Algorithms and their Implementation for Optimal Hydro Plants Scheduling and Dispatch

Diploma Thesis

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Statutory Declaration

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Graz, ...

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Abstract

In order to be comparative in electric power production an optimal scheduling of power plants, minimizing the costs, is necessary. The first step during optimization is long-term scheduling, considering a whole year and dividing it into multiple stages. As the input information for long-term scheduling also depends on erratic environmentally caused values, stochastic optimization has to be used to achieve best results. Especially in stochastic optimization the number of variables can get enormous when using a linear problem formulation. Depending on the solver it can therefore take a very long time to get a result, or solving the problem can even get impossible.

The thesis deals with multi-stage stochastic optimization applied to hydro scheduling. Dynamic programming, a method to find an optimal solution for multistage stochastic optimization problems based on an approximation of the expected future cost function expressed as a piecewise linear function, is used. This approximated function is composed out of the dual solutions of the optimization problem for each stage and can be interpreted as a Benders optimality cut in a nested decomposition algorithm. Despite of slow convergence for certain cases this approach has proven to be performing satisfactorily.

Due to the rising use of renewable energy the optimal use of hydro power plants is getting increasingly important. Special practical examples show the applicability of the program for these modern environmental-relevant tasks.

Kurzfassung

Um wettbewerbsfähig in der elektrischen Energieerzeugung zu sein ist eine kostenminimierende Fahrplanerstellung für die Kraftwerke nötig. Der erste Schritt der Optimierung ist die Langzeitfahrplanerstellung. Dabei wird ein gesamtes Jahr in mehrere Stufen unterteilt. Die Daten für eine Langzeitfahrplanerstellung hängen von unregelmäßigen, umweltbedingten Einflüssen ab. Aus diesem Grund wird, um die besten Ergebnisse zu erzielen, eine stochastische Optimierung eingesetzt. Dabei kann für die lineare Formulierung die Anzahl der Variablen sehr groß werden. Abhängig vom Solver kann es sehr lange dauern, bis man Ergebnisse erhält. Es kann auch vorkommen, dass das Problem gar nicht gelöst werden kann.

Anhand der Einsatzplanung von Speicherkraftwerken beschäftigt sich diese Diplomarbeit mit der mehrstufigen stochastischen Optimierung. Die dabei angewandte Methode zur Lösung eines mehrstufigen Optimierungsproblems basiert auf der Annäherung der zukünftigen Kosten durch eine stückweise lineare Funktion. Diese Approximationsfunktion wird aus den dualen Ergebnissen jeder Optimierungsaufgabe der verschiedenen Zeitabschnitte zusammengesetzt und kann als optimaler Benders Cut in einem verschachtelten Zerlegungsalgorithmus verstanden werden. Abgesehen von einer langsamen Konvergenz in speziellen Fällen konnte gezeigt werden, dass der Algorithmus zufriedenstellend ausgeführt werden kann.

Durch die stärkere Nutzung erneuerbarer Energieträger gewinnt der optimierte Einsatz von Speicherkraftwerken immer mehr an Bedeutung. Entsprechende Anwendungsbeispiele zeigen die Anwendbarkeit des Programms für diese modernen umweltrelevanten Aufgabenstellungen.

Contents

1. Introduction

In the deregulated energy markets the optimal use of the power production as well as the reduction of costs are very important in order to be competitive. The electrical power system is divided into power production, transmission and distribution. Each subsystem is controlled, monitored and optimized by itself, but they are connected and depend on each other. The main function of all three parts is the economical supply with electric energy, within the constraints of service security, power quality and environmental impact. [1]

In the majority of power systems various energy sources are used. These are fossils for thermal plants (coal, oil, gas) and renewables for hydro, solar and wind power plants and biomass plants. In the classical power systems predominantly the hydro-thermal interconnected operation is used.

Nowadays shares of wind and solar power are growing pretty rapidly. This is not least caused by energy political pressure, as the *20-20-20-goals* of the European Union and national laws (for example the *EEG, Erneuerbare-Energien-Gesetz* in Germany and the *ÖSG, Ökostromgesetz* in Austria). However, these new energy forms are highly stochastic with strongly varying power inputs, which can cause real technical transmission problems in the grids and high efforts for regulation and compensation of possible outages and bottlenecks. Also the distribution between the control areas has to be taken into account. Thus the scheduling of the different power plants depends on the well known economical demand of cheapest generation cost considering an increasing effort for managing the technical conditions like availability, security and quality of supply.

To solve this complex problem several approaches can be used. One important part in this area is the computerized optimization of scheduling different kinds of power plants. During these optimizations economical aspects are mostly contained in the main function and the technical conditions are taken into account using constraints. The purpose is to find an optimal solution which can be computed as fast as possible. Because of the complexity of the exact model and the limitations of calculation power and run time, approximations have to be used in most cases. [2, p.8,11]

Typically there exist three time intervals with characteristic load curves: years, weeks and days. Therefore the optimization is mostly divided into these three parts. In this thesis the optimization is done for one year and named long term scheduling as no consistent naming for the different planning horizons is defined in literature. [2, p.7,15]

Usually in long term scheduling it can be distinguished between deterministic and stochastic optimization. Especially in hydro-thermal distributed systems the load forecasts, the availabilities of the generating units, the fuel prices and the inflows have a stochastic uncertainty. The even more stochastic power input from renewable energy, like wind and solar power, causes new requirements (i.e. a more flexible power plant portfolio), whereby the main concept of optimization did not change. [2, p.27,28] [27, p.3]

Long-term scheduling can be used especially to find the optimal strategy for large hydroelectric systems management.

Because of the stochastic natural variations of the inflows, using yearly planning horizons and decomposing them into different stages can be advantageous. The approach which is used for deterministic dual dynamic programming (DDDP) and stochastic dual dynamic programming (SDDP) is based on an approximation of a future cost function by a piecewise linear function.

1.1. Objectives and requirements

The objectives and requirements for this project were:

- The optimization software should be written in C++, using a linear programming solver of the XA optimization library.
- The program should be able to deal with input data varying in size.
- Linear Programming (LP), Stochastic Dual Dynamic Programming (SDDP) and Deterministic Dual Dynamic Programming (DDDP) should be compared with each other.

In a first step, the dual dynamic programming is performed for the deterministic case. In a second and third step, the stochastic dual dynamic programming and the linear programming are performed subsequently. As the sizes of the problems can lead to several millions of elements taking part in the optimization, a special class in C++, being both, flexible, fast and memory saving, is implemented.

In chapter 2 an overview of the most important types of power plants, being of relevance for this work, is given. These are thermal and hydro power plants as well as wind and solar power stations. Additionally some information regarding the costs of power production and the market conditions is given. At the end of the chapter the basics of long term and short term scheduling are described.

An overview of several fields in which optimization problems can occur and techniques to solve them is given in chapter 3. The basics of the fields in which the problem can be used and the techniques to solve are explained in general and with short examples.

The implementation of the whole system is presented in the first part of chapter 4. The Decomposition Approach and Benders decomposition are described in the second and third parts. The fourth part describes the flexible array class used to deal with the problem of the variable size of the optimization problem when working with C++. The XA optimisation library is described in the last part.

The inputs for the simulations and their solutions are presented in chapter 5, followed by a final discussion in chapter 6.

2. Technical, economical and scheduling basics

2.1. Power Plants

Different methods to produce electrical energy have been invented in the past. Some of the most used power plants are for example:

- Thermal Power Plants
  - Fossil Burning Power Stations
  - Gas and Steam Power Stations
  - Nuclear Electrical Power Plants
  - Biomass Power Plants
- Hydro Power Plants
  - Run-of-River Power Plants
  - Storage Power Plants
  - Pump Storage Power Plants
- Wind Power Stations
- Solar Power Plants

Still most of the electrical energy world wide is produced in thermal power plants. [4, p.271]

Another way to categorize power plants is based on their use for covering different load conditions: [6, p.5]

- Minimum Load Power Plants,
  which can supply energy during the whole year with a large number of full load hours
- Medium Load Power Plants,
  their production is reduced or turned off in times of lesser load
- Maximum/High Load Power Plants,
  which are used to cover the peak load

The base load is covered with thermal and run-of-river power plants. In the last years the share of wind power was increased, so that also wind power stations can be used to cover a part of the base load. The economy of wind power depends on the intensity and the regularity of available wind. In countries with hydro resources run-of-river power plants and storage power plants are more important. As the power can fluctuate, back-up power plants are needed to compensate this. Thermal power plants can do that up to a certain amount, but in this type of power stations changes in the production lead to higher costs and cause higher fuel consumption. Therefore the use of storage power plants is preferred for this purpose, as the variable costs are less. But due to the restricted storage capacity only a certain amount of electrical energy can be produced continuously. Thus storage power plants are mostly used for short periods in times which allow a high profit to be generated. [3, p.114] [13, p.325] [4, p.13]

2.1.1. Thermal Power Plants

Thermal power plants transform thermal energy into electrical energy. As mentioned before thermal power plants can be classified by the type of fuel they use. That can be fossil fuel, like coal, oil or gas, nuclear or biomass fuel. But also solar thermal electric, geothermal and waste incineration plants are thermal power plants.

Most of the thermal power plants are based on the Clausius-Rankine process. The condensed water is pumped out of a condenser and flows through a feed water pump, which increases the pressure up to the high-pressure level. During this stage there are nearly no changes in the temperature (T) and the volume. The water is then pre-heated up to its boiling point using feed water heaters to increase the efficiency.

