Erosion of Channel Beds Covered by Cohesive Sediments

Hesam Fouladfar[1], Mahmoud Shafai Bajestan[2]
1- PhD, Head of river engineering department, IWPCO
2- Professor of faculty of Water Science Engineering of Ahvaz Chamran University

Email: h.fouladfar@iwpco.ir

Abstract

A physically based model is developed for estimation of surface erosion flux of cohesive bed channels. Merit of the model, with respect to well-known formulas, is application of new approaches for estimation of erosion flux coefficient and critical shear stress. This model also accounts for stochastic behavior of the actual bed shear stress. It is assumed that the erosion flux coefficient is the same as entrainment flux of fluid mud layer. The critical shear stress has also taken to be proportional to bed shear strength. Rayleigh probability function has been proposed as the distribution of bed shear stress. The physical parameters needed for the model are effective sediment cohesion, volumetric concentration, entrainment velocity and gelling concentration. The statistical parameter of the model, named scale parameter, is applied to correlate the model, for practice. The experiments were conducted in a straight recirculating flume filled with natural cohesive sediments. It has been shown the scale parameter, regardless degree of consolidation, has a strong power relation with relative critical bed shear stress. This parameter can easily be determined by two fast erosion tests. The model estimated erosion flux has shown well agreement with experimental measurements.

Keywords: cohesive sediment, erosion flux, erosion coefficient, entrainment velocity, gelling concentration, shear stress, Rayleigh distribution, critical shear stress, effective cohesion, volumetric concentration.

1. INTRODUCTION

In most widely used resuspension formulations, the user calculates the erosion flux as a function of excess bed shear stress and up to three empirical material coefficients. Some frequently used approaches, define excess bed shear stress as current induced bottom shear stress minus critical shear stress below that no erosion occurs. Usually fitting models to short-term erosion experiments determine the empirical coefficients [1]. However, interpretation of data is often at least partially subjective, and frequently the results are specific to physical configuration and time history of the procedure used. These differences can lead to significant differences between derived parameters, even when the same sediments are tested [2]. Moreover, most of classical resuspension formulations try to give deterministic description of resuspension process. The bed shear stress as a main factor is however a stochastic variable. The relation between resuspension and bed shear stress is not necessarily linear. The stochastic behavior of turbulence is probably an important aspect in phenomenological description of resuspension. For this, the erosion flux results, obtained from different formulas, are very scattered and do not converge to reality. The aim of this study is to develop and verify a new physically based formulation for estimation of resuspension flux of beds covered with cohesive material. The proposed formulation must require minimum field and laboratorial measurements for a reasonable prediction of resuspension.

2. MATERIALS AND METHODS

It is proposed that resuspension flux formula framework to be the same as Partheniades' formula structure [3], which is multiplication of the erosion flux coefficient (M) and the probability of exceedence of actual instantaneous bed shear stress (τ_0) from critical shear stress (τ_{cr}).

$$E = M \times p(\tau_0 > \tau_{cr}) \qquad (1)$$

In which, M equals to fluid mud entrainment rate, τ_{cr} equals to actual bed shear strength at the interface, τ_0 is instantaneous bed shear stress and p is the probability density function.

There is not any theoretical relationship for the erosion flux coefficient M. Clearly, this parameter is not larger than product of entrainment rate w_e (if the bed were fluid mud) and concentration of entrained fluid $c_{gel} = \rho_s \times \phi_{gel}$. The initial entrainment rates w_e of a fluid mud layer, has been investigated, both numerically and by means of experiments [4]. The M in the Partheniades equation is net erosion flux, while the M used in proposed resuspension formulation describes resuspension rate provided the bed shear stress exceeds the bed yield stress.

So one can expect the M used in this formulation to be higher than the traditional M. Some equations have been derived for entrainment of fluid mud layer by turbulent water layer above ([4], [5] [6]). An initial estimation for w_e is $w_e \approx 0.28u_*$ for low Richardson number [4].

It is proposed that the Rayleigh Probability Density Function (RPDF) is the distribution of actual bed shear stress at one moment in time and at one infinitesimal area of the bed, τ_0.

$$f(\tau_0;\sigma) = \frac{\tau_0}{\sigma^2}\exp\left(-\frac{{\tau_0}^2}{2\sigma^2}\right), \qquad \tau_0 \geq 0, \qquad \mu = \sigma\sqrt{\frac{\pi}{2}} \tag{2}$$

For simplicity, a new scale parameter, β (Beta) is defined as:

$$\beta = \frac{1}{2\sigma^2} \tag{3}$$

So the RPDF can be rewritten as:

$$f(\tau_0,\beta) = 2\beta\tau_0\exp(-\beta{\tau_0}^2) \qquad \tau_0 \geq 0, \qquad \mu = \frac{1}{2}\sqrt{\frac{\pi}{\beta}} \tag{4}$$

The cumulative density function is:

$$F(\tau_0) = 1 - \exp(-\beta{\tau_0}^2) \tag{5}$$

In the RPDF, when actual shear stress τ_0 is normalized by the regularly used bed shear stress τ_b, then regularly bed shear stress is related to the Beta as follows:

$$\mu = \frac{1}{2}\tau_b\sqrt{\frac{\pi}{\beta}} \tag{6}$$

One can see that the regularly used bed shear stress τ_b is not necessarily equal to the average of RPDF. Clearly, RPDF cannot simulate the turbulence flow accurately, but in practical point of view, RPDF contains only one argument, Beta. Thus only one parameter, has to be estimated when no data of the turbulence field are available. This reduces the chance of large mistakes. This distribution does not allow negative bed shear stress to act. So one has to deal with a probability of exceedence at one side of the bed shear stress spectrum. This makes the evaluation of the probability exceedence function easier.

In the proposed formulation, it is assumed that the critical shear stress to be the actual bed shear strength at interface. Obviously, the actual bed shear strength near the bed surface τ_y is at least equal to the critical shear stress and generally much higher. The reason of this assumption is that, one can calculate the τ_y as a function of density, effective stress and cohesion. Therefore, it can be considered as deterministic variable. Moreover, the burst mode of turbulence which is dominant phenomenon of resuspension on smooth bed like beds covered with cohesive materials happens rarely and corresponds to exceedence probability of higher value for critical shear stress. A relationship is derived between volumetric concentration of cohesive sediment ϕ and drained yield shear stress τ_c, in the failure layer in terms of microscopic properties [7]. The fractal description of material at micro-level is the basis of this relation. τ_c acts in the failure plane and reads:

$$\tau_c = c'\phi + \frac{1}{3}\tan\varphi'(1+2K_0)K_{\sigma,0} + \tan\varphi'\sigma'_v \tag{7}$$

The term $c'\phi$ is function of physico-chemical properties. τ_c is the yield stress in failure layer, while the τ_y is the yield stress parallel to the bed. Some simplifications are applied to Eq.7. The formulation will use the τ_y at the interface. At this level, the effective stresses are almost zero, so the effective stress term can be discarded. When substituting the constitutive relation of [8], $\sigma'_v = K_\sigma\phi^n - K_{\sigma,0} \approx 0$, Eq.7 becomes:

$$\tau_y = c'\phi + \frac{1}{3}\tan\varphi'(1+2K_0)K_\sigma\phi^n \tag{8}$$

The angle of internal friction φ' varies between 5° and 40°, and K_0 varies between 0.3 and 3.0. It means that $\tan\phi'(1+2K_0)/3$ is of order one. Near the interface the effective stresses are zero, so the term $\sigma'_v = K_\sigma\phi^n$ is very small at the interface and the cohesion term $c'\phi$ dominates there. To this end, the proposed relation for critical shear stress, including a reduction factor, becomes:

$$\tau_{cr} = 0.001c'\phi \tag{9}$$

Water Research Institute of Ministry of Energy collected water and sediment samples from Sefidrud Dam Reservoir located 200 km north-west of Tehran, Iran. The laboratories of this institute measured physico-chemical properties, mineralogical composition, mechanical properties and grain size distribution of cohesive sediment samples.

A straight recirculating flume with width of 45cm and length of 12 m is used for experiments. Water depth was 0.2 m. Observing reach had length of 2 meter situated in downstream of the flume. The flume was filled with sand to improve the boundary layer in upstream of the observing reach. During erosion experiments, when a sediment layer was present, the water depth in the flume was constant and did not change during erosion process. A sediment mixing tank equipped with pumping system also was set.

Two series of tests were carried out to determine erosional characteristics of the sediment. The first experiment was performed on a sediment layer with consolidation period of one day and layer thickness of 0.07 m. In the second case, the consolidation period amounted to seven days resulting in a sediment layer of 0.055 m. The sediment beds were formed by deposition in still water after a mixing period of approximately 12 hours. The concentration of initial suspension was 50 kg/m^3. The erosional behavior of each sediment layer were determined by increasing bed shear stress in successive steps of one hour**.**

The gelling concentration (structural density) was measured using settling column. The column had an internal diameter of 0.1 m and height of 1.5 m.

A turbidity meter, Analite 160, was used for measuring suspended sediment concentration during erosion tests. Furthermore, velocities were measured with an electromagnetic current meter. The bed shear stress was calculated in the flume using the relation between the depth-averaged flow velocity U and the friction coefficient C_f:

$$\tau_b = C_f \frac{1}{2} \rho U^2 \tag{10}$$

The Colebrook-White formula was used to estimate friction coefficients in fully-developed turbulent pipe and open channel flow. This formula for free-surface flow is presented as follows [8]:

$$\frac{1}{\sqrt{C_f}} = -4.0 \log\left(\frac{k_s}{12R} + \frac{1.25}{\mathrm{Re}\sqrt{C_f}} \right) \tag{11}$$

Where k_s is the roughness height. The ultrasonic displacement sensor was used to measure surface profile of the bed after draining water from the flume. The volumetric concentration of the bed was determined in different consolidation period, by sampling of the bed after each test and measuring dry density of the bed. The bed sediment cohesion was measured by consolidated drained triaxial test.

3. RESULTS AND DISCUSSION

The resulting new resuspension expression reads:

$$E = 0.28 \times u_* \times C_{gel} \times \exp\left(-\beta \left(\frac{0.001 c' \phi}{\tau_b} \right)^2 \right) \tag{12}$$

Using Eq. 12, the following equation can be derived for Beta:

$$\beta = \ln\left(\frac{0.28 \times u_* \times C_{gel}}{E_{observed}} \right) \times \left(\frac{0.001 \tau_{cr}}{\tau_b} \right)^{-2} \tag{13}$$

The above equation shows power formed relation between Beta and relative critical shear stress, τ_{cr} / τ_b, with a bit difference with ordinary power equations. The difference is that, first term of equation is not constant for successive erosion tests and different shear stress. To evaluate the power regression between Beta and and relative critical shear stress, the experimental data of [9], [10] and [11] were considered.

Fig.1-(a) illustrates the erosion flux as function of regularly bed shear stress for both experimental and formulation results, and for one-day consolidation. It is seen that there is good agreement between measured erosion flux data and formulation results. Root mean square error, RMSE = 4.2E-09 and Mean absolute relative error, MARE = 0.43. Fig. 1-(b) illustrates erosion flux as a function of regularly bed shear stress for both experimental and formulation results in seven days consolidation. Again, it is seen that there is good agreement between measured erosion flux and the formulation data. RMSE = 4. 1E-6 and MARE = 0.12. Fig. 1-(c) shows the erosion flux as a function of RBSS for both experimental and formulation results in seven days consolidation based on one day beta values. As it can be seen, still there is relatively good agreement between measured erosion flux and the formulation data. RMSE =2E-5 and MARE = 0.52.

4. Conclusions

A new formulation has been proposed for estimation of resuspension for channel beds covered with cohesive sediments. In the formulation, with more realistic and physical bases than before, one can estimate input parameters including resuspension flux coefficient and critical shear stress. In addition, the formulation has involved stochastic behavior of bed shear stress. The statistical scale parameter of the formulation which is a calibrating factor has a strong power relation with the relative critical shear stress. Since the degree of bed consolidation did not show a significant effect on the scale parameter, one can estimate it with two fast resuspension tests in laboratory or field. Comparison of the formulation estimated resuspension flux with current measurements and previous studies has shown a well agreement.

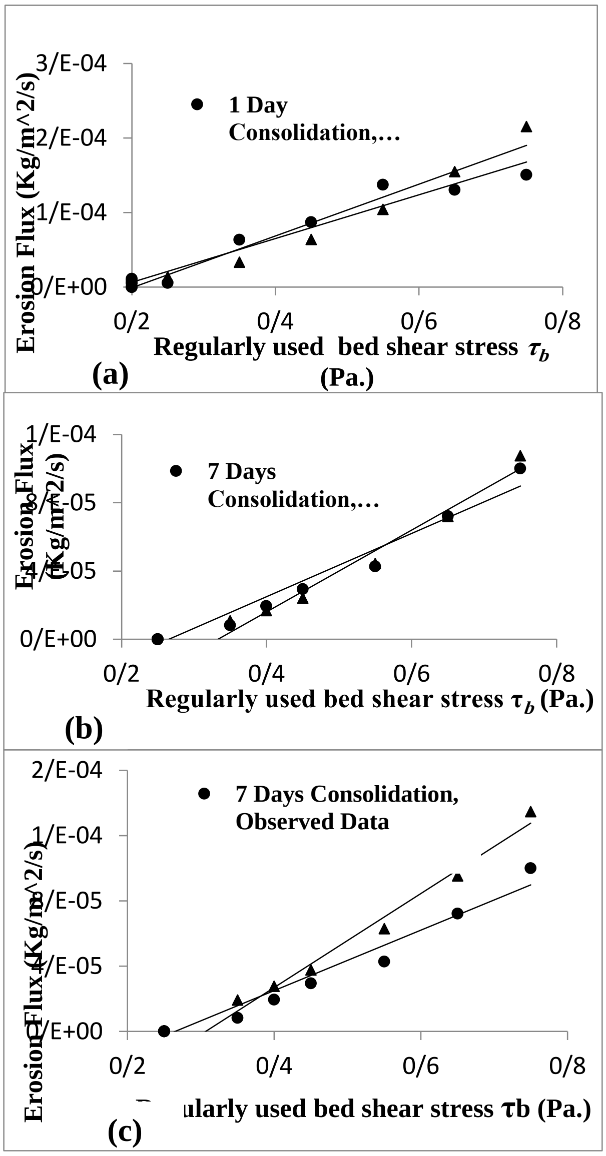

Fig.1 Comparison between experimental data of resuspension flux and formulation results

5. References

1. Winterwerp, J.C. and Van Kesteren, W.G.M. (2004). "*Introduction to the physics of cohesive sediment in the marine environment.*" Elsevier.
2. Sanford, LP. and Maa, JPY. (2001). "*A unified erosion formulation for fine sediments.*" Marine Geology, Vol. 179
3. Partheniades, E. (2009). "*Cohesive sediments in open channels*". Elsevier
4. Kranenburg, C., and Winterwerp, J.C. (1997). "*Erosion of fluid mud layers, I: Entrainment model*." ASCE, Journal of Hydraulic Engineering, 123 (6): 504-511.
5. Kranenburg, C. (1994). "*An entrainment model for fluid mud. Communications on Hydraulic and Geotechnical Engineering.*" Faculty of Civil Engineering, Delft University of Technology, Report 94-10.
6. Bruens, A.W. (2003). "*Entraining mud suspensions.*" PhD thesis, Delft University of Technology, Department of Civil Engineering and Geosciences, The Netherlands.
7. Merckelbach, L.M. (2000). "*Consolidation and strength evolution of soft mud layers.*" Delft University of Technology, Faculty of Civil Engineering and Geosciences, Report 00-2, ISSN 0169-6548.
8. Henderson, F. M. (1966). "*Open Channel Flow*" Macmillan Publishing Co - New York.
9. Kuijper, C., Corneliesse J.M., and Winterwerp, J.C. (1990). "*Erosion and deposition characteristics of natural muds sediments from the Hollandsch Diep*, Rijkswaterstaat, Delft Hydraulics, Report 27.
10. Krishnappan, B. G., and Engel, P. (1994). "*Critical shear stresses for erosion and deposition of fine suspended sediments in the Fraser river.*" National Water Research Institute, Ontario, Canada.
11. Schaaff, E., Grenz, C., Pinazo, C., and Lansard, B. (2006). "*Field and laboratory measurements of sediment erodibility: A comparison.*" Journal of sea research, Vol 55.