothesis. From a theoretical and practical perspective this assumption is not applicable, regarding startups which are not listed on a stock market.

The “Subjective” approach and the “MAD” approach use subjective judgement in predicting the future development of the startup. The benefits of using the “MAD” approach are its versatility in valuing European options as well as American options. The discount rate derivation problem is existing in both approaches because the “Subjective approach” recommends the DCF for deriving the underlying startup value as well. The possibility of valuing American options make the “MAD” approach superior in comparison to the “Subjective” approach which is bound strictly to European options due to the usage of the Black and Sholes framework.

The “Revised Classic” approach uses a decision tree which demands discrete scenarios with attached probabilities, in case the replication assumption does not hold. Mapping out distinct scenarios and estimating probabilities for each scenario in light of a dynamic and high uncertain development of a startup is not trivial. With more uncertainties present, the decision tree quickly increases in size and gets complex to compute. This approach additionally complicates its usage through the need of a changing discount rate throughout the tree for being valid. These mentioned problems lead to a more complex usage compared to the “MAD” approach which incorporates several uncertainties into a combined volatility value. Additionally, the binomial lattice of the “MAD” approach has the advantage of using a constant risk-free discount rate instead of a changing discount rate, as used in the decision tree.

The “Integrated” approach is similar to the “Revised Classic” approach, except that it has the possibility of incorporating market data about the uncertainties. This can generally be seen a benefit in comparison to the “Revised classic” approach although without the availability of usable market data, this advantage disappears. The drawbacks of this method are the same as in the “revised classic” approach, namely the changing discount rate problem and the demand for estimating distinct scenarios. Therefore, it can be concluded that theoretically even though this method is superior to the “Revised classic” approach, it is inferior in comparison to the “MAD” approach regarding the calculation methodology.

The problems of all approaches analyzed, regarding assumptions, data availability and calculation methodology lead to the conclusion that the “MAD” approach is the most applicable approach for valuing non-exchange traded startups.

4.4 Optimizing the Chosen Approach

The main problems concerning the usability of the “MAD” approach are the derivation of the discount rate for calculating the underlying, using the DCF method and the subjective estimation regarding the future development of the startup, used for generating data. The risk adjusted discount rate is generally derived with the WACC using the CAPM, which is problematic when applied on a startup.

A model for risk adjustment of uncertain cash flows, using a certainty equivalent, was developed by Zhang (2010). This model tries to objectively calculate a discount rate in a different way compared to the CAPM.

Regarding the data requirements and the theoretical assumptions, the result generated by this approach is suitable to risk adjust the CFs for a proper DCF valuation and therefore deriving the startup value and incorporating the uncertainties. The model is briefly explained below and later operationalized in chapter 5.

Risk Equivalent Model

The basic notion of this model:

Certainty equivalent = X_d = Expected Value X – risk equivalent (1)

This is based on the rational that a Certainty equivalent X_d equals the expected value minus the value reduction caused by risk. The underlying assumption is, that the real value is distributed normally around the estimated expected value and that the risk equivalent, which is the risk of falling short of the normal distributed expected value, can be captured with a put option.

Risk is the danger, that the actual value is smaller than the expected one. To value the certainty equivalent in this equation, a European put option is used. Using the put option a certainty equivalent coefficient is computed which has a range: $\{CEC \in \mathbb{R} | 0 \leq CEC \leq 1\}$.

Formula 2 below shows the basic approach using a put option at t=0:

$P_o = X * e^{-r_f * T} * N(-d_2) - S_0 * N(-d_1)$ (2)

S… Current value of the forecast
X…Forecast value at maturity time of the forecasted cash flow
T…Maturity time at which X occurs
σ… Annual standard deviation of forecasting value

Because S is used as the present value of the variable X, the connection between these two leads to formula 3 below:

$S_0 = X * e^{-r_f * T}$ (3)

Formula 2 and 3 combined leads to formula 4:

$P_o = X * e^{-r_f * T} * [N(-d_2) - N(-d_1)]$ (4)

Substitution of statistical terms d_1 and d_2 using the relation introduced in the Black and Sholes discussion in chapter 3.4.1 equals to formula 5:

$P_o = X * e^{-r_f * T} * [2N\left(\sigma\sqrt{\frac{T}{4}}\right) - 1]$ (5)

P_o computed in formula 5 is captured at t=0, according to the basic notion of this risk adjustment method. The risk equivalent, the certainty equivalent and the expected value occur at a future time shown in formula 1. Therefore, the present value of the put option value has to be transferred to the expected future date shown below in equation 6:

$Risk\ equivalent = P_o * e^{r_f * T}$(6)

Equation 5 and 6 combined lead to the formula for the risk equivalent below in formula 7:

$Risk\ equivalent = X * [2N\left(\sigma\sqrt{\frac{T}{4}}\right) - 1]$ (7)

Using formula 1, the relation between the risk and the certainty equivalent can be established which leads to formula 8:

$Certainty\ Equivalent = X - X * [2N\left(\sigma\sqrt{\frac{T}{4}}\right) - 1]$ (8)

To compute a coefficient instead of an absolute number which can be integrated easily into other calculations, the forecast value X at maturity is set to 1. The certainty equivalent coefficient d then equals to formula 9:

$d = 2 * [1 - N\left(\sigma\sqrt{\frac{T}{4}}\right)]$ (9)

This way the risk equivalent coefficient ν can be computed in formula 10:

$\nu = 1 - d = 2N\left(\sigma\sqrt{\frac{T}{4}}\right) - 1$ (10)

The model has two variables: The volatility or uncertainty σ and the time T . Testable at this model are the values at the limit of the certainty equivalent coefficient and the risk equivalent coefficient:

$\lim_{T\to\infty} d = 0\,, \lim_{\sigma\to\infty} d = 0$ (11)

$\lim_{T\to\infty} \nu = \infty$, $\lim_{\sigma\to\infty} \nu = \infty$ (12)

Formula 11 can be interpreted as the behavior of the certainty equivalent coefficient d in case the time, till the value is captured and the uncertainty of the value captured, move towards infinity. The certainty equivalent d ,in both cases, converges to 0. These are reasonable behaviors of the certainty equivalent coefficient because it shows the influence, time has on the coefficient as well as the uncertainty has on the certainty equivalent. The risk equivalent in formula 12 has the opposite behavior which is consistent.

Discounting of cash flows mainly consists of two steps: Adjusting the expected value with a risk premium to reach the certainty equivalent and discounting this certainty equivalent at the risk-free-rate. Both steps are necessary to account for the time value of money and the uncertainty about the cash flows.

This model is appealing for the calculation of the underlying in the “MAD” approach because it simplifies the risk incorporation into the valuation of the underlying to two variables: The time of cash flow occurring T and the return volatility of the estimated startup development σ which gets calculated through a MCS of the projected cash flows.

5 Valuation of an Example

In order to show the valuation approach on a realistic example, startups in the IT sector were searched for, due to the existence of multiple companies which fit to the defined startup characteristics. “Runtastic” was identified as being a startup and having a business plan available for usage (Runtastic, 2009). This plan incorporates multiple product lines which would further complicate the estimation task without having impact on the applicability or the calculation framework. Therefore, the business plan is simplified in concentrating the business model of the startup to one product, which is also a realistic assumption regarding startups in the IT sector. Focusing on one core product is “the” startup strategy, as emphasized by Portnoy (2014). The startup example used in this valuation is named differently, because of the slight adaptions. Although the same business model is used and a similar development path is projected.

Startup Example

Startup: Biketastic

Field: Mobile application for sport tracking

Business Plan:

- Development of a software for capturing, storing and sharing sports data over the internet
- Incorporating GPS and pulse tracking hardware already existing on the market
- Providing software with advertisements implemented to the customer for free and providing a paid version without advertisements.

Customers: Due to the quantity and the composition of the market, starting locally on a very limited scale to test the product, then expanding globally.

Stage of Startup: Major part of software already developed by the entrepreneurs, at startup initiation, software is rolled out locally. Early stage, startup with no revenues yet.

Real Option Structure: It is assumed, that the strategy of expanding globally and attracting new customers is possible to be implemented for three years starting at time 0 and this expansion is estimated to cost 4 million Euros. After this time, competition is assumed to have taken over the market and the startup is worthless. The option strike price is the investment of the global product rollout, which is considered the substantial amount in this startup financing.

Business model: “App” is downloadable onto a mobile phone and free to use in a standard version but with advertisements. The “premium” version has an initial charge but is advertisement free in usage. The advertisement space is sold.

The calculation of the real option value is done in two steps using the calculation procedure of the “MAD” approach:

1. Calculation of underlying value and the volatility of its returns
2. Valuing the real option using binomial lattice

5.1 Calculation of the Underlying Value and Volatility

Building on the uncertainty factors driving startup value identified in chapter 2.5 and using the AHP approach, the future development of the startup is estimated. This is done with an estimation of “Inflexible” future Free Cash Flows and possible deviations from the “fixed” estimates due to the identified uncertainties. Zhangs risk adjustment model is incorporated for calculation of the discount rate.

Risk Identification

Based on the categorized uncertainty factors in chapter 2.5, sub risks for each category are estimated. Risks are identified regarding their financial impact:

- Product and Technology:
    - Problems of software postpone market launch
    - Failure of final software development
    - Problems using other on the market existing hardware with own software
    - No market need for software at launch
- Management and Organization
    - Disharmony among team
    - Strategic issues
    - Execution problems
    - Marketing issues
- Funding
    - Additional unplanned financial needs for development and launch
- Competition
    - Better product already in place at launch
    - Competition catches up

The identified firm specific risks are then analyzed:

Risk Analysis

➔ AHP method used

AHP Risks above numbered:

- R1: Product and Technology:
- R2: Management and Organization
- R3: Funding
- R4: Competition

The Scale in Table 1 is used to compare the risks of the four identified risk categories and to quantify their relative importance concerning the impact on the startups financials. First, the AHP Matrix below in Table 4 is filled comparing one risk with another, the reciprocal of the comparison is used when comparing their inverse relationship. The risk factors compared are evaluated for their “Financial impact”.

Risk	**R1**	**R2**	**R3**	**R4**	**Nth root of product**	**Eigenvector ω**
R1	1,00	3,00	7,00	2,00	2,55	0,49
R2	0,33	1,00	4,00	2,00	1,28	0,25
R3	0,14	0,25	1,00	0,20	0,29	0,06
R4	0,50	0,50	5,00	1,00	1,06	0,20
Total					5,17	1,00

Table 4: AHP Matrix

After factors for comparison of the risks are estimated and the matrix is filled, the Nth root of the product of each row is calculated. This product acts as a basis for calculating the Eigenvector ω which can be interpreted as a relative importance of the individual risks in comparison to each other. The Eigenvector ω for each risk is calculated with dividing the Nth root of product each row through the Total of this column.

A sanity check is conducted following this calculation to determine whether or not the individual judgements are consistent to each other or not, in order to increase the trustworthiness of the result.

The consistency ratio CR is calculated and compared to variables provided by Saaty (1980) below in Table 5. AHP theory says $A * \omega = \lambda_{max}$,therefore $A * \omega$ has to be calculated. The last column of Table 4 represent the Eigenvector ω which has to be multiplied with the entries of each row to get a new vector:

$A_1 * \omega_1$ =1,00*0,49+3,00*0,25+7,00*0,06+2,00*0,2=2,04
A*ω equals (2,04; 1,04; 0,23; 0,86)

To derive λ_{max} ,the above calculated product $A_x * \omega_x$ has to be divided by the corresponding Eigenvector ω. λ_{max} is the arithmetic mean of these values.

2,04/0,49=4,14
1,04/0,25=4,23
0,23/0,06=4,08
0,86/0,20=4,18
Arithmetic Mean: 4,16 = λ_{max}

One side note: The values individual λ values have to be bigger than the size of the matrix, in this case n=4 which leads to a requirement of λ>4. This is the case in this calculation.

The consistency index (CI) of the matrix is calculated now and compared to the CR:

$CI = (\frac{\lambda_{max}-n}{n-1})$ = 0,05

Saaty (1980) provides a comparison scale to evaluate our judgment, shown below in Table 5:

n	1	2	3	4	5	6	7	8	9	10
RI	0,00	0,00	0,58	0,9	1,12	1,24	1,32	1,41	1,45	1,49

Table 5: Average Random Index Values (Saaty, 1980)

Our matrix has 4 entries, therefore RI= 0,9 in Table 5 is the right value to choose.

CR= CI/RI= 0,05/0,9= 0,06

The higher the judgement, the more untrustworthy the result because it mimics random guessing. The consistency ratio (CR) states the error in consistency between judgements, if the number is smaller than 10%, the result is acceptable according to this method. Our result of 6% inconsistency is well below this limit.

The importance of the risks identified, which have an impact on CFs were calculated in Table 4 as the Eigenvector ω in the last column. Then they were checked for consistency in judgement and are listed below in a hierarchical order. The Eigenvector ω can be interpreted as the importance of the risks influencing the future cash flows:

R1: Product and Technology	49%
R2: Management and Organization	25%
R4: Competition	20%
R3: Funding	6%

The future cash flow, the startup is going to generate is captured with the DCF analysis. Due to the “MAD” approach, the development of the startup has to be estimated in two ways: Without flexibility, to derive a basic underlying value and with flexibility, to capture the uncertainties. Due to the dynamic nature of a startup, this period should not expand too far into the future in order to minimize estimation errors. A 5-year period is chosen for estimating the development path of the startup.

The option for expanding the business is assumed to expire in three years based on initial assumptions at the beginning of this chapter and the option is exercisable anytime.

MCS is used for simulating the impact of the uncertainties in the development of the startup. Incorporating these uncertainties as distributions of the forecasted cash flows, returns a value of the underlying as well as a volatility of the return of the underlying, which is needed in the next step.

A spreadsheet is used to conduct the DCF valuation through estimating the FCF and discounting them back to the present. The following components are recommended and used by Copeland and Antikarov (2001a) in the “MAD” approach:

- Revenue
- Costs
- Taxes
- Capital Expenditures (CAPEX)
- Increase in working capital

The calculation to determine the FCF:

$$FCF = Revenue - Costs - CAPEX - Increase\ in\ working\ Capital$$

To simplify the estimation task of the uncertain future development, the factors are adjusted to fit the startup:

- Revenue= Number of estimated users is multiplied with an estimated revenue per user
- Costs
- Tax rate= 20% of positive FCF
- CAPEX is assumed to be included in costs
- Working capital change assumed to be minimal at a software startup

To determine the underlying value, a base forecast of the startup is needed which does not incorporate flexibilities in the development. The key factors which drive the uncertainty regarding the outcome of the startup were identified using the AHP risk hierarchy and are Product and Technology risks. This factor is assumed to have the biggest financial impact. The impact of the Product and Technology risk, leads beside the other three risk factors to an uncertainty in revenues. Costs are assumed to be without variability because the major software development is already finished and the uncertainties therefore limited to minor expenses.

A Terminal Value (TV) is used to capture the value after the projected cash flows. Due to the specific business model, forecasts for “Revenue per customer” and “Number of customers” are made for the expected revenue.

Copeland and Antikarov (2001a) emphasize, that for each of the two uncertainties, three values have to be estimated:

- Value at the beginning
- Expected value in year 5
- Worst or best case with 95% confidence in one of the years for deriving volatility of the estimated development

The expected values are point estimates, the upper or lower values for each interval are estimated with a 95% confidence interval. A confidence interval describes the range of plausible values for an unknown parameter. The confidence level describes the probability that the unknown parameter is in the proposed range Dekking et. al (2005)

For example: In the beginning, revenue/customer is expected to be 0,5€ and in year 5 it is assumed to grow to 2€. With a 95% confidence interval, the worst case is expected of not being lower as 1€ per customer in year 5.

The subjective estimates are shown in Table 6 below, marked in red. The other values can be calculated based on these:

Year	**0**	**1**	**2**	**3**	**4**	**5**
Revenue/Customer						
Worst Case		0,48	0,56	0,67	0,82	1,00
Expected	0,5	0,66	0,87	1,15	1,52	2,00
Good		0,9	1,35	1,97	2,82	4,00
# of Customers						
Bad		9794	13792	20619	31793	50000
Expected	10 000	18206	33145	60342	109856	200000
Good		33842	79650	176592	379597	800000
Cost						
Expected	20000	24022	28854	34657	41628	50000
Tax 20%						

Table 6: Forecasted Startup Development

The following methodology is used for calculating the cash flow development:

The development of the expected values is assumed to follow a constant compounding rate.

$E_{t+\Delta t} = E_t e^{r\Delta t}$ (13)

E_t…Expected value at time t
r…Compounding rate

The boundary estimate leads to calculation of volatility.

If the upper boundary is estimated, formula 14 is used with V_T^{upper} as the upper estimated boundary value and V_0 as the initial value at t=0:

$\sigma = \frac{\ln\left(\frac{V_T^{upper}}{V_0}\right) - \sum_{i=1}^{n} r_i}{2\sqrt{T}}$ (14)

In case the lower boundary is easier to be estimated, formula 15 is used V_T^{lower} as the lower estimated boundary V_0 as the initial value at t=0:

$\sigma = \frac{\sum_{i=1}^{n} r_i - \ln\left(\frac{V_T^{lower}}{V_0}\right)}{2\sqrt{T}}$ (15)

For the MCS, the excel add-in software “ModelRisk” developed by “Vosesoftware.com” is used.

The standard formula for calculating the PV at t=0 is shown below in formula 16:

$PV = \sum_{t=0}^{5} \frac{FCF_t}{(1+WACC)^t} + TV$ (16)

In order to incorporate the risk adjustment approach developed by Zhang (2010), formula 16 has to be adapted. The FCF_t are adjusted for risks in a way, that the certainty equivalent is calculated, which is then discounted back to t=0 at the risk-free-rate to compensate for the time value of money:

$PV = \sum_{t=0}^{5} \frac{CE(FCF_t)}{(1+r_f)^t} + TV$ (17)

The TV is an important part of the NPV of a startup because profitability is assumed to persist due to the going concern assumption. There are several ways for computing the TV, the one chosen here is the “Constant Growth Model” further described by Lütolf, Pirnes and Indaimo (2011b). It assumes a constant growth g, which is estimated to be a conservative of 3% after year 5. The free cash flows in year 6 are assumed to be the cash flows from year 5 with a growth rate g of 3%:

$FCF_6 = FCF_5(1+g)$ (18)

This leads to a TV shown below in formula 19:

$TV = \frac{FCF_5 * (1+g_\infty)}{(WACC_\infty - g_\infty)}$ (19)

For the $WACC_\infty$ the capital costs for a mature business should be chosen. Damodaran (2020) published a list where the costs of capital of US companies are outlined per sector. Our startup best fits to the sector “Software (Internet)” which leads to the following average capital costs of mature businesses: Cost of equity 10.62%. Because the startup is assumed to be solely equity financed also at maturity in order to keep it simple, the $WACC_\infty = C_e = 10.62\%$.

Table 7 below shows the inflexible FCFs based on the forecasted and calculated values in Table 6:

Year	1	2	3	4	5	TV
FCF	**-12 011**	**0**	**27 726**	**99 906**	**280 000**	**3 784 777**

Table 7: Inflexible FCF

Compared to the size of the other cash flows, the terminal value is a major contributor to the total value of the startup.

In an environment with negative interest rates, finding a risk-free interest rate in the market is problematic. For simplicity in the calculation, a risk-free-rate of 3% rate is assumed.

For the subsequent Real Option Valuation, the standard deviation of the return of the startup σ is needed. An assumption of the Black and Sholes equation and of the recombining binomial tree approach, used in this Real Option Valuation is, that the volatility remains constant over time.

The volatility used for Real Option Analysis is the volatility of the return from the PV. To derive this volatility using MCS, the following relation is used shown in formula 20:

$r = \ln\left(\frac{PV_1 + FCF_1}{PV_0}\right)$ (20)

The present value PV_1 is calculated analog as the PV_0, using formula 17, except that the values are discounted over different periods. The return r is simulated and for the PV_1 , the actual FCF_t distributions are used. For the PV_0 value, the expected forecast value without flexibility is used, which is constant.

The risk adjustment method developed by Zhang (2010) is influenced by the risk-free rate, the time and the volatility of the returns of the startup. The volatility needed in this calculation is the expected standard deviation of returns, which is the standard deviation of the return distribution calculated by simulating equation 20. This data is not available at this state of the calculation, which leads to **Problem 1**:

5.1.1 Problem 1

- The volatility for calculating the risk adjustment is not available when needed for the calculation of the underlying value.

The volatility is based on the change of the underlying startup value over time. Problematic with this is the order the underlying value is calculated. The volatility is needed for calculating a discount rate and subsequently to discount the cash flows, but the volatility is calculated out of the result of this DCF.

One possible solution for this issue is to analyze the sensitivity of the distribution for simple discount rate changes. In case sensitivity is low, it would be possible to use an estimated return volatility for deriving the underlying value and based on this, the exact return volatility without losing important accuracy.

For this sensitivity analysis, estimated startup development in Table 6 are used. To isolate the problem, costs were neglected in this calculation because they present an issue by themself, as discussed in “Problem 2”, further below.

For comparing the variation of the standard deviation, caused by a change in the discount rates, MCS was conducted with 10000 samples and five different discount rates, shown below in Table 8. The return standard deviation is simply the measured standard deviation of the simulated return using “ModelRisk”:

Discount rates	**Return standard deviation**
3%	76,161%
10%	76,271%
50%	76,653%
100%	76,855%
150%	76,950%

Table 8: Discount Rates

The result presented in Table 8 indicates a low sensitivity of the return standard deviation by a change of the discount rate. This shows, that the major influence factor regarding the distribution is not the discount rate, rather the distribution of the cash flows, as shown below in Illustration 12:

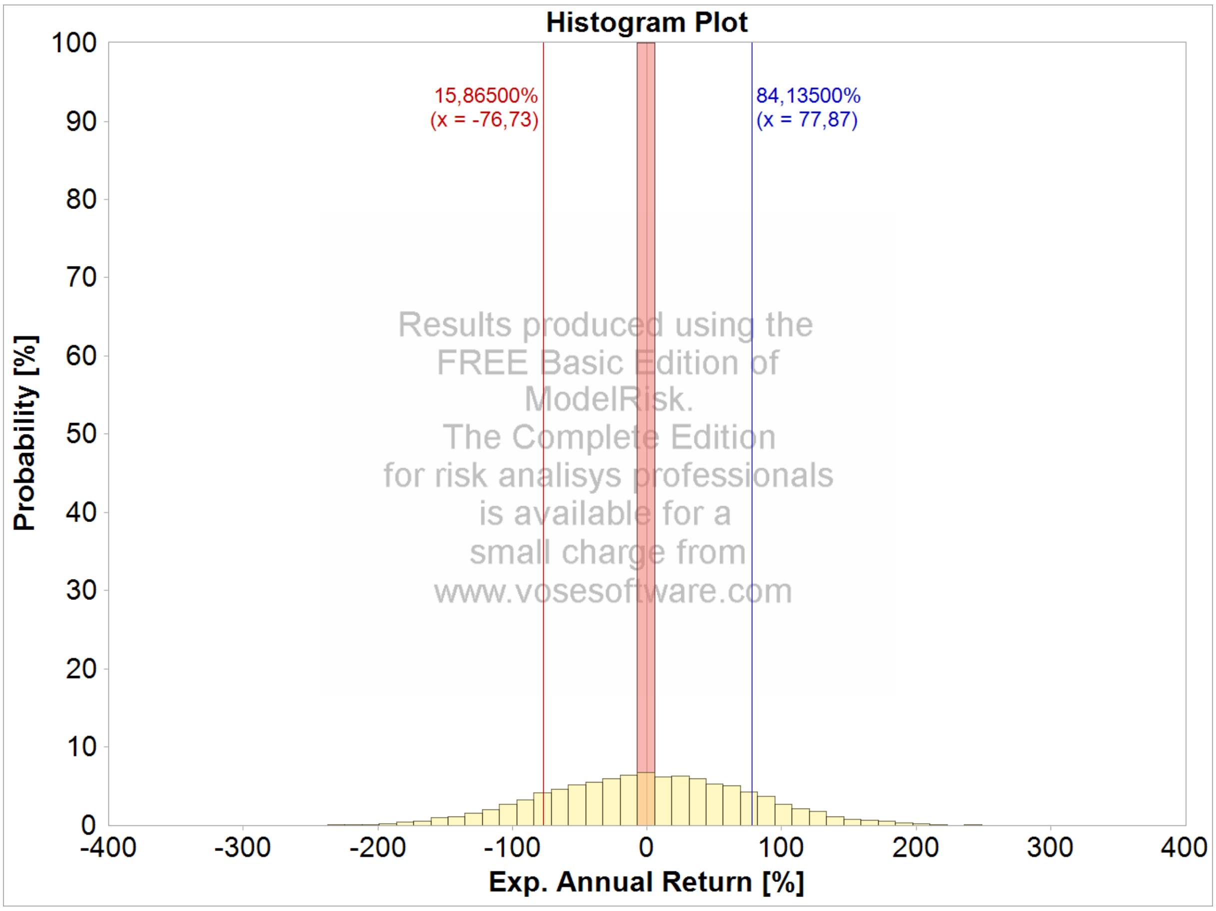

Illustration 12: Return Distribution

The yellow distribution in Illustration 12 is the actual distribution of returns using the data from Table 6 and a discount rate of 50% , only the costs were neglected in the FCF calculation. The reason for this is further explained later at “Problem 2” and would only distort the point made in presenting this chart. The red “distribution” is the inflexible projection of FCF development of the startup. This illustration was generated using 10000 iterations. The thin blue and red line indicate the interval of one standard deviation to the left and one standard deviation to the right of the mean of the yellow distribution. The interval of one standard deviation of the red function collapses into one line because the volatility is zero. Visible in this Illustration is the difference in standard deviations between the distribution of the cash flows and of the “inflexible” forecasted cash flows. This supports the point made with Table 8 and leads to the solution of problem 1 in picking an almost random discount rate for calculating the return distribution in order to use Zhangs model for risk adjustment. Because the change in the standard deviations of returns is not zero while changing the discount rate, ideally the “random” discount rate should be chosen in a realistic range of 50% for a startup to minimize the error.

The risk adjustment of the cash flows using the model by Zhang (2010) can be calculated now. The certainty equivalent coefficient is computed in the following Table 9:

Year	1	2	3	4	5
Volatility of return	0,76653				
CEC	0,7015	0,5878	0,5068	0,4434	0,3914

Table 9: Certainty Equivalent Factor for PV_0

For calculation of the PV_1, the CEC discount factor has to be recalculated because the time periods till the CF occurs has shifted, as shown in Table 10 below:

Year	1	2	3	4	5
Volatility of return	0,76653				
CEC	1,0000	0,7015	0,5878	0,5068	0,4434

Table 10: Certainty Equivalent Factor for PV_1

The costs were neglected in the former simulation in order to reach a mathematically correct result, the reason why the cost components were not included, is the possibility of reaching negative values. This is the case in startups with regularly negative cash flows at the beginning. Using equation 20 in the MCS for calculating the return, erroneous results can occur in cases where either the numerator or the denominator turns negative. The logarithm is not defined at 0 or in negative territory. This leads to **Problem 2:**

5.1.2 Problem 2

- Negative cash flows lead to mathematical problems and a breach of the “MAD” approach assumption which is a normal distribution of the return.

Formula 20 divides the present value of the estimated future cash flows $PV_1 + FCF_1$ which includes a variation of cash flows, with the PV of CFs of the startup without variation or “flexibility”:

$$r = \ln\left(\frac{PV_1 + FCF_1}{PV_0}\right) \quad (21)$$

In case the estimated PV_0 is negative or the sum of $PV_1 + FCF_1$ is negative, deriving a result for the return is not possible using formula 20 due to the use the logarithm on a negative value. The MCS calculation software “Model Risk” computes errors in these cases. These computational errors and the instances where the return value is close of being an error, influences the return distribution. The result is a distribution which deviates from a normal distribution, expressed in statistical terms: Excess skewness and kurtosis. This is shown in Illustration 13 below. The MCS used 10000 iterations, costs are included in the calculation and risk adjustment is done using the values in Table 9 and Table 10.

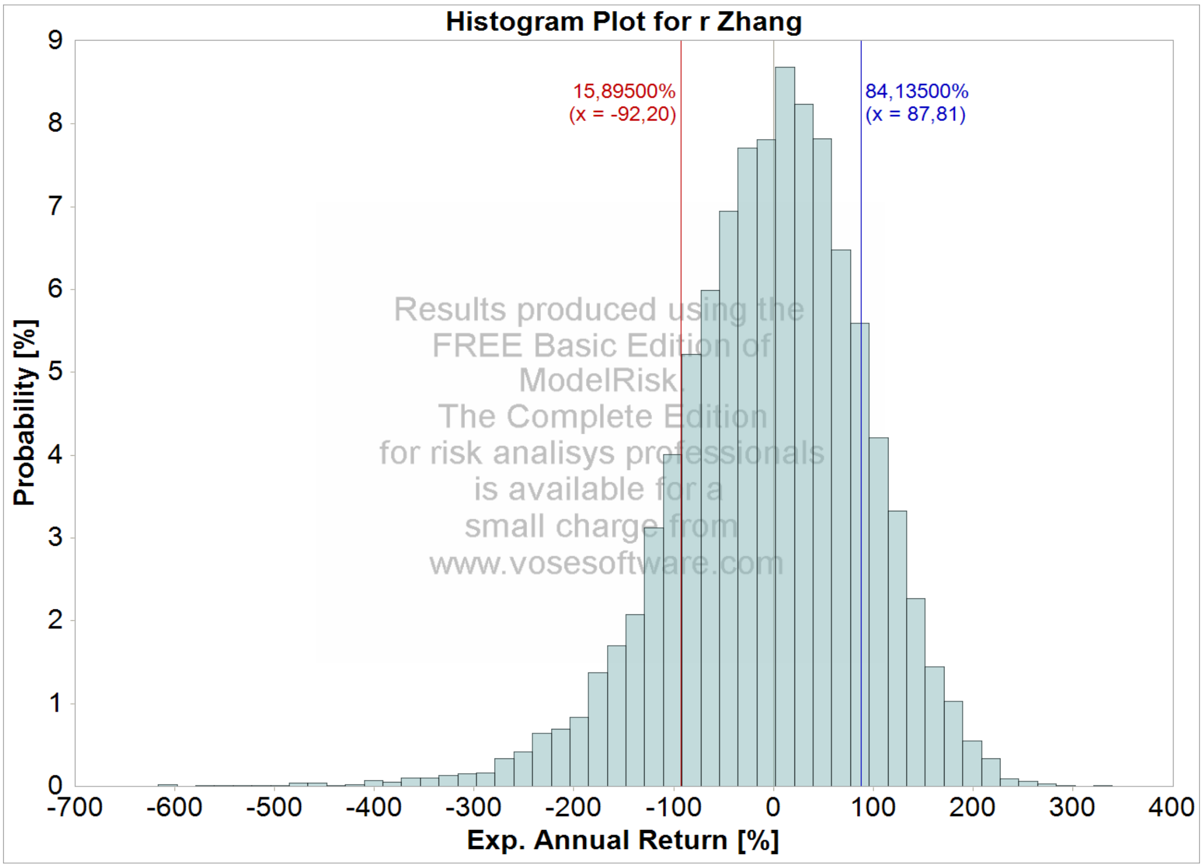

Illustration 13: Return Distribution with Negative CFs

The statistics of Illustration 13 are listed in Table 11 below:

Number of errors	45
Skewness	-0,6639
Kurtosis	4,6671

Table 11: Illustration 13 Statistics

A normal distribution is symmetric, therefore has no skewness and a kurtosis of +3. The kurtosis of the result is 4,6671 which can be considered “excessive”. The skewness of -0,6639 is also far of a normal distributed skewness. These results are troublesome regarding the assumption of the binomial lattice framework which requires a normally distributed return. In order to prevent the distribution deficiencies and the mathematical errors, a rule for the “MAD” approach must be defined for deriving the volatility must be defined below in formula 22:

$$\frac{PV_1+FCF_1}{PV_0} > 0 \quad (22)$$

Additionally, negative cash flows, which get risk adjusted and discounted back with the DCF are also problematic from a theoretical perspective. When risk adjusting negative cash flows into certainty equivalent ones, using a certainty equivalent coefficient, they decrease in size and therefore get less negative. This implies that more risk, decreases the adverse effect of negative cash flows. This is different to the positive cash flow risk adjustment, which is commonly done, and which decreases the positive impact on value, the more uncertainty present. This issue was picked up by Luehrman (1998b) who points out that this is a common mistake of DCF valuation. He emphasizes that “…*expenditures are rarely subject to the same operating and product-market forces that make project cash flows risky*” (p.21) and future cash outlays should not be discounted “…over optimistically” (p.21). He uses the risk-free-rate for discounting negative cash flows. This insight is used when deriving the value of the underlying.

One possible solution, approaching the negative cash flow issue with using MCS, is to discount the costs, which are the biggest factor leading to negative cash flows and are assumed to be fixed, at the risk-free-rate and to deduct them from the positive PV at time zero. This is possible because cash flows are additive. The cash flows occurring remain positive and the return distribution is normal. Because the mean value of the return is not of importance for deriving the volatility of the return, this would be a reasonable approach. For calculation of the underlying value, the costs definitely have to be included.

The return distribution of Illustration 13, after dropping the costs of the calculation, is shown below in Illustration 14:

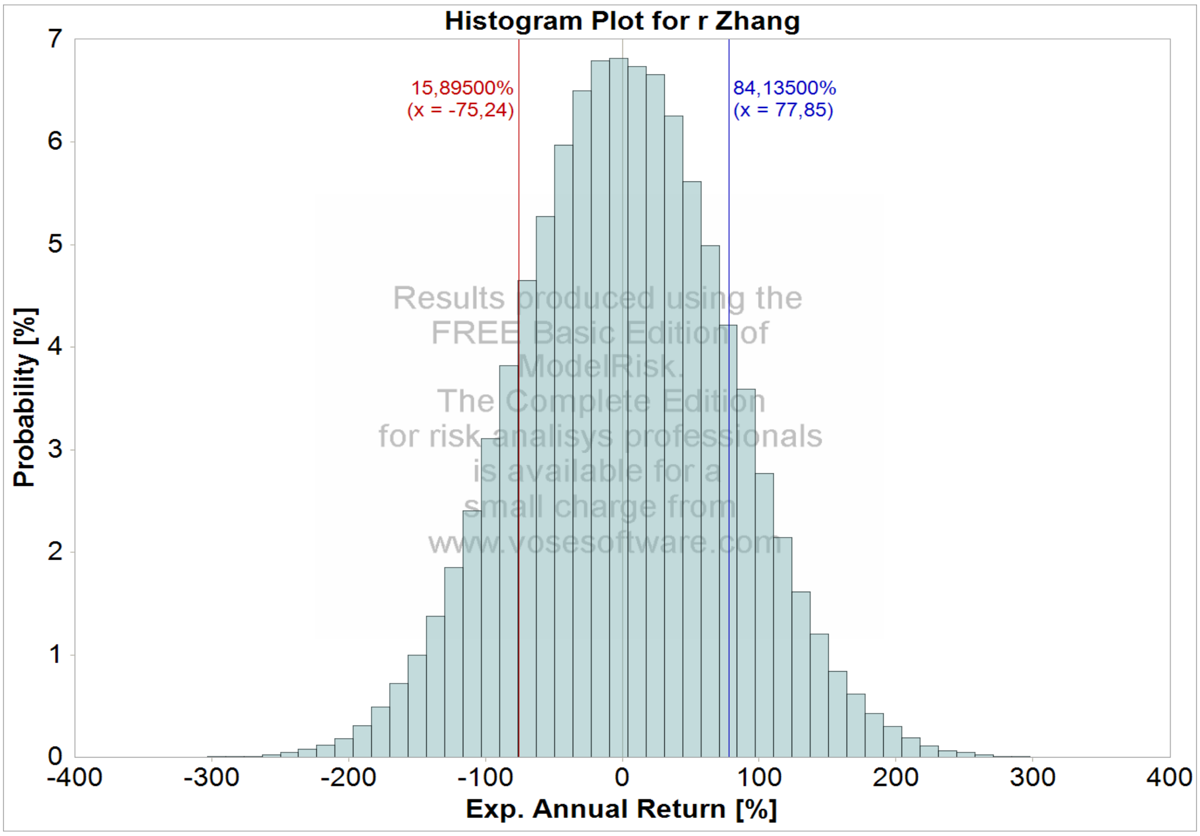

Illustration 14: Expected Annual Return with Costs excluded

The return distribution in Illustration 14 has the following properties shown below in Table 12:

Number of errors	0
Skewness	0,0222
Kurtosis	3,0149
Iterations	100 000
Volatility	0,7677

Table 12: Illustration 14 Statistics

The values in Table 12 are close to an ideal distribution, after repeating the MCS several times the descriptive statistical numbers change around the ideal values of skewness 0 and kurtosis +3.

This leads to the final standard deviation of returns used as the volatility in the binomial lattice approach later on:

Expected annual return volatility **σ**	76,77%

The value of the underlying at time $t = 0$ is the PV of the startup incorporating the “inflexible” development. The additional value due to the volatility is captured with the binomial lattice approach in the next section.

The estimated “inflexible” development of the startup in Table 6 is used and risk is adjusted with the model of Zhang in Table 9 and Table 10. To calculate the PV_0, the $PV_{0\,Revenue}$ of “inflexible” future cash flows is used and the costs $PV_{0\,Cost}$ are deducted to reach the PV_0, shown below in formula 23:

$Value\ of\ underlying = Inflexible\ PV_0 = PV_{0\,Revenue} - PV_{0\,Cost}$ (23)

Problematic here is the tax deduction. It would be improper to only deduct taxes from the revenue without the costs in the same period. This would lead to an incorrect amount of taxes being paid.

Standard approach for deducting taxes, of the simplified startup, is shown below in formula 24:

$FCF_t = (Revenue_t - Cost_t) * (1 - Tax\ rate)$ (24)

Costs are deducted before the taxes are being calculated. Because a decrease of costs would increase the taxes being paid, formula 25 would be wrong:

$FCF_{t\,Revenue} = (Revenue_t) * (1 - Tax\ rate)$ (25)

One way of solving this issue and deducting the taxes properly each year is defined in formula 26 below:

$FCF_{t\,Revenue} = Revenue_t - right\ taxes\ paid = Revenue_t - ((Revenue_t - Cost_t) * Tax\ rate)$
(26)

Now it is reasonable to discount the costs separately and deduct them later. Taxes are only paid when FCFs are positive, therefore in year 1 and year 2 where FCF_t are negative and zero, this adjustment must not be made.

The result of the individual PV_0 components are listed below in Table 13:

$PV_{0\,Revenue}$	€ 4 005 057
$PV_{0\,Cost}$	€ 162 353
Investment Cost I_0	€ 4 000 000

Table 13: NPV Components

The $Inflexible\ PV_0$ value of the startup at t=0 is calculated using formula 23:

$Inflexible\ PV_0$	€ 3 842 704

Investing now in the startup and rolling out the software on an international scale is estimated to cost 4 Mio €. Using this to calculate the NPV of the startup:

$Inflexible\ NPV_0 = PV_0 - I_0$ (27)

Result of formula 27:

$Inflexible\ NPV_0$	€ -157 296

Using the NPV decision criterion introduced in section 3.4.1, the investment would not be undertaken.

5.2 Building the Binomial Lattice

A binomial lattice is built to value the startup following the “MAD” methodology. The decision for expansion of the startup can be made every year up to year three. If the startup does not develop favorably, the major investment for expanding internationally is not conducted. The real option is left unexercised and the startup is assumed worthless without an abandonment value.

For the evolution of the binomial lattice, the earlier calculated underlying value and volatility are used. In Table 14 below, essential variables for building the binomial lattice using the risk neutral probability approach, are listed.

Underlying Value	PV_0	3 842 704€
Volatility	σ	0,7677
Risk-free rate	r_f	3%
Up move	$e^{\sigma\sqrt{\delta t}}$	2,1548
Down move	$e^{-\sigma\sqrt{\delta t}}$	0,4641
Risk neutral probability	$\frac{e^{\mathrm{rf}(\delta t)} - d}{u - d}$	0,3350
Time step	Δt	1 year

Table 14: Binomial Lattice Variables

The two-step binomial lattice approach first creates a tree based on the evolution of the underlying value and then a second tree is derived of the first one in order to capture the real option value. The lattice evolution of the underlying is displayed in Illustration 15 and starts with the PV of future cash flows in year 0 and develops with an expected annual volatility of returns up to year 3. The time steps Δt between the nodes were chosen to be one year. More nodes would lead to more precision, as was explained in chapter 3.4.1.. For showing the applicability of the framework on startups, as well as showing the operationalization of the altered methodology, this imprecision is tolerable but should to be improved in further research.

Below in Illustration 15, the evolution of the underlying value over time is shown:

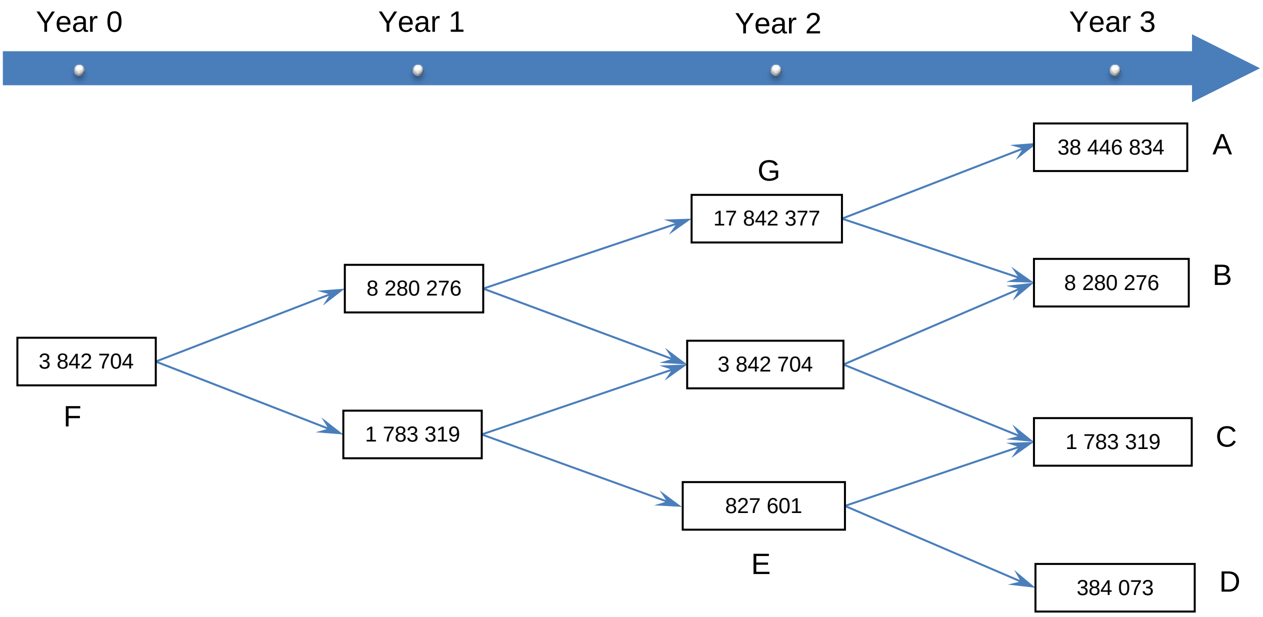

Illustration 15: Lattice Evolution of the Underlying

The strike price for exercising the option and the abandonment value, if the investment is not conducted and therefore the real option expires unexercised, are shown below in Table 15:

Strike price X	4 000 000
Abandonment value	0

Table 15: Exercise Price and Abandonment Value

Deducting the exercise price from the nodes in year 3 of Illustration 15 leads to the values shown below in Table 16:

Node	A	B	C	D
Exercise Value	34 446 834	4 280 276	-€ 2 216 681	-€ 3 615 927

Table 16: Year 3 Value

Because the exercise of the option is voluntary for the option holder and the value of the startup, if the option is not exercised in year 3, equals 0, the value maximizing decision in node C and D is to let the option expire. The decision possible at the nodes in the lattice is either to investment or to wait, shown below in formula 28:

$MAX\ value = MAX(\ Investment, Wait)$ (28)

In Illustration 16, this value maximizing approach of the Real Option framework is shown. The value of the startup at node A, B, C, D is then discounted back to the present. Because the option can also be exercised in year 0, year 1 and year 2, these nodes are checked for the maximum value between exercise and waiting. The highest value is picked at each node to compute the real option value. These highest values at each node are shown below in Illustration 16:

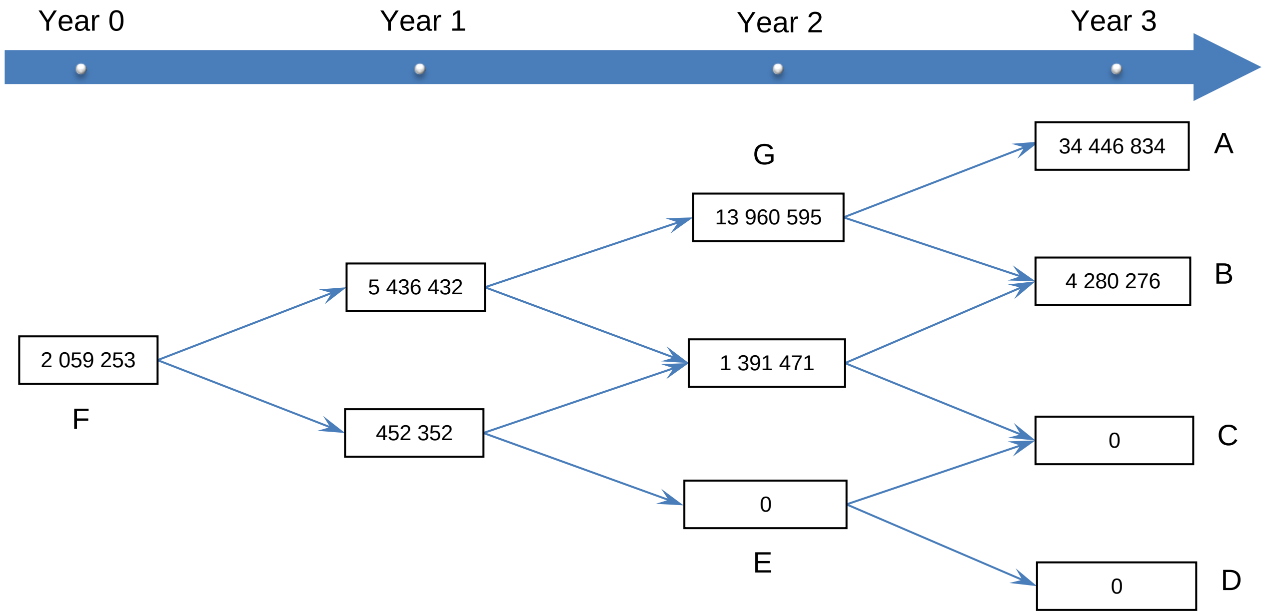

Illustration 16: Option Valuation Lattice

For example, at node "G", the exercise is compared to the waiting decision: The value of the startup in node "G" of Illustration 15, minus exercise price is compared to the discounted value of keeping the option open.

$$MAX(\,Investment, Wait) = MAX\left(17842377 - 4000000, \left[\mathrm{P}(34446834) + (1 - \mathrm{P})(4280276)\right] \exp(-\mathrm{rf} * \mathrm{dt})\right) \tag{29}$$

Equation 29 equals: $MAX(\,Investment = 13\,842\,377, Wait = 13\,960\,595)$. The economical decision is not to exercise the option at Node "G".

Rolling back the tree this way, leads to the real option value for the startup:

Real Option Value	2 059 253

A comparison between the real option value and the inflexible NPV for the startup in Illustration 17 is shown:

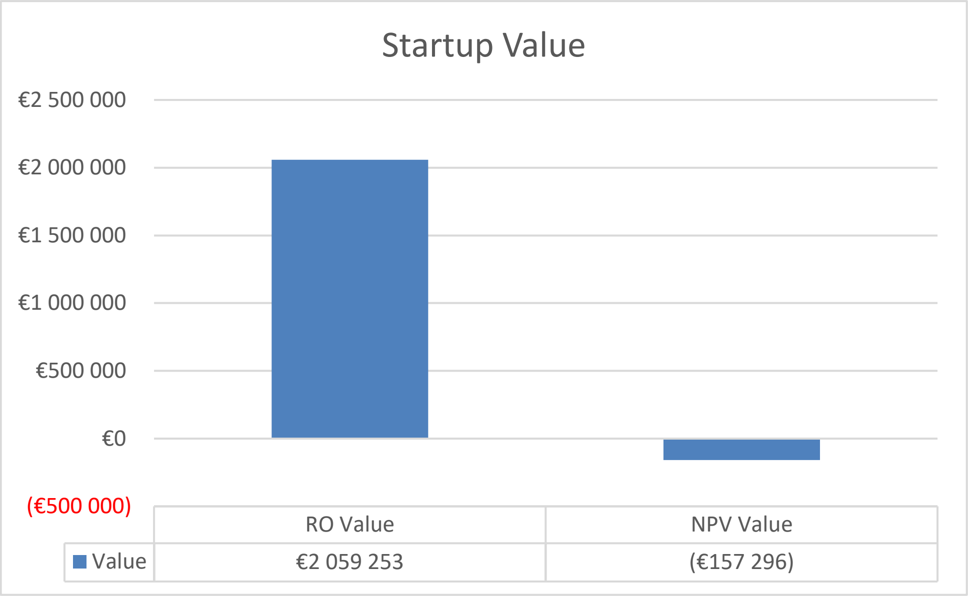

Illustration 17: Value Comparison

Illustration 17 shows the value of the difference in valuation between the Real Option Valuation and the NPV approach. The flexibility incorporated into the Real Option Approach can be made explicit using formula 30:

$Flexibility\ Value = ROA\ Value - NPV\ Value$ (30)

This leads to the flexibility value of 2 216 549 €.

Basically, making an investment or abandonment decision at the beginning when the startup is launched, as required by the NPV decision rule introduced in chapter 3.4.1, leads to a loss of 157 296€ due to the uncertainties in the future development of the startup. The real option value is substantially higher because the investment can be postponed and is only conducted in case the startup develops favorably. The difference between these two values is the value of the flexibility, for postponing the decision to a later date.

To summarize the decision strategy for owning the option of investing into “Biketastic” within the first three years, Illustration 18 below is used. Using the mathematical decision rule of formula 28 at each node, a value maximizing strategy can be identified.

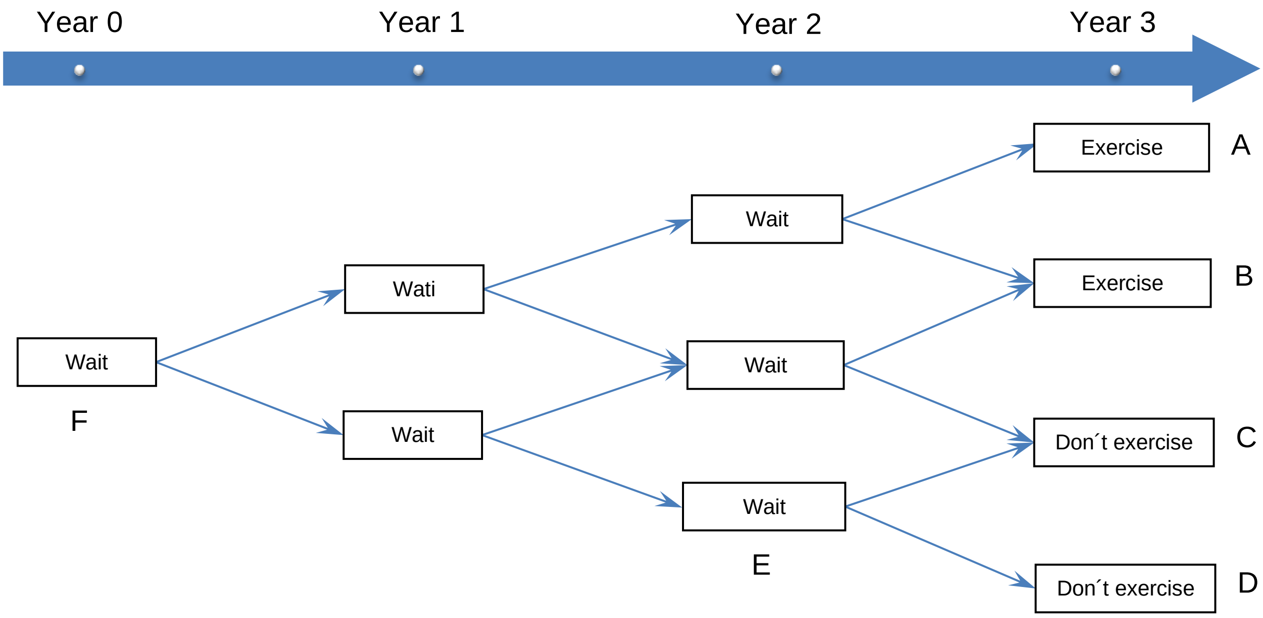

Illustration 18: Optimal Decision Strategy

This lattice graphically shows the optimal decisions at each node regarding the real option.

6 Conclusion

This research aimed to value non-market-traded startups using the Real Option framework. First, existing valuation approaches used in valuing startups were analyzed and their major factors impacting startup value were identified. The next step was to find a Real Option Approach suitable for application on startups regarding the prevailing assumptions of the individual frameworks, the scarce data availability present at a startup and the calculation methodology. After finding a theoretically suitable approach, valuation on an example was conducted, shortcomings were identified, researched and solutions were proposed. Finally, the real option value of the startup was calculated.

To identify major value drivers impacting the value of a startup, several ways for valuing startups were identified. Depending on the length of the startups operating history, different quantities of data are available, which influence the choice of the valuation approach and therefore the way uncertainties are captured. Explicit factors were identified analyzing qualitative valuation approaches. Through a cross comparison, major uncertainties impacting startup value were identified. The key factors are the following: Product/Technology, Management/Organization, Funding and Competition. Using the AHP method, a hierarchical order of risk importance was established, which was used to aid in the estimation process of the future startup development.

The real option area was subsequently analyzed for finding a suitable approach in order to value a startup regarding three important factors: the data availability, underlying assumptions and calculation methodology. After ruling out the commonly used market replication assumption due to its problematic theoretical basis and practical application regarding startups, the “MAD” approach was identified to be feasible for valuing a non-exchange-traded startup.

One issue from the theoretical perspective was addressed before the operationalization was conducted. The “MAD” approach for Real Option Valuation is based on the DCF methodology which uses the WACC model for deriving the discount rate. This is problematic regarding the valuation of a startup because using the CAPM for deriving the equity capital costs implies an identification of a replicating asset with the same volatility as the venture. The replication assumption was dropped earlier, therefore an alternative way of detecting the discount rate had to be identified. A potential solution was found in a newly developed framework for risk adjustment developed by Zhang.

Operationalizing the valuation of a startup example led to two problems and two insights regarding the usage of this “optimized MAD” approach. The first problem originated in the calculation methodology. The risk adjustment approach needs the calculated volatility information, which is derived using the risk adjustment approach, this is a problem because information is needed not present at the calculation stage. A sensitivity analysis was set up to solve this issue. The new insight concluded: The discount rate has only minor influence on the standard deviation of returns (volatility), which can therefore be estimated for calculating the volatility. The next problem identified was located in the negative nature of cash flows, common at startups, which led to problems while calculating the volatility using the Monte Carlo Simulation. The negative cash flows led to mathematical errors in the calculation, as well as problems in the return distribution which would render the subsequent Real Option Valuation questionable. The cost component was identified to be the single biggest factor driving the cash flows negative. It was shown that neglecting the cost component for solely calculating the volatility is possible. For calculating the PV of the “inflexible” cash flows, which is the value of the underlying startup, Zhang’s risk adjustment model was used.

The following Real Option Valuation showed the additional flexibility value of the startup in comparison to the common NPV valuation.

The gap in knowledge regarding the valuation of startups, whose shares are not traded on the stock market, was closed with the identification and optimization of a Real Option Approach and its operationalization. Several above-mentioned discoveries and conclusions were made along the way in answering the research questions.

As this research points out, the Real Option framework is usable in valuing non-exchange-traded startups. The identified “MAD” approach was shown to be applicable using the incorporated optimizations. The demanding estimation of the future startup development remains. The identified major factors impacting the development of the startups combined with the use of the AHP method can only aid in this process, but substantial amount of subjective estimation is still needed.

The limits of this research are the data availability of the valuation example and the assumptions which were defined in the operationalization. Regarding the data of the startup used, more data about the flexibilities present in the development and the actual implemented strategy as well as a price information would further enhance the conclusions drawn in this research. The second limitation are the simplifications used in defining the free cash flows, additional variable cash flow components could be incorporated.

Further research could test this valuation approach either through comparing the result with different other startup valuation approaches or with the actual transaction price of the valued startup in order to have a comparable price information.

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Illustrations

Tables

Abbreviations

CAPM	Capital Asset Pricing Model
CF	Cash Flow
DCF	Discounted Cash Flow
FCF	Free Cash Flow
FFF	Founder, Friends and Family
GBM	Geometric Brownian Motion
MAD	Market Asset Disclaimer
MCS	Monte Carlo Simulation
PV	Present Value
NPV	Net Present Value
WACC	Weighted Average Cost of Capital