The steam (S) is produced in a steam boiler. During this stage the temperature stays nearly the same. To further enhance the efficiency the steam is overheated in a re-heater, increasing the temperature and the specific volume. The steam flows through a high pressure steam turbine. In the turbine the enthalpy change is transformed into kinetic energy. Afterwards the steam is again reheated and flows through a middle-pressure and a low-pressure turbine. The turbines and the generator are connected through a shaft. In the generator the kinetic energy is transformed in electrical energy. The cooled and expanded steam flows through a condenser and the cyclic process starts again. [14, p.37-38] [4, p.272-274]

The power produced by the turbine depends on the efficiency η_{th}, the mass flow of the steam $\dot{m}$ and the enthalpy change ΔH:

$$P_{th} = \eta_{th} \cdot \dot{m} \cdot \Delta H \tag{2.1}$$

The change of the enthalpy primarily depends on the change of the steam temperature in the turbine. [17, p.68]

Gas and steam turbines are often used side-by-side or in a tandem construction inside a thermal power plant. In this case the exhaust gases of a natural gas fired turbine pass through a heat exchanger and are used to heat up water in order to produce steam inside a steam boiler. This combined process enhances the efficiency of the thermal power plant. [14, p.58] [4, p.282]

The degree of efficiency η is dependent on the temperature gradient between the high temperature input T_{high} to the turbine and the low temperature output T_{low}. It can be regulated by the mode the steam producer is operated. Normally the value is between 30% and 60%. [19, 2/p.6]

$$\eta = 1 - \frac{T_{high}}{T_{low}} \tag{2.2}$$

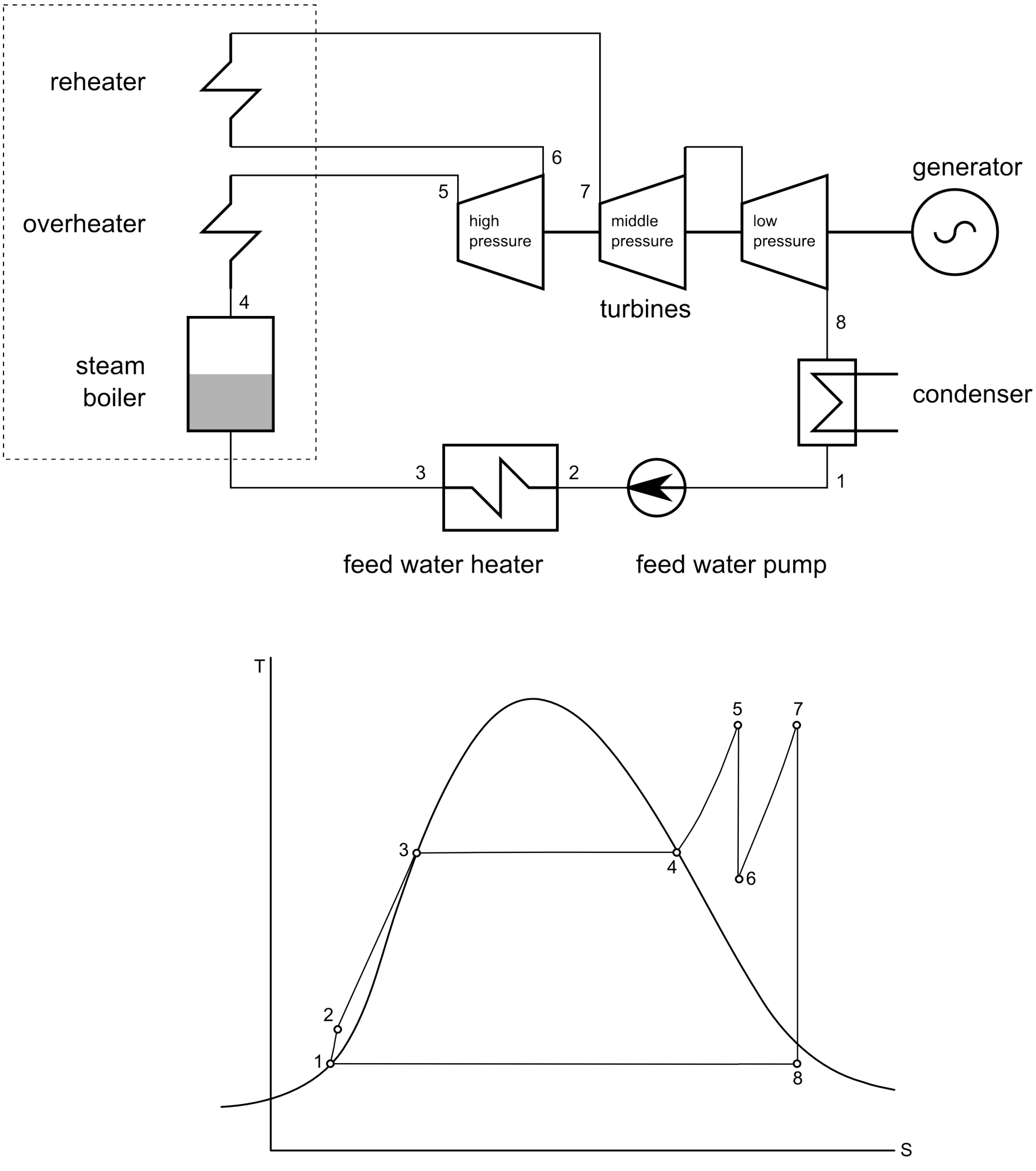

Figure 2.1.: Scheme of a steam power plant with its associated S/T-diagram [14, p.37]

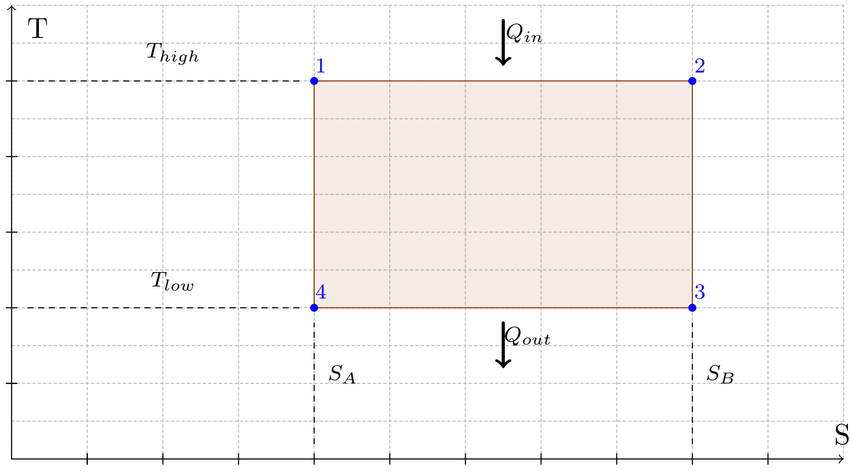

Figure 2.2.: Carnot cycle, illustrated by the temperature-entropy (T/S) diagram [19, 2/p.6]

Even if the efficiency of modern gas- and steam power plants is 58%, more than 40% are not used, so it is important to use the remaining thermal energy. It can be used for example to provide facility or community heating. This way the efficiency is increased up to 75% to 85%. [19, 8/p.14] [18, p.5]

The time to raise the temperature in the steam boiler from cold up to the operating temperature is several hours, so thermal power plants are usually used to cover the base load (except for plants using gas turbines). Also the shut down is executed in a certain time so that the turbine and other elements do not create cracks because of too rapid drops in temperature. The starting time of nuclear power plants can take days. [6, p.42]

If thermal power plants are used for covering fast load changes, different techniques are used. Because the temperature changes should be small and the change of the enthalpy depends on the change of the steam temperature, the control of the output can be done by changing the flow rate of the steam, which is proportional to the cross section of the inflow. Depending on the technique this can be done fast, whereby the efficiency is lower, or it can take up to 60 seconds. [17, p.68]

The advantage of thermal power plants is the deterministic continuous production of electricity in comparison to storage power plants, wind turbines or solar photovoltaic power plants, which depend on natural, erratic and discontinuous values.

On the other hand the disadvantages are the long start and stop times of the plants, the dependency on price increases of fossil fuels or the need of fuel imports. Also the emissions pose big problems for the environment.

2.1.2. Hydroelectric Power Plants

Hydroelectricity is a way of producing electrical energy by using the potential and kinetic energy of water.

The potential energy of the water is contained in the reservoir, except for run-of-river power plants, where little or no water is stored. For both the same method is used: the water is running down a pipe and driving a water turbine connected to a generator which generates the electrical energy.

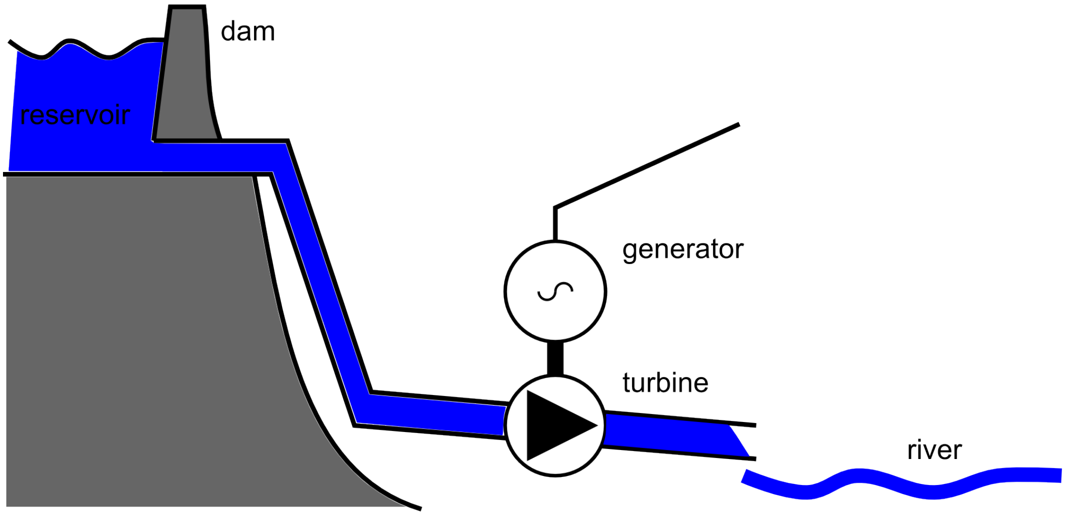

Figure 2.3.: Schematic of a hydroelectric power plant (storage power plant)

The electrical output can be calculated by formulas 2.3 and 2.4,

$$P = Q \cdot h \cdot c \tag{2.3}$$

$$c = g \cdot \rho \cdot \eta \tag{2.4}$$

where Q is the flow rate of the water, h is its head and c is a coefficient composed of the gravitational acceleration g, the water density ρ and the overall efficiency of the system η. [3, p.31-32]

The normal output of a hydroelectric power plant is in between several kWs and MWs.

The energy which can be generated depends not only on the available volume but also on the height difference between the source and the water outflow (called head). The potential energy of the water is directly proportional to the head. That is the main reason for building the dams of storage power plants as high as possible to maximize the generation of electrical energy. The second dependent variable is the water flow rate through the turbine.

The main advantage of hydro systems is their independence of fossil fuel, eliminating fuel costs and dependencies on changing prices and imports. So they give a hand in reducing CO2 emissions, an aspect which got more important with the Kyoto Protocol. [3, p.24]

Hydro power plants have a higher expected useful lifetime than fuel fired power plants. Since the generating units can be started and stopped quickly, they can follow fast rising system load changes efficiently. The starting time is divided into the run-up time of the turbine and the time needed for synchronization with the grid. [6, p.42]

Hydro power plants can be categorized differently, for example depending on the head:

- low head hydroelectric power plants
  with a head up to 15 meters
- medium head hydroelectric power plants
  with a head between 15 and 50 meters
- high head hydroelectric power plants
  with a head of more than 50 meters

Low head hydroelectric power plants

They are directly built in the course of a river, a little storage can be created in the head water. The energy yield is maximized if the whole useful height of a river is divided into several stages. In most cases multiple hydro power plants cascaded on the same river influence each other, as the outflow of one power plant increases the inflow in the following reservoir. So their operation has to be balanced. This controlling occurs automatically, based on complex control- and simulation models using stochastic and deterministic algorithms. Also a certain depth of water has to be provided for shipping traffic and fish way (European Water Framework Directive). [3, p.99-103]

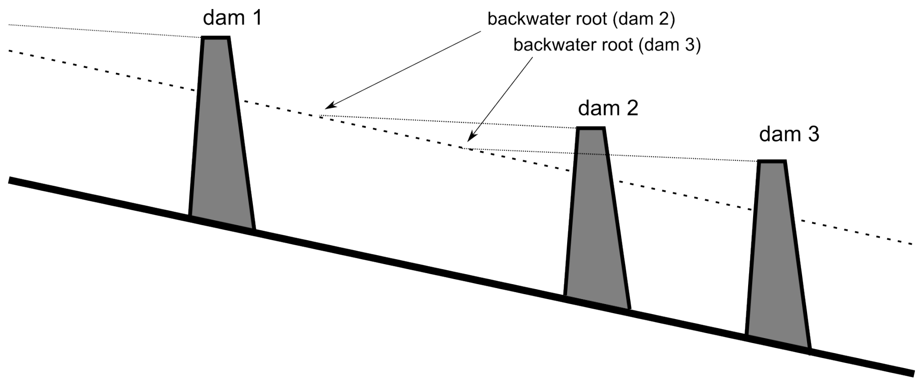

Figure 2.4.: Schematic of a chain of run-of-river power plants

Medium head hydroelectric power plants

These can be storage power plants with low dams or run-of-river power plants with high weirs. The facilities can be differentiated by their way of use. Besides being exclusively used for producing electrical energy they are often also used for example for high-water protection, to provide a drinking water reservoir, for shipping or to create recreational areas. [3, p.110]

High head hydroelectric power plants

Storage power plants have a smaller flow rate, they mainly use the high head to produce energy. The compensation of the stochastic inflow can be done by appropriate scheduling using one or more storages. The main advantage of using a storage is that the amount of energy which can be produced is not directly depending on the current inflow. Because of environmental reasons a direct flow of the water into the lower courses is not allowed, the water has to be latched first. [3, p.111-114]

The natural inflow depends on the rivers that feed the reservoir which follows the four seasons over the year. For example in spring and summer the inflow is higher, because of the snow melt and higher rainfall. The storages can be daily-, weekly-, yearly- or long-term-storages, being used even over multiple years. For the construction of all hydro power plants the amount of available water is very important. The amount is fluctuating over the year, so like for storage power plants, also for run-of-river plants built in alpine water, caused by the snow-melt, the amount of water being available during summer is much higher than during winter. This can be compensated by using yearly storages, but even in this case a specific amount of water, defined by the responsible authorities, has to flow downstream. Additionally there are long-term fluctuations of the amount of water available, resulting in dry, normal and wet years. [4, p.221-228]

Pumped storage power plants

Pumped storage power plants do not produce additional electricity, except if there is an additional natural inflow. Their main purpose is a balancing of the load with an optimal efficiency. In this type of power plants water is also stored in a lower elevation reservoir. During periods of low-cost electric power, water is pumped to a higher elevation reservoir, while the stored water is used to produce electrical energy during periods of high demand. [3, p.115]

Each storage power plant has a minimum spillage for different reasons. The spillage can be uncontrollable, as the evaporation and the seepage in the reservoir, or it can be controlled by humans, as to have a minimum flow in the river to maintain the ecology of the environment.

Keeping a minimum storage also can be of relevance for different reasons, for example environmental or touristic ones.

2.1.3. Wind Power Station

In the last years wind power got more important, as it was discussed together with climate changes, the Kyoto protocol and the EU's climate targets. Wind power stations transform the kinetic energy of the wind into mechanic energy by the rotor blades and the rotor. Via a gear drive the generator is powered and transforms this mechanic power into electric power. [14, p.35,48]

The theoretical maximum generation capacity is

$$P_0 = \frac{1}{2} \cdot A \cdot \varrho \cdot v_0^3 \tag{2.5}$$

where ϱ is the density of the air, v_0 is the wind velocity and A is the surface the wind is passing through. [4, p.341]

Global winds can be explained by the different solar radiation, the rotation of the earth and the different distribution of land and water masses. Local winds are influenced from the different types of surface, i.e. mountain and valley breezes, sea-land breezes and katabatic winds. [14, p.34]

For an optimal performance constant wind conditions are necessary. In reality, especially near ground level, wind power is extremely stochastically changing. This can be influenced for example by surrounding buildings and woods. [14, p.34-35]

When choosing a site for wind power stations, to define the nominal power, the frequency distribution of wind velocity (daily, weekly and monthly distribution) has to be known [14, p.35]. The potential of wind power can be evaluated using a local wind-atlas.

As the economy of wind power stations depends on the intensity and regularity of the wind velocity medium wind speeds of 5-6 m/s are necessary, these conditions are available especially in coast and mountain regions. On-shore and off-shore wind power stations are used, whereby off-shore installations have an higher usage factor. [4, p.13]

The way wind power plants are constructed is also influenced by the fact that winds get more constant as well as stronger with rising height. Therefore increasing the hub height together with the rotor diameter increases the generation capacity.

The correlation between wind velocity and electrical output is shown in figure 2.5.

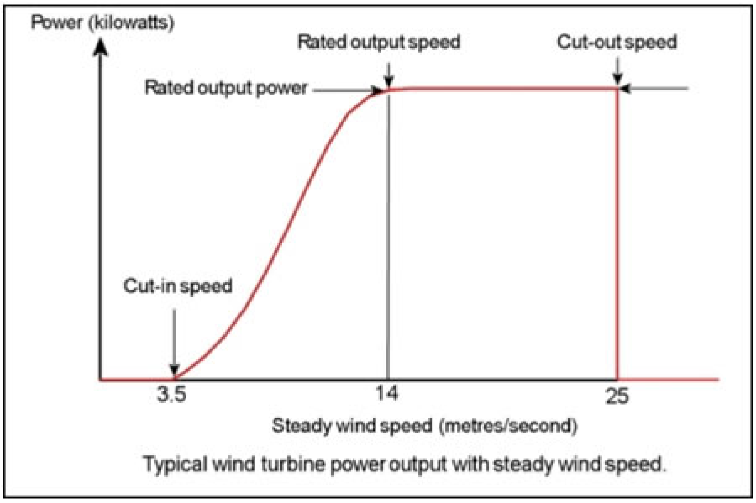

Figure 2.5.: Wind turbine power output over wind speed [35]

During the first phase the wind velocity is less than the cut-in speed, the electrical output therefore is zero. In the next phase the converter starts, the electrical output increases with v^3 until the nominal speed is reached. During the third phase higher wind velocities are compensated, the converter therefore generates constant output until the cut-off velocity is reached. With wind strengths higher than the cut-off velocity the converter needs to be retarded and the rotor blades have to be turned out of the wind. Based on a 1.5 MW exemplary values would be 2.5 to 3.5 m/s for the cut-in speed, 12 to 14 m/s for the nominal speed and 25 to 30 m/s for the cut-out speed. [14, p.50]

Stall and pitch regulation are two methods which are used to adapt to winds of different strength. The first one is used together with fixed rotor blades, with natural stalling occuring at defined windspeeds. When using pitch regulation, being more common, the blades are actively turned into or out of the wind depending on its strength. [14, p.51-53]

2.1.4. Solar power plants

Solar power plants convert the solar radiation into electrical energy either directly or indirectly.

The solar radiation is strongly fluctuating temporally as well es regionally. Besides the day and night rhythm this is caused by the elliptic orbit of the earth and its slightly inclined axis. On the outer border of the earth's atmosphere the average solar radiation is 1367 W/m^2. In July (greatest sun distance) the value is 1235 W/m^2, in January (smallest sun distance) 1420 W/m^2. [15, p.4]

The density of the solar energy is decreasing until reaching the earth surface, where the maximum is 1 kW/m^2. The radiation is influenced by diffusion and absorption. Diffusion is caused by air molecules, steam and dust, which lead to reflection into space and diffuse solar radiation. Parts of the sun light are also absorbed by ozone in the ultraviolet range and by steam in the infrared range. [16, p.7]

In Austria, for example in Graz, 2/3 of the radiation arrive between May and September and only around 13 % between November and February. [16, p.11]

The solar radiation can also fluctuate caused by the weather, i.e. the clouds, which can also cause fluctuations in a very short time when passing over a power plant. Reflection and diffuse radiation are also influenced by the snow.

Solar thermal power plants

To produce electrical energy through a steam process parabolic troughs, or solar power towers for greater power, are used.

Using parabolic mirrors the direct part of the solar radiation is focused (diffuse solar radiation can not be used). Parabolic trough power plants reach temperatures of 100-400 °C. The water is heated up in a pipe which is located in the focal axis of the trough. For higher temperatures paraboloids or heliostats are used.

For solar power towers the radiation is concentrated to the top of the tower using flat mirrors (heliostats). These are moved continuously to always have the right angel to the sun. The receiver transfers the heat to a working fluid (steam, helium, liquid natrium). Temperatures of 500 - 1200 °C can be reached, this allows to drive gas and steam turbines, which produce electricity in conventional form. [4, p.15]

Photovoltaic power plants

In photovoltaic power plants the solar radiation is directly transformed to electric energy through solar cells, which are made of semiconducting material, which absorbs a part of the photon flux of the sunlight and transforms this energy into electrical energy. This is called photovoltaic effect. [15, p.11]

2.2. Cost Account

For production, transforming, transmission and distribution of energy costs are developed. [4, p.58]

Some costs tend to remain the same even during busy periods (fixed costs), while others rise and fall depending on the volume of work (variable costs).

The fixed costs are associated with the business administration and do not change during quiet or busy times:

- amortization costs, interests
- fixed operating costs
    - taxes

  - insurances
  - personnel costs
  - costs of repairs
  - disposal costs

Variable costs are associated with productive work, and naturally rise and fall with business activity:

- fuel costs
- auxiliaries
- labour-related part for service and maintenance
- labour-related part for disposal
- CO2 certificates

The investment related part of the fixed costs for each year can be calculated using the annuity method. The annuity factor for the plant costs α_{cap} depends on the interest factor q or the interest i respectively, where $q = (1 + i)$, as well as on the expected lifetime:
[4, p.54-58]

$$\alpha_{cap} = \frac{q^n \cdot (q-1)}{q^n - 1} = \frac{i}{1 - \frac{1}{(1+i)^n}} \tag{2.6}$$

The annual capital costs then result in being the product of the initial investment cost c_{plant} and the annuity factor α_{cap}:

$$c_{cap/a} = c_{plant} \cdot \alpha_{cap} \tag{2.7}$$

The fuel costs can be calculated based on the fuel price in €/kg, the lower heating value LHV in kcal/kg combined with an conversion of factor 860 kcal/kWh, the efficiency η of the power plant and W, the amount of energy produced:

$$\text{fuel costs} = 860\ \text{kcal/kWh} \cdot \frac{\text{fuel price}}{LHV \cdot \eta} \cdot W \tag{2.8}$$

Run of river power plants and storage power plants nearly don't have variable costs. In pumped storage power plants the purchased energy for pumping is creating costs. This pumping incurred costs in relation to the energy produced afterwards depend on the purchase price for the electrical energy used for pumping in €/kWh, the efficiency of the pumping operation η_p as well as the operation of the turbine η_t, and of course the amount of energy produced:

$$\text{pumping costs} = \frac{\text{purchase price}}{\eta_t \cdot \eta_p} \cdot W \tag{2.9}$$

Construction costs including all related costs can be quite different. They depend on the gross electrical output, the location and the market situation. [13, p.232]

Electricity production costs are based on the power which can be sold, so the own use and the losses are not included. They depend on the type of power plant, the fuel and the size of the power plant. [13, p.234]

An example of a detailed calculation of electricity production costs for some fossile fired thermal power plants is shown in table 2.1 (german market, 2005). [13, p.237]

	unit	brown coal	hard coal	gas- and steam turbine	gas turbine
technical parameters					
gross electrical output	MW	1100	600	400	150
net electrical power	MW	1040	556	394	149
firing thermal capacity	MW	2558	1333	727	441
gross electrical efficiency	%	43.0 %	45.0 %	55.0 %	34.0 %
net electrical efficiency	%	40.6 %	41.7 %	54.2 %	33.7 %
emissions / MWh fuel	kg/MWh$_{fuel}$	410	342	202	202
emissions / MWh electricity	kg/MWh$_{el}$	1009	821	373	600
technical and economical data					
time of construction	months	48	36	24	12
calculated active life	a	35	35	25	25
calculated interests real	%	7.5	7.5	7.5	7.5
fuel price in LHV	€/MWh	3.97	9.12	20.01	23.76
petroleum tax (exempt from tax because of mineral oil tax act)	ct/KWh H$_u$	0	0	0	0
operating personnel	persons	80	70	30	5
personnel costs	k€/man-year	90	90	90	90
maintenance (fix, relating to investment)	%/a	1.6	1.5	0.7	0.5
maintenance (variable)	€/MWh$_{el}$	0	0	3	3
auxiliaries and disposal	€/MWh$_{el}$	1.65	1.3	0.5	0.5
insurance and overheads	%/a	0.5	0.5	0.5	0.5
allocations CO2 emissions	t/MWh$_{el}$	750	750	365	580
certificate costs	€/t	18	18	18	18
capital expenditure					
engineering/procurement/construction price (EPC-price)	M€	1210.0	540.0	180.0	45.0
builder/engineer/misc price	M€	90.8	40.5	13.5	3.4
decommissioning costs	M€	6.1	2.7	0.9	0.2
interest of EPC-price	M€	120.4	41.7	8.9	1.2
sum	M€	1427.2	624.9	203.3	49.8
energy and emission balance					
full load hours	h/a	7500	5500	5000	1250
electricity generation	GWh/a	7796	3056	1970	186
fuel consumption	GWh/h	19186	7333	3636	551
CO2 emission in total	kt/a	7866	2508	735	111
CO2 emission for certificate	kt/a	2019	216	15	4
electricity production costs - fixed costs					
capital cost	M€/a	116.3	50.9	18.2	4.5
maintenance	M€/a	21.5	9.0	1.4	0.2
personal (1%/a rate of increase real)	M€/a	8.0	7.0	2.9	0.5
insurance and overheads	M€/a	6.1	2.7	0.9	0.2
sum	M€/a	151.9	69.6	23.5	5.4
electricity production costs - variable costs typical for the time of usage					
fuel costs	M€/a	76.1	66.9	72.8	13.1
petrol tax costs	M€/a	0.0	0.0	0.0	0.0
maintenance for gas turbine	M€/a	0.0	0.0	5.9	0.6
auxiliaries and disposal	M€/a	12.9	4.0	1.0	0.1
CO2-certificates	M€/a	36.3	3.9	0.3	0.1
sum	M€/a	125.3	74.8	79.9	13.8
sum of annual costs	M€/a	277.20	1444.40	103.40	19.20
specific costs for the typical time of usage	€/MWh	35.55	47.26	52.48	103.69
power costs	€/(kW*a)	146.11	125.34	59.52	36.54
energy costs	€/MWh	16.07	24.47	40.57	74.46

Table 2.1.: Example of a detailed calculation of electricity production costs [13, p.237]

Great uncertainties are the fuel prices, which do not have a trend. They can leap up and down. Also the CO2-certificates can not be foreseen. [13, p.239,241]

For hydro power plants the investment costs depend on topographical and geological facts, the building technique, the amount of water, the height of the head and the type of the turbine. The costs can be quite different. The technical lifetime of the turbine and the electrical parts is around 40 years, the lifetime of the dam is usually estimated with 80 years. An additional part of the operation costs which are not to be neglected are the fees for water usage. The electricity production costs can also be quite different and depend on the investments and the local conditions.[13, p.265-266]

head	turbine	output	costs in €/kW
30 m	Francis	20 - 80 MW	3530 - 4970
8 m	Kaplan	<15 MW	4600 - 6200
		15 - 50 MW	3750 - 4950
		>50 MW	2690 - 3720

Table 2.2.: Examples for investment costs for large run-of-river power plants [13, p.266]

2.3. Market Conditions

Most of the energy is sold by long-term contracts on the forward marked. This contracts are based on the results of long-term scheduling. Just a part of the energy is sold in the spot or intra-day market.

The merit order is a curve sorted by the marginal costs of energy production. Based on this curve it is always possible to see which power plants can cover the needed load economicly. [7, p.1-2]

The sequence in the merit order is related to the variable costs of production, in an ideal case they are the marginal costs of the power plants. So they are just the production costs without fixed costs. [6, p.35]

The marginal costs are:
[7, p.1,3]

$$\text{marginal costs} = \frac{\text{fuel price}}{\eta} + \text{certificate} \cdot \frac{\text{specific emission factor}}{\eta} + \text{variable costs} \tag{2.10}$$

For each hour of the next day an auction is done on the day-ahead spot market of the EEX. All offers for sale are sorted ascending by the marginal costs and all offers for buying are sorted descending. The intersection of the two curves defines the market price and the amount of energy traded. [7, p.2]

All winning bids get or have to pay the same market clearing price. If the market clearing price is higher than the marginal costs, a profit contribution to the fixed costs is achieved. The profit contribution is the difference between the offered price and the market price. The last used power plant can only cover the variable costs. [6, p.37]

While producers and power traders are in competition with each other, the grid is still a natural monopoly. This is because it is not possible to build several high voltage lines and substations for each provider. The costs for transmission and distribution are cleared with regulated prices by an authority. [24, p.48]

For the optimization of costs different models and techniques can be used. Some examples are described in the following chapter.

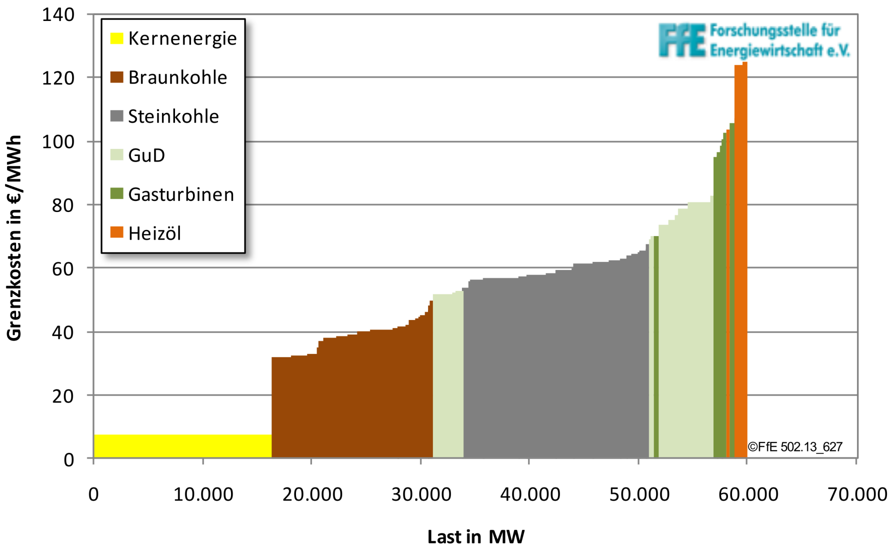

Figure 2.6.: Example of marginal costs as a function of load power, with the colours indicating the plant types used for production according to their merit order [7, p.3]
nuclear power: yellow, brown coal: brown, hard coal: dark gray, gas and steam: light gray, gas: green, oil: orange

2.4. Scheduling of Power Plants

Power production, transmission and distribution have the task to supply electrical energy within several technical, quality and environmental constraints. The objective function for system optimization is the cost effectiveness. To solve such a complex model, it has to be divided into different domains. The graph below shows the main classification. [2, p.8-9]

While scheduling focuses on economic aspects, the operational management concentrates on the power quality which has to be provided.

In scheduling the resource planning is decided, so that the load can be covered. Also the exchange between the trading zones or control areas has to be taken into account. The scheduling includes constraints that cross over all time intervals, for example minimum downtimes, so an optimization has to be run for at least some days. If storage power plants or delivery contracts are included, the time horizon should be extended to the accordant interval. Most power plants have long-term contracts for primary energy. [2, p.8-10]

In scheduling a compromise between detailed models, approximations and available calculation power should be found. [2, p.10]

In Austria the transfer power is given in 30 minutes averages, so a model for one year would have 17.520 time intervals. The number of variables and the necessary calculating time would be enormous. [5, p.6]

As a consequence the constraints have to be restricted. However, this might create infeasibilities in the short term scheduling. [2, p.33]

The optimization can be split into three main parts: days, weeks and years.

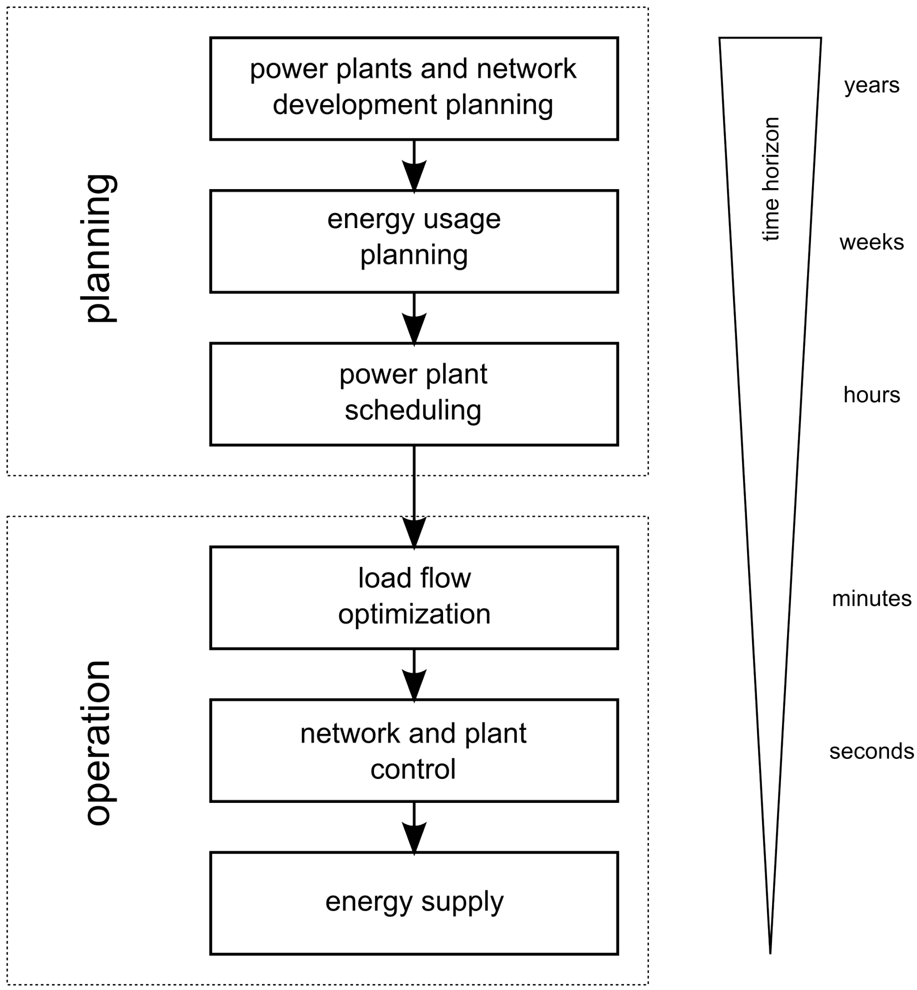

Figure 2.7.: Overview of tasks in planning and operation [2, p.9]

The optimization objective and its mathematical formulation can use different approaches. The costs can be minimized or the profits maximized.

At first one possible scheduling approach in a vertically integrated energy company is explained, afterwards the differences to the liberalised energy marked are mentioned.

In vertically integrated energy companies (old monopolies) the production as well as the grid were managed by the same organisation.

The different stages of scheduling can be categorized for example by the relevant time horizons: [4, p.607]

- Long Term Scheduling
  scheduling for periods of one year or more
- Middle Term Scheduling
  scheduling for weeks or months
- Short Term Scheduling
  scheduling for one or more days

- Instantaneous Optimization

Long Term Scheduling

The objective of long term scheduling is to run a group of power plants with minimal costs, including the management of yearly and multi-year storage reservoirs and the scheduling of thermal power plants. [4, p.626-627]

Long term scheduling is used for a planning period of one year or even longer and the planning periods are usually divided into months or weeks (12 or 52 stages). In this scheduling strategy long-term subscriber and supply agreements can be included. It can also be used for determining the fuel supply for the thermal blocks and the optimal start-up and shut-down times. It is important to know the maintenance schedule of the power plants. Forecast values (i.e. for load and fuel price) used as planning basis can introduce additional uncertainty. [5, p.6]

Above all long term scheduling is used to develop an optimal strategy for the reasonable use of large hydroelectric systems.

While the management of annual storage reservoirs is optimized, daily and weekly storage reservoirs are treated like run-of-river power plants. For the transmission system the transmission losses are neglected. [4, p.627]

Since the inflow is not constant over the year and can be different from expectation, the simulation has to use more than one scenario. It is important to take the whole set of fluctuating water inflows into account and to formulate this in a very detailed model. On the other hand this extends the model and makes it more complicated and time consuming to find a feasible solution. So stochastic optimization is important. [4, p.627]

The two control variables used in models for simulating hydro power plants are the spillage and the electrical output. The storage is a state variable, since it is derived when the other two values are determined. This fact will be described in chapter 5.

To improve the results of long-term scheduling, it is repeated in continuous intervals. New inputs can improve the output, lost chances can never be made up. [4, p.627]

Middle Term Scheduling

After long-term scheduling the planned use of the thermal power plants and the amount of water being available during different weeks or months are known. Also decisions about long-term delivery contracts are done during long-term scheduling. In middle-term scheduling the optimization is going to be more detailed. The weekly or daily reservoirs are taken into account. The uncertainties of the forecasts for the available water and the availabilities and costs for thermal blocks are still given, even if they are less than during the long-term scheduling. [4, p.628]

As the outputs of middle-term scheduling are used as inputs for short-term scheduling, the optimization is usually repeated several times per week, to improve the results. It can also be based on stochastic optimization.

Short Term Scheduling

Short-term scheduling is normally done on a daily basis and taking into account much more details than in long- and middle-term scheduling. Where different power plants where summarized into groups during previous optimizations steps, they are now considered individually. The advantage is, that the forecasts are more detailed and secure. The expected load curve for the following day is given in half-hour intervals. The grid and the allocation of reactive power can be taken into account, being important because of its influence to the costs of the transmission and the amplitude of the voltage. [4, p.628]

Instantaneous Optimization

The real network load never follows the values of the forecast created by short-term scheduling. So the instantaneous optimization quickly responds to changes caused by fluctuations in the needed load, blackouts, drop-outs of power plants or outages in transmission networks. For the power-frequency control the current values are monitored. The frequency response reserve has to balance between the need and the production. It is tried to allocate it between different power plants. Especially (pump-)storage power plants are used. With the spinning and non-spinning reserve it is tried to restore the scheduled exchange between the different trading zones or control areas and to split the load-difference over several power plants in the trading zone or control area. If just a few power plants are used to balance the differences, it is risked to leave the economic optimum. So the best solution is an on-line optimization for the load. [4, p.629] [17, p.76,78]

Since the energy market is liberalized nowadays, the objective function for optimization has changed. Instead of minimizing the costs for providing the needed electrical energy, maximizing the profit, which of course also includes a minimization of the production and procurement costs, is important now. Power plants or groups of plants are independent of the transmission and distribution network, the costumer can choose any producer, the grid is like a neutral instance just transmitting the energy. [20, p.40][4, p.630]

The power producers are in competition with each other and try to maximize their benefit. There can be more than one producer on the market, all performing their own optimizations within their constraints. But not only the costs are important, also their success in the marketplace.

Not the whole system can be optimized because different power plants, the transmission system and different distribution networks are often owned by several independent and competing companies. This means, an optimum for the overall system can not be reached even if each producer is working at his optimum. It may happen, that a more economic producer can not sell his energy because of other mechanisms on the market. This is the risk of the system because it is supposed that the cost pressure in the competition overcompensates this differences to the total optimum. [4, p.635]

The main structure of the planning process is not changed, it is still started with the long-term energy supplies and the revision timetables, which correspond to stochastic values. New elements which have to be taken into account are the variability of the prices on the energy market as well as the prices for fuel, which can fluctuate more. The horizons are not changed, just new aspects are added. So additional degrees of freedom within the planning horizon got introduced, because the producers can also buy energy on the spot market instead of producing it by themselves or sell energy there. [4, p.636-637]

In the last years more renewable energy with fluctuating production is used. This did not change the optimization algorithms, but introduced more uncertainties. Also emission certificate trading took part of the optimizations. After the unbundling of the grid the management of the system got changed, so now the network operator can use auctions to ask for frequency response reserve and spinning and non-spinning reserve. [27, p.5-7]

Nowadays the requirements on optimization algorithms are that they are more detailed, solve the problem as fast as possible and deliver the information in a way allowing it to be used very fast.

3. Mathematical Basics

3.1. Optimization, Mathematical Algorithms

In general the objective of an optimization algorithm is to find the extremum, which means to maximize or minimize the objective function with respect to various constraints. The unknown variables should be solved. There are a lot of optimization subfields and techniques to do this job. In this thesis, some of them are explained, corresponding to the fields and the techniques used by XA, an optimization library used for this purpose.

Optimization problems are given in several fields for different reasons. For example:

- in economies to minimize costs or in transportation companies to calculate the best routes in a complex transport system
- in farming to calculate the amounts of different kinds of seeds to be used to achieve best harvests
- in nutritional science to calculate the smallest amount of food with the highest nutritional content
- in the petrol industry optimization is applied to production problems, which means that the oil production, the allocation and the processing should be done for the lowest costs
- in the telecom sector optimization algorithms are used to calculate the most useful connections between different cities for the lowest costs
- in the steel industry the optimal use of the rolling trains is of relevance
- in the chemical industry as well as in the pharmaceutical industry optimizations can be applied to compound problems
- in aviation the whole transportation network has to be optimized

Fields

Depending on the type of input and output data and the mathematical description of the problem a categorization into several fields can be done, like for example:

- Linear Programming: the objective function and all constraints are linear, all variables are continuous variables
- Integer Programming: the objective function and all constrains are linear, all variables are integer variables
- Mixed-Integer Programming: the objective function and all constrains are linear, some variables are continuous, some are integer variables

- Non-linear Programming: the objective function and the constrains are non-linear
- Quadratic Programming: a special case of non-linear programming
- Stochastic Programming: taking into account uncertainties of fluctuating values
- Dynamic Programming: decomposing large problems into nested subproblems

Techniques

The actual optimization can then be done using different techniques, depending of the problems:

- Simplex/Vertex Method: the optimization is done by the walking along the vertices of the feasible region
- Branch and Bound: is used for finding integer solutions by branching and bounding
- Interior Point Method: the optimization is done by moving within the feasible region
- Ellipsoid Method: the optimum is found by constructing ellipsoids around the target point
- Steepest Descent: the optimum is found by following the steepest descent of the surface of the function

In the electricity industry, one or more of these optimization algorithms are used for the scheduling of power plant systems. Since the liberalization of the energy market, the minimization of the marginal-costs has become necessary to optimize power plants.

In different trading zones or control areas a certain number of power plants have to cover the total load in the system, depending on

- the kind of power plants
- the prices of the production
- the possibility of the optimal use of the power plants
- the spot prices

Additionally the import of energy might be less expensive than the costs incurred on producing it locally. In other cases, it might be better to produce more energy and export it during periods of higher prices.

In hydro-thermal scheduling the thermal power plants should be used for covering the base load. Because the turn-on and turn-off times for hydro power plants are shorter in comparison to thermal plants, they can easily be switched on or off for short periods of time. They should therefore be used mostly for peak-shaving. Also the costs for producing hydro energy are low. Additionally if it is “green energy” it can be sold on spot markets at a certain price. The main problem with hydro power plants is that the natural inflow of storage power plants or the flow rate of run-of-river plants has to be taken into account in the generation of electrical energy and these values fluctuate during the year and can not be forecasted with high accuracy.

The problem of scheduling can be formulated as a linear problem, all the constraints and formulas have to be linearized. Depending on the number of power plants, the problem might become too big to solve it within reasonable time, so it has to be separated into multiple stages and solved using decomposition approaches. Taken into account fluctuating load and inflow conditions of different years the system has to be solved with stochastic programming.

3.2. Fields

3.2.1. Linear Programming

In linear programming the main purpose is to find the extremum of the objective function. This objective function and all constraints are linear, using continuous variables. The constraints can be equations and inequations with $\leq$ or $\geq$ conditions. [25, p.124]

Linear programming is one of the most used methods in operations research. [28, p.142]

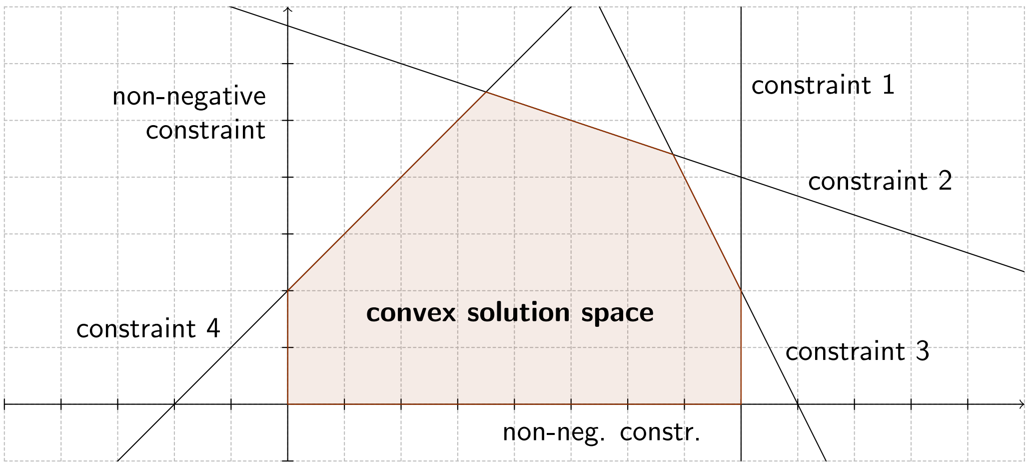

Figure 3.1.: Convex solution space bounded by several constraints [28, p.143]

The constraints form an convex solution space (convex polyhedron). They restrict the solution space as shown in the graph above. The optimum is on one of the vertexes. [28, p.142]

Another constraint is the demand for a linear objective function. Taking several linear functions, which are only used in a certain interval of the objective function, non-linear functions can be approximated. [28, p.112]

The simplest way of describing a linear system in the general form is:

$$f(x) = c_1 \cdot x_1 + c_2 \cdot x_2 + ... + c_n \cdot x_n \tag{3.1}$$

$$c, x ... \mathbb{R}^n \tag{3.2}$$

Because the variables can have values within the range from $-\infty$ to $+\infty$, constraints have to be defined to find a possible solution.

Considering this constraints it can be rewritten as:
[28, p.145]

$$min : f(x) = c^T \cdot x \tag{3.3}$$

subject to:

$$A \cdot x \geq b \tag{3.4}$$
$$x \geq 0 \tag{3.5}$$

with:

c	$\epsilon\ \mathbb{R}^n$	cost-vector
x	$\epsilon\ \mathbb{R}^n$	vector of the opimization variable
A	$\epsilon\ \mathbb{R}^{m \times n}, m < n$	matrix, coefficients of the constraints
b	$\epsilon\ \mathbb{R}^m$	vector, known values

It does not matter if the objective function should be maximized or minimized, because a search for a maximum can also always be interpreted as a search for a minimum by multiplying the objective function by -1 and vice versa. [26, p.40]

To solve a problem in its linear form, all the constraints and formulas have to be linearized. Interpreted geometrically the constraints describe a polyhedron, the so called feasible region. To find a solution it is important that the constraints do not contradict each other, the polyhedron is completely bounded and there is only one minimum or maximum. If one of this requirements is not met, it is not possible to find a solution. This is shown in the following examples. [26, p.39]

Empty solution space

If there are contradicting constraints, the solution space is empty.

$$x_1 > 4 \tag{3.6}$$
$$x_1 < 1 \tag{3.7}$$

Unbounded solution space

If the solution space is not completely bounded, the problem may also not have a unique solution.

$$max : x_1 + x2 \tag{3.8}$$

subject to:

$$x_1 + x_2 \geq 3 \tag{3.9}$$
$$x_1, x_2 > 0 \tag{3.10}$$

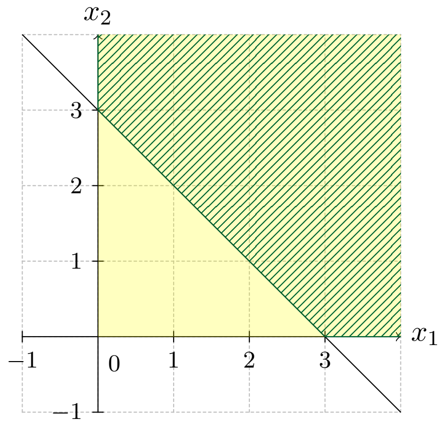

Figure 3.2.: Solution space (shown in green) which is not completely bounded

Ambiguous solutions

If there exists no explicit extremum a set of ambiguous solutions is possible.

$$max : x_1 + x_2 \tag{3.11}$$

subject to:

$$x_1 + x_2 \leq 3 \tag{3.12}$$
$$x_2 \leq 2 \tag{3.13}$$
$$x_1, x_2 > 0 \tag{3.14}$$

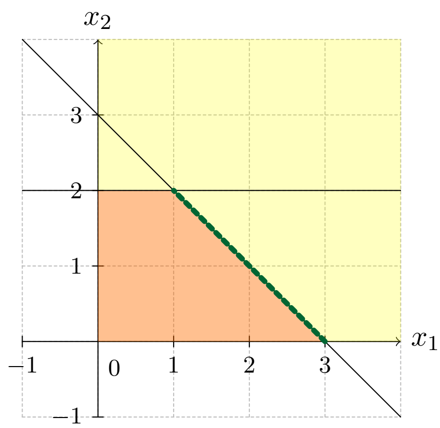

Figure 3.3.: Optimization problem without explicit extremum (possible solutions for x_1 and x_2 are shown as green line)

The great advantage of linear programming compared to other methods is its simplicity and the possibility to solve problems very fast.

Example - mathematical problem definition

The following example should explain the way of finding the feasible region by drawing constraints. In the following example two variables are used. With more variables the dimensionality rises and it is much harder to visualize the problem.

The problem definition is taken from [28, p.143]:

$$max : 4 \cdot x_1 + 3 \cdot x_2 \tag{3.15}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 \leq 6 \tag{3.16}$$
$$3 \cdot x_1 - 2 \cdot x_2 \leq 3 \tag{3.17}$$
$$2 \cdot x_2 \leq 5 \tag{3.18}$$
$$2 \cdot x_1 + x_2 \leq 4 \tag{3.19}$$
$$x_1, x_2 \geq 0 \tag{3.20}$$

The constraints define a convex area. The initial solution space defined by by the conditions $x_1 > 0$ and $x_2 > 0$ is shown as yellow area. The first two constraints define the area marked red in the solution space. The third constraint does not restrict the solution space at all, therefore it is called redundant. The solution space defined by the fourth constraint is shown as blue area. The final solution space is the overlapping area of both sets of constraints. It is shown in the diagram using violet.

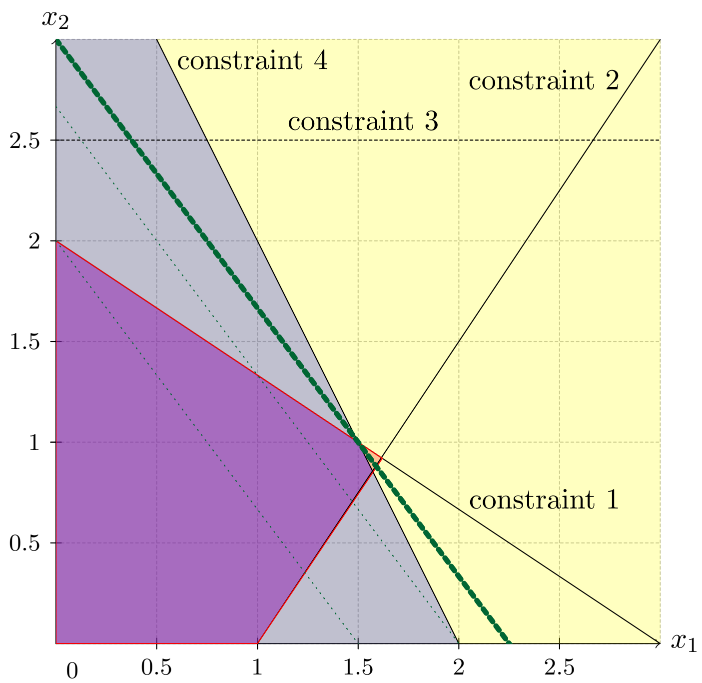

Figure 3.4.: Graphical solution of the optimization problem

Finally it is necessary to find the maximum of the objective function in the solution space. To do this a straight line representing the function $4 \cdot x_1 + 3 \cdot x_2$, shown in green, is introduced. This straight line is moved parallel towards the extremum until it intersects the solution space at exactly one point. The solution for x_1 and x_2 is the maximum of the objective function.

The optimal solution is:

$$x_1 = 1.5 \tag{3.21}$$
$$x_2 = 1 \tag{3.22}$$
$$f(x) = 9 \tag{3.23}$$

Duality

Based on the duality principle, each linear programming problem (the primal problem) has its corresponding dual problem. Between the primal and the dual problem exists a close and defined relation. This relationship can be used to easier find a solution for linear programming in some cases.

The relation is:
[29, p.343-345]

- If the primal problem (LP) is a maximization problem where all constraints are $\leq$-inequalities, so the dual problem (DP) is a minimization problem with $\geq$-inequalities and vice versa.
- The coefficients from the objective function of the LP are the coefficients on the right hand side of the constraints of the DP in the same order.
- The coefficients on the right side of the constraints of the LP are the coefficients of the objective function in the DP, also in the same order.
- The left side coefficients of the constraints of the LP read in horizontal direction are the left side coefficients of the DP in vertical direction (it is a transposed matrix).

So a linear programming (primal) problem with m constraints and n variables results in a dual problem with n constraints and m variables.

This means, the primal problem

$$max : c^T \cdot x \tag{3.24}$$

subject to:

$$A \cdot x \leq b \tag{3.25}$$
$$x \geq 0 \tag{3.26}$$

results in the dual problem

$$min : b^T \cdot y \tag{3.27}$$

subject to:

$$A^T \cdot y \geq c \tag{3.28}$$

$$y \geq 0 \tag{3.29}$$

In some cases it can make sense to repeat this process to construct a second dual problem based on the first dual problem generated out of the original linear programming problem, which would result in:

$$min : y^T \cdot b \tag{3.30}$$

subject to:

$$y^T \cdot A \geq c^T \tag{3.31}$$

$$y^T \geq 0 \tag{3.32}$$

Because of the close relationship between the primal and the dual problem the value of the objective function at the optimum is the same for both formulations, though the objective function itself looks quite different. This called strong duality and is true in most cases.

As an example converting the primal problem

$$max : 2 \cdot x_1 + 3 \cdot x_2 + x_3 \tag{3.33}$$

subject to:

$$2 \cdot x_1 + x_2 + x_3 \leq 20 \tag{3.34}$$

$$x_1 + 2 \cdot x_2 \leq 30 \tag{3.35}$$

$$x_1, x_2, x_3 \geq 0 \tag{3.36}$$

with three coefficients and two constraints results in the dual problem

$$min : 20 \cdot y_1 + 30 \cdot y_2 \tag{3.37}$$

subject to:

$$2 \cdot y_1 + y_2 \geq 2 \tag{3.38}$$

$$y_1 + 2 \cdot y_2 \geq 3 \tag{3.39}$$

$$y_1 \geq 1 \tag{3.40}$$

$$y_1, y_2 \geq 0 \tag{3.41}$$

with two coefficients and three contraints.

The solution of the primal problem is $x_1 = 0$ and $x_2 = 15$ with a value of 50 for the objective function, for the dual problem it is $y_1 = 1$ and $y_2 = 1$, also with a value of 50 for the objective function. How to solve a linear problem using primal and dual problem formulations is going to be described in chapter 4.3

3.2.2. Integer programming

If all unknown variables are integers the problem is called an integer programming problem. Sometimes this might be necessary, because in several cases the results can only be represented by integer values, for example in the case of a number of people, a number of cars or when on/off-decisions are represented by integer values. The main problem of integer programming compared to linear programming problems with continuous variables is that it takes much more time to find a solution or the problems can not be solved in the worst case. Therefore special integer programming techniques are used to solve such problems. [26, p.9-10]

Example - mathematical problem definition

Again this can be demonstrated looking at the example introduced when explaining linear programming.

$$max : 4 \cdot x_1 + 3 \cdot x_2 \tag{3.42}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 \leq 6 \tag{3.43}$$
$$3 \cdot x_1 - 2 \cdot x_2 \leq 3 \tag{3.44}$$
$$2 \cdot x_2 \leq 5 \tag{3.45}$$
$$2 \cdot x_1 + x_2 \leq 4 \tag{3.46}$$
$$x_1, x_2 \geq 0 \tag{3.47}$$
$$\tag{3.48}$$

Also in this case the solution can be determined graphically. The constraints are drawn in the graph and the solution space is displayed. The possible integer values for x_1 and x_2 are demonstrated with the red points. The best solution for x_1 and x_2 with respect to the solution space and the objective function would be at $x_1 = 1.5$, but as both, x_1 and x_2, should be integer values, the final solution also matching this additional criteria is at $x_1 = x_2 = 1$ with $f(x) = 7$, even if this is not the possible optimum within the whole solution space.

3.2.3. Mixed Integer programming

Additional complexity is added if only some of the unknown variables are required to be integer values, but the rest are continues variables. In such cases the problems are called mixed-integer problems. In practice most optimization problems are mixed-integer problems. [28, p.163]

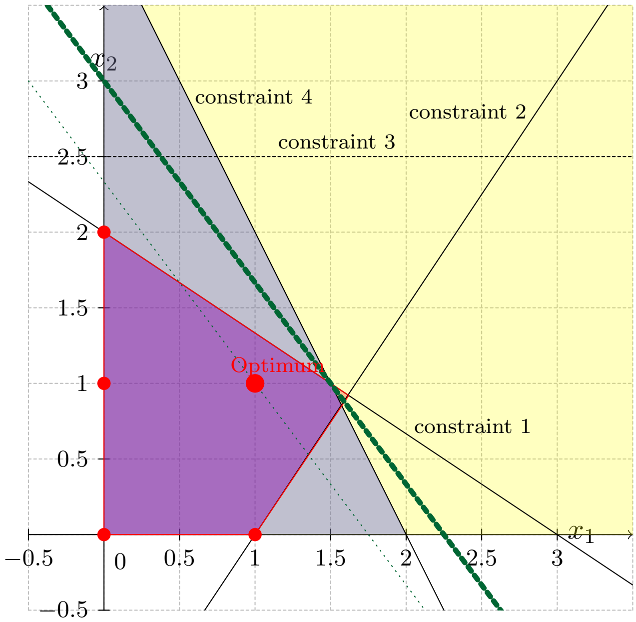

Figure 3.5.: Graphical solution (integer programming)

Example - mathematical problem definition

Again, using a simple, 2-dimensional optimization task, finding the solution can be demonstrated graphically:

$$max : 4 \cdot x_1 + 3 \cdot x_2 \tag{3.49}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 \leq 6 \tag{3.50}$$
$$3 \cdot x_1 - 2 \cdot x_2 \leq 3 \tag{3.51}$$
$$2 \cdot x_2 \leq 5 \tag{3.52}$$
$$2 \cdot x_1 + x_2 \leq 4 \tag{3.53}$$
$$x_1, x_2 \geq 0 \tag{3.54}$$

If only x_1 needs to be an integer value while continuous values are allowed for x_2 the possible solutions are not restricted to the intersections of grid lines representing integer values on both axes. All points on all grid lines having integer values on the x_1-axis within the area bounded by the constraint functions are potential candidates in this case. This is shown in figure 3.6 with the red dot representing the optimum. The optimal solution is at $x_1 = 1$ and $x_2 = 1.33$ with $f(x) = 8$.

3.2.4. Stochastic programming

In stochastic programming also uncertainties of data or fluctuating values which may occur with several optimization problems are taken into account. Real world problems very often have to be

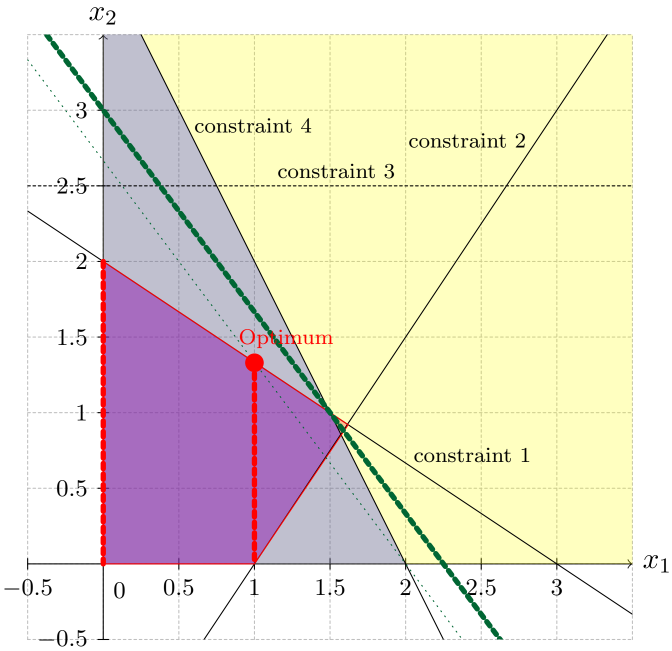

Figure 3.6.: Graphical solution (mixed integer programming)

formulated including unknown parameters, or at least based on information which is only known with a certain accuracy. Stochastic programming is a way to deal with such problems and still allow the optimization to be run.

The main purpose is to find a solution which is feasible for all sets of input data, and to do this within a certain computational tolerance and within a given total computing time or maximum number of iterations. The generated solutions can be at a global as well as a local optimum.

Solving such problems is based on using stochastic or random variables with unknown values but given probabilities. [25, p.483]

The general form of a two stage stochastic problem looks like:
[31, p.10-11]:

$$min : c^T \cdot x + E_\xi \cdot Q(x, \xi) \tag{3.55}$$

subject to:

$$A \cdot x = b \tag{3.56}$$
$$x \geq 0 \tag{3.57}$$

in combination with:

$$Q(x, \xi) = min : q^T(\xi) \cdot y \tag{3.58}$$

subject to:

$$W(\xi) \cdot y = h(\xi) - T(\xi) \cdot x \tag{3.59}$$
$$y \geq 0 \tag{3.60}$$

In the first stage the decision vector $x \in \Re^n$ has to be optimized without knowing the whole information on some random events. This stochastic input data is represented by the random vector ξ, which is formed by the components of q^T , h and T. E_ξ is the mathematical expectation with respect to ξ.

As the results of the first stage are optimal with respect to the probability distribution of the random input data which was taken into account, they are not optimal for each single scenario. The second stage optimization is done at a later point in time, when the values of the stochastic inputs are already known. Its goal is to make the best out of the particular scenario, which can not be influenced anymore. The second stage variable vector is $y \in \Re^m$.

Example - a farmers problem

A farmer wants to decide how much of his 500 hectares of land he should use for grain, for corn and for sugar beets. [31, p.4-10]

The restrictions are, that he needs 200 tons of wheat and 240 tons of corn for animal feeding. This amounts can be raised on the farm or partially or even completely bought from a wholesale. In cases of an overproduction the remainings can also be sold. The sugar beet production is only done for selling and there are regulatory restrictions, which result in a dramatically reduced price for any production which exceeds 6000 tons. The purchase and selling prices for the different products are listed in table 3.1.

product	selling price	purchase price
wheat	\$ 170 / ton	\$ 238 / ton
corn	\$ 150 / ton	\$ 210 / ton
sugar beet (first 6000 tons)	\$ 36 / ton	-
sugar beet (above 6000 tons)	\$ 10 / ton	-

Table 3.1.: Purchase and selling prices

Further known input data are the costs for preparing the land for the production of the different agricultural products and the yield which can be expected in each case. These costs are listed in table 3.2.

product	preparation costs	expected yield
wheat	\$ 150 / ha	2.5 tons / ha
corn	\$ 230 / ha	3.0 tons / ha
sugar beet	\$ 260 / ha	20.0 tons / ha

Table 3.2.: Preparation costs and expected yield for each product

culture	acreage	yield	sales
wheat	120 ha	300 tons	100 tons
corn	80 ha	240 tons	-
sugar beets	300 ha	6000 tons	6000 tons

Table 3.3.: Optimal solution with linear programming

After solving this problem with a linear solver program, the farmer obtains an optimal solution with an total profit of $ 118,600 as shown in table 3.3.

Unfortunately nature introduces a stochastic effect, as weather conditions, especially rain and sunshine, vary from year to year. So harvests which are 20 % higher or lower than the average are typical. This not only leads to higher or lower profits, but also changes the distribution of land usage which has to be done to reach the best possible profit.

culture	acreage	yield	sales
wheat	183.33 ha	550 tons	350 tons
corn	66.67 ha	240 tons	-
sugar beets	250.00 ha	6000 tons	6000 tons

Table 3.4.: Optimal solution for years with a plus-20%-harvest

culture	acreage	yield	sales
wheat	100.00 ha	200 tons	-
corn	25.00 ha	60 tons	-
sugar beets	375.00 ha	6000 tons	6000 tons

Table 3.5.: Optimal solution for years with a minus-20%-harvest

Thus the total profit when optimally using the land in these other scenarios would be $ 167,667 in good (+20%) years and $ 59,950 in bad (-20%) years.

As it is not possible to make weather forecasts accurate enough, it is also not possible to make perfect decisions which match the future weather conditions. To still have a chance to come to an decision, which is good for all possible scenarios, all these scenarios have to be considered together with the probability of their appearance. This is where stochastic programming plays its role.

In the case of the given example there are three scenarios with the same probability of occurrence, one for normal weather conditions, one for better and one for worse harvests.

Passing the stochastic formulation of the problem through a linear solver gives us an optimal solution of using 170 hectares for wheat, 80 hectares for corn and 250 hectares for sugar beets. This values are the primary output of the optimization, the so called first stage output, the values for sells and purchases depