

# SUBSPACE DECOMPOSITION IN THE FREQUENCY DOMAIN

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**ABSTRACT:** Measuring brain activity with non invasive techniques as EEG and MEG allows to detect oscillatory sources related to neural processes. Covariance-based spatial filters determined by linear subspace methods allow to extract narrow band sources whose band power correlates with a given target variable in single trial. Since knowledge about the frequency band of interest usually is unknown, filterbank strategies are commonly used. They rely on time domain filtering of the signals to predefined frequency bands. We suggest that the implementation can be optimized by computing the covariance matrices directly in the frequency domain, thus rendering the iterative time-domain filtering unnecessary. Our contribution shows that the implementation in the frequency domain is computationally more efficient than the classic approach. We evaluated the novel approach in the context of source power co-modulation (SPoC) and give indications, how it can be extended to other subspace methods such as common spatial patterns (CSP) [1].

## INTRODUCTION

Measuring electrical oscillatory activity of the brain by using electroencephalography (EEG) provides functional information about the underlying neural processes [2]. Extraction and analysis techniques of such oscillatory components have been developed in the context of brain-computer interfaces (BCI) [3], [4] and neuroscience [5]. As low signal-to-noise ratio (SNR) and volume conduction impedes the EEG analysis, spatial de-mixing approaches are widely used in order to extract oscillatory subspace components. For this purpose, unsupervised techniques are widely used, with independent component analysis (ICA) [6], [7] being most prominent in the field. With specific prior knowledge, however, more specialized methods like spatio-spectral decomposition (SSD) [8] or slow feature analysis [9] may prove valuable. If discrete labels are available, however, then a supervised method like common spatial patterns (CSP) [1] can improve the subspace representation, as the spatial decomposition can be guided by the label information. CSP is applicable when discrete labels are given (e.g. class labels in a motor imagery task, hits vs. misses in a perception task). CSP determines the projecting components based on channel-space covariance matrices, that maximize the contrast of oscillatory activity between conditions.

In other paradigms, the additional information is provided in the form of continuous labels rather than discrete class labels. A regression approach – like Source Power Co-modulation (SPoC), introduced by Dähne et al. – is able to exploit these continuous labels in order to extract spatial components [10], [11]. Both supervised covariance-based subspace methods, CSP and SPoC, have been designed to extract oscillatory components whose band-power is informative. However, while CSP expects discrete two-class labels and maximizes contrast, SPoC requires a continuous target signal and identifies spatial components, which co-modulate in their power with this known continuous univariate target signal. For applications of SPoC on neural signals please refer to [12], [10], [11], [13].

Choosing a suitable frequency band is a critical hyperparameter for these methods, since they require a narrow band frequency filter to be applied to the data prior to starting the search for subspace components [14], [15]. If knowledge about expected informative frequency bands is not available, a generic filter bank approach can be used, as proposed by Ang et al. for CSP [16]. It runs CSP separately on several versions of the data, each one pre-filtered to a different frequency band. Finally, the outcomes of the bands are merged, e.g. by a subsequent feature selection or regression step as proposed by Nove et al. [17].

Typically, filterbank strategies are implemented by filtering the signals in the time domain. However, since trial-wise stationarity of the signals is assumed for most applications, the explicit representation of the temporal dynamics within a trial may actually not be necessary.

In this regard, we propose the implementation of a more computationally efficient filter-bank approach for subspace methods that is based on the calculation of a stationary frequency domain representation of the data. We present results of a study carried out in the context of SPoC for real EEG data.

## METHODS

### *Forward Model of EEG Generation*

Let  $\mathbf{X} \in \mathbb{R}^{N_c \times N_t}$  be a multivariate signal describing data of brain activity measured in the EEG sensor space, where  $N_t$  is the number of time samples and  $N_c$  the number of sensors. Furthermore, let  $\mathbf{S} \in \mathbb{R}^{N_s \times N_t}$  describe the time course of  $N_s$  neural sources, where

$N_s$  describes the number of hidden neural sources considered. We assume a linear generative model, which maps the source space to the sensor space as follows:

$$\mathbf{X} = \mathbf{A} \mathbf{S} + \mathbf{E} . \quad (1)$$

In this model, matrix  $\mathbf{A} \in \mathbb{R}^{N_c \times N_s}$  describes the projection of the sources to the sensor space, where the columns of  $\mathbf{A}$ ,  $\mathbf{a} \in \mathbb{R}^{N_c}$ , are referred to as *spatial patterns*. Furthermore, the matrix  $\mathbf{E}$  contains spatially and temporally uncorrelated noise to model measurement noise.

An estimation of the time course of a source component  $\hat{\mathbf{s}}$  can be extracted from the measurements by applying a *spatial filter*  $\mathbf{w} \in \mathbb{R}^{N_c}$ , which projects the data from sensor space into source space. Thus we have  $\hat{\mathbf{s}} = \mathbf{w}^\top \mathbf{X}$ . For many problems, such a spatial filter  $\mathbf{w}$  is not known a priori and must be estimated from the data. However, once a spatial filter (or an entire set thereof, denoted by  $\mathbf{W} \in \mathbb{R}^{N_c \times N_s}$ , where each column  $\mathbf{w}$  represents a single spatial filter) has been obtained, an estimate of the corresponding spatial patterns can be obtained via  $\hat{\mathbf{A}} = \mathbf{C} \mathbf{W} (\mathbf{W}^\top \mathbf{C} \mathbf{W})^{-1}$ , where  $\mathbf{C} \in \mathbb{R}^{N_c \times N_c}$  denotes the spatial covariance matrix of the data. See [18] for further details on the relation between spatial filters and spatial patterns.

#### Source Power Co-Modulation — SPoC

The multivariate analysis method called source power co-modulation (SPoC) by Dähne and colleagues [10] utilizes a supervised regression approach in order to estimate a set of spatial filters  $\mathbf{W}$ . The method assumes that the recorded data  $\mathbf{X}$  has been pre-filtered to a narrow frequency band, which contains the oscillatory source of interest.

Based on data of multiple epochs  $e$ , a filter  $\mathbf{w}$  is optimized such that the power of an epoch  $\Theta_x(e) = \text{var}[\hat{\mathbf{s}}](e)$  of the spatially filtered data  $\hat{\mathbf{s}} = \mathbf{w}^\top \mathbf{X}$ , maximally covaries with a known, epoch-wise defined univariate target variable  $z(e)$ . For the sake of simplicity in the notation,  $\hat{\mathbf{s}}$  will be noted as  $\mathbf{s}$ , hereafter.

It can be shown that solving such an optimization problem is equivalent to solving the generalized eigenvalue problem [10]

$$\mathbf{C}_z \mathbf{W} = \mathbf{\Lambda} \mathbf{C} \mathbf{W} \quad (2)$$

where  $\mathbf{C}_z = \langle \mathbf{C}(e) z(e) \rangle$  and  $\mathbf{C} = \langle \mathbf{C}(e) \rangle$ .  $\langle \mathbf{C}(e) \rangle$  and  $\langle \mathbf{C}(e) z(e) \rangle$  provide the ( $z$ -weighted) covariance of  $\mathbf{X}$ , averaged across epochs  $e$ :  $\mathbf{C}(e) = \mathbf{X}(e) \mathbf{X}(e)^\top$ . Matrix  $\mathbf{\Lambda} \in \mathbb{R}^{N_c \times N_c}$  contains the corresponding eigenvalues in the main diagonal.

Given a spatial filter  $\mathbf{w}_{tr}$  determined on training data  $tr$ , the true target variable  $\mathbf{z} = [z(1) \dots z(N_e)]^\top$  can subsequently be approximated/estimated as  $\hat{\mathbf{z}}$  on a single-trial basis for unseen test data ( $te$ ) epochs  $\mathbf{X}_{te}$  via  $\hat{\mathbf{z}}(e) = \text{var}[\mathbf{w}_{tr}^\top \mathbf{X}_{te}(e)]$ . While in most scenarios a small number of filters is utilized, we limit our analysis for the remainder of this contribution to the one spatial filter  $\mathbf{w}$  which corresponds to the biggest eigenvalue of the aforementioned decomposition.

#### Filterbank SPoC

Until now, we have assumed that the target frequency band is known. Unfortunately, this is typically not true, thus exploring the full available spectrum is necessary. SPoC can then be extended by using the filterbank concept proposed by Ang and colleagues for the filterbank CSP algorithm [16]. Here, a set of  $N_{filt}$  frequency bands are defined, for which the subspace decomposition method is applied separately. In the context of SPoC, this approach shall be termed filterbank SPoC (**FB-SPoC**) hereafter. FB-SPoC results in a set of  $N_{filt}$  different estimations of  $\hat{\mathbf{z}}$ . We define these intermediate band-wise estimations  $\hat{\mathbf{z}}_i$ . To obtain a final  $\hat{\mathbf{z}}$ , a linear model combining all the estimations of the target variables can be applied:

$$\hat{\mathbf{z}}(e) = \sum_i^{N_{filt}} \beta_i \hat{\mathbf{z}}_i(e) \quad (3)$$

where the weights  $\beta$  are determined by solving the optimization problem:

$$\arg \max_{\beta} \|\hat{\mathbf{z}} - \mathbf{z}\|_2^2 + \lambda \|\beta\|_p^2. \quad (4)$$

In Eq. 4,  $\lambda$  is a positive real-valued regularization parameter and  $p$  defines the type of regularization applied to the model, with  $p = 2$  corresponding to the classic Tikhonov regularization and  $p = 1$  a sparseness promoting prior, termed LASSO.

#### Computation of Covariance Matrices in the Frequency Domain

In implementations of filterbanks for different algorithms (as CSP, spatio-spectral decomposition [8], among others), data initially is bandpass filtered in the time domain using IIR or FIR filters. Thus, the computational cost grows linearly with the number of bands.

However, since the aforementioned methods are based on the computation of the covariance matrix of the signal, which is assumed to be stationary in the analyzed epochs, the actual computation of such covariance matrices could alternatively be executed directly in the frequency domain. According to the Plancherel theorem [19], the dot product of two signals in the time domain is equal to the inner product of their frequency representation. Consequently, the covariance matrix  $\mathbf{C}$  may be computed for a specific frequency band  $f$  as

$$\mathbf{C}_{i,j}^f = \text{re}(\langle \mathcal{X}_i^f, \mathcal{X}_j^f \rangle) \quad (5)$$

where  $\text{re}(\cdot)$  is the real part of the argument,  $\mathcal{X}_i$  are the coefficients of the Fourier transform of channel  $i$  in  $\mathbf{X}$ , and superindex  $f$  indexes the frequency bin corresponding to frequency bands of interest. The intuition behind neglecting the imaginary part of the dot product is that it provides information about the mean phase difference between the considered distributions,

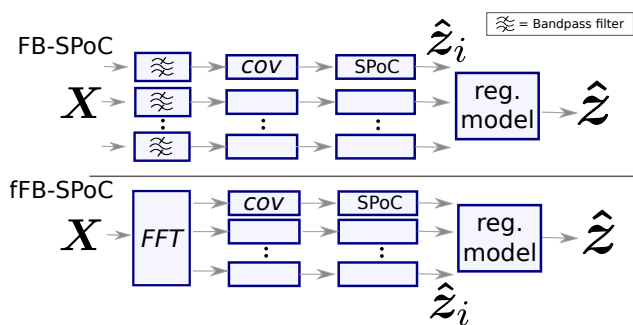


Figure 1. Schematic representation of the filter bank strategy applied to SPoC, FB-SPoC, compared to the proposed approach of computing the covariance matrix directly in the frequency domain (fFB-SPoC).

therefore it does not contribute to the co-varying power information provided by the covariance matrix.

Computing the covariance matrix in the frequency domain requires a single calculation of the fourier transform of  $X$ , and this computational effort is independent of the number of frequency bands included in the filter bank. Furthermore, a single copy of the signal is required in memory, whereas for time domain filter bank approaches,  $N_{filt}$  versions of it are necessary. Thus, the frequency domain representation optimizes memory access operations and allows cheaper caching in hierarchical memory architectures. The proposed approach of computing FB-SPoC using the frequency representation of data  $X$  is termed **fFB-SPoC** hereafter. Figure 1 shows a schematic representation of the implementation differences between FB-SPoC and fFB-SPoC

## EXPERIMENTAL SETUP

The proposed approaches, FB-SPoC and fFB-SPoC, were tested using real EEG data. Signals were recorded from 64 passive Ag/AgCl electrodes (EasyCap / BrainAmp DC amplifiers) placed according to the 10-20 system and referenced against the nose. Data were recorded while performing an auditory oddball experiment with an interstimulus onset of 1 s. Information about the paradigm was not used in the subsequent analysis. Signals were sampled at 1 kHz, then a band-pass filter with a cut-off frequency of 0.7-90 Hz and a notch filter at 50 Hz were applied to the data. Afterwards, it was downsampled to 250 Hz.

A target source  $s$ , which would serve as the ground truth source in a following simulation, was determined by projecting the preprocessed EEG data onto a single source. The corresponding filter  $v$  for this purpose was chosen pseudo-randomly. The projected signal was then filtered to the alpha band (8 – 12 Hz) and its envelope was extracted via the Hilbert transform.

The final dataset for running performance comparisons was obtained by segmenting the EEG and the ground truth target source  $s$  data into 1 s windows with 50% overlap. The ground truth target variable  $z(e)$  for each

epoch  $e$  was defined as the average of the envelope of the target source  $s$  for that epoch.

## Performance Metrics

In order to quantitatively assess the performance of the considered methods, the following performance metrics were considered:

**Correlation – corr:** This metric evaluates the quality of the final regression model. More precisely, it describes the correlation between the target variable estimated by the regression model  $\hat{z}$  and the true modulating signal (target variable)  $z$ . A higher absolute value suggests a better estimation.

**Best Band Correlation – corr:** This metric evaluates the quality of estimation for the best performing frequency band. More precisely, it describes the correlation between the target variable estimated by the best filter  $\hat{z}_i$  and the true modulating signal (target variable)  $z$ . A higher absolute value suggests a better estimation.

**Earth Mover’s Distance – emd:** This metric can be used to characterize the most important frequency band. It measures the dissimilarity between the estimated and the true spatial pattern  $\hat{a}$  and  $a$  within a single frequency band. The lower the value of  $emd$ , the more accurate the estimation. As fFB-SPoC and FB-SPoC yield one pattern per frequency band,  $emd$  is calculated in the frequency band achieving the highest  $corr$  performance.

**Angle Between Patterns – angle:** Analogous to  $emd$ , this metric describes the angle between the estimated and true spatial patterns  $\hat{a}$  and  $a$ . The lower the value of  $angle$ , the more accurate the estimation. Since FB-SPoC and fFB-SPoC yield patterns corresponding to more than one frequency band,  $emd$  is calculated using the pattern related to the most relevant frequency band, according to the  $corr$  achieved.

**Elapsed Time – et:** Computational cost is compared in terms of walltime required to compute the final estimation of  $\hat{z}$ .

## Parameter Sensitivity Analysis

The aforementioned metrics are assessed in a parameter sensitivity analysis. For this, a random search was performed, where the parameter space is defined by (1) the number of bands in the filter bank, (2) the type of spacing (grid) between passing bands of the filter bank and their corresponding width, which can be linearly or logarithmically spaced, and (3) the regularization type for the regression model in Eq. 4. Random search of the parameter space was performed using the random-search tool provided by the publicly available sequential model-based algorithm configuration (SMAC) toolbox<sup>1</sup> [20], whereas the parameters sensitivity analysis was performed using functional ANOVA (FANOVA)<sup>2</sup> [21]

<sup>1</sup><http://www.cs.ubc.ca/labs/beta/Projects/SMAC/>

<sup>2</sup><https://github.com/automl/fanova>

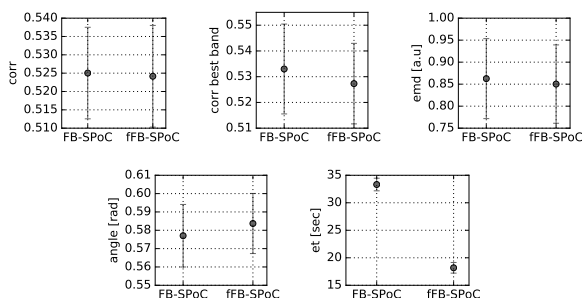


Figure 2. Marginalized performance of FB-SPoC and fFB-SPoC for the considered metrics.

## RESULTS

### *FB-SPoC vs fFB-SPoC: Overall Performance*

Figure 2 shows the marginalized performance of FB-SPoC and fFB-SPoC as computed with FANOVA. As expected, correlation values achieved and the spatial accuracy of the spatial patterns (as assessed by emd and angle), are not significantly different for the considered methods. On the other hand, marginalized walltime was significantly lower for fFB-SPoC compared to FB-SPoC.

### *FB-SPoC vs fFB-SPoC: Parameter Sensitivity Analysis*

Figure 3 shows marginalized effects of different configurations of the parameter space upon the performance, both for FB-SPoC and fFB-SPoC.

The number of frequency bands is the parameter that had the greatest impact on the correlation achieved. The grid type used for the definition of the filter bands plays a less critical role, with the linear model showing a small advantage over the logarithmic grid. The regularization method of the regression model did not affect the performance in terms of the marginalized correlation.

The effects of the parameter configuration upon emd and angle are very similar. Comparably to the effects upon the correlation metrics, the number of bands used in the filter bank is the parameter that has the strongest impact upon the spatial patterns, where as the grid type and the regression model do not seem to be critical.

For both, the accuracy of the target variable (corr) and the spatial pattern estimation, the optimal number of frequency bands for the considered scenario are approximately 5 bands. Using more than 5 bands does not improve the performance, according to any of the considered measures.

Finally, the bottom row of Figure 3 demonstrates the computational advantages of fFB-SPoC, where the walltime required increases at a much slower rate than for FB-SPoC. It is worth mentioning that between 1 and 3 bands, the metric et grows with the same rate for both algorithms.

## DISCUSSION

In this contribution, we extended the use of a filterbank approach to the context of source power co-modulation

analysis, SPoC. Furthermore, we propose to perform the covariance matrix calculation in the frequency domain to speed-up the computation of filterbank-based subspace techniques.

- i A filterbank strategy for SPoC is a suitable approach to estimate target variables that co-modulate with the power of hidden neural sources. The proposed approaches are specially valuable in scenarios where the frequency band of interest is not known and, consequently, a full exploration of the available spectrum is necessary. Such applications have already been reported in the literature, for example [13], [12].
- ii Under the realistic scenario considered, the number of frequency bands is the most important hyperparameter considering the high final correlation with the target variable and a good reconstruction of the true spatial pattern  $\alpha$ . This might be explained by the fact that a coarse segmentation of the frequency spectrum leads to mixing of informative and noisy frequency bands into the same filters, thus degrading the performance. This is also observed in the variance of the performance itself — a sudden reduction of the variance is observed once the number of frequency bands becomes greater than five. It is important to point out that the optimal number of frequency bands should be defined individually and for each application scenario, since the width of the informative frequency band of the target source and its location within the spectrum is not known a priori.
- iii For our data, where no label noise was involved, the grid type and regression model had little influence. Similarly to the number of frequency bands, the grid type is likely to be application-dependent. The regression model, on the other hand, is likely to be independent of the frequency characteristics of the target neural source, but may be sensitive to the level of noise contained in its labels. In future work, we will investigate the interaction of label noise with different regression models.
- iv We have also shown that the computation of the covariance matrix using the frequency representation of the EEG data is a suitable approach for filter bank strategies. The computational advantage is not only caused by the single-time computation of the FFT compared to the  $N_{filt}$ -many (sequential) time-domain filtering steps. It also affects the calculation of the covariance matrix itself, which has quadratic runtime. When computed in the time domain, each of the entries of the covariance matrix  $C$  corresponds to the dot product of two time series, each with length  $N$ . In contrast, when computed in the frequency domain, each entry of  $C$  corresponds to the dot product of two vectors containing a subset of  $M$  frequency bins obtained after the Fourier transform, with typically  $N \gg M$ . It is important to point out that the covariance is computed simul-

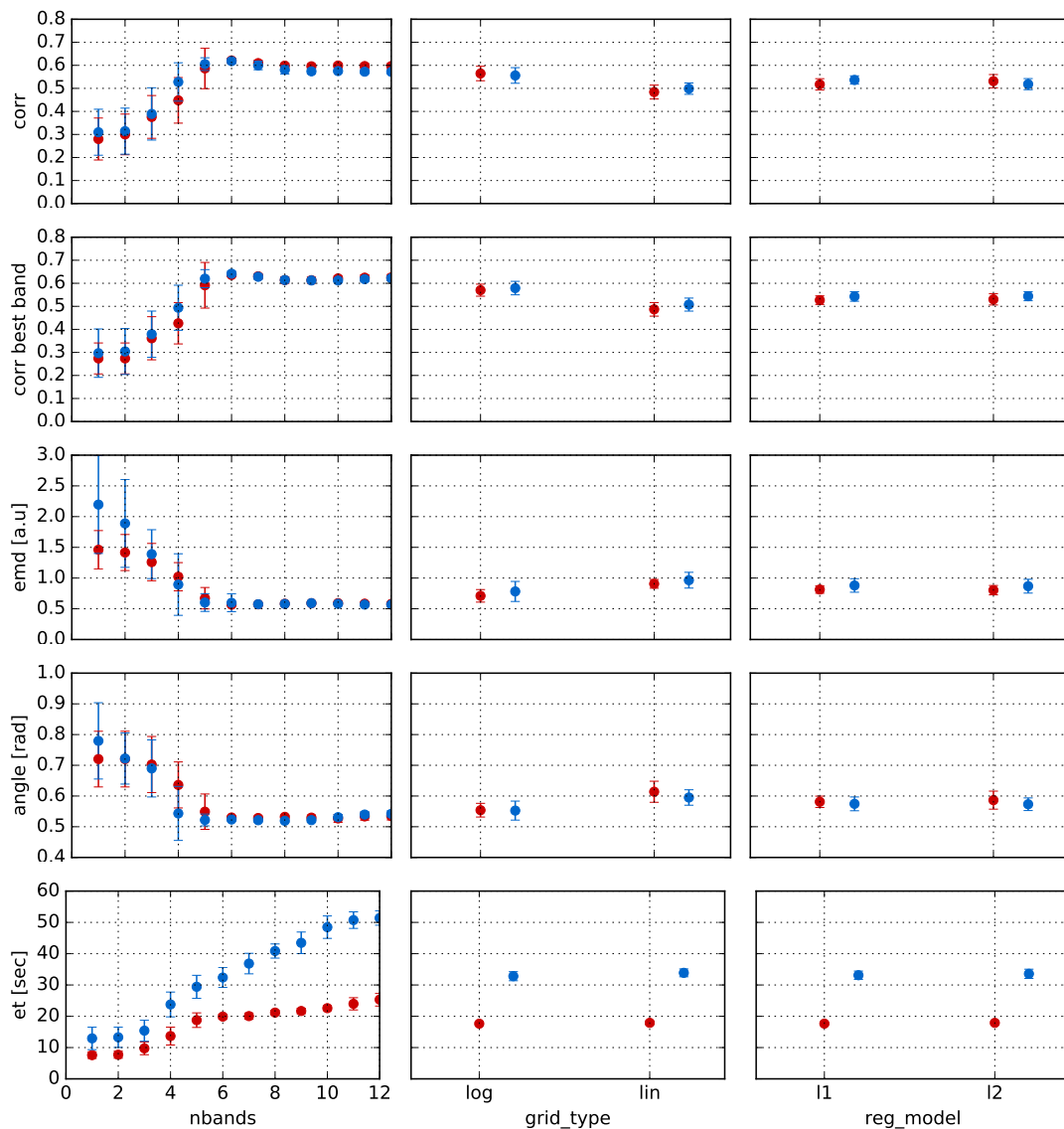


Figure 3. Marginalized parameter sensitivity analysis comparing FB-SPoC ● and fFB-SPoC ●, in terms of the considered performance measures.

- taneously on the entire subset of  $M$  frequency bins in a single step, and not for each bin individually.
- v The main limitation of the frequency-domain filterbank approach is the coarse granularity of the frequency bands considered. Their resolution is limited by the number of frequency bins resulting from the Fourier transformation, while filters in the time domain can be defined in high precision.
  - vi Another limitation of applying SPoC in the frequency domain is the smaller number of SPoC components which can be derived per frequency band. Specifically, the rank of the covariance matrix is limited by the number of frequency bins contained in the analyzed band. However, this limitation may not be a relevant one in practice, as a full rank SPoC decomposition often is not required and usually only the first-ranked, most informative components are

used.

- vii Finally, the proposed frequency domain approach for filterbank analysis should easily extend to other covariance-based subspace methods such as CSP or SSD.

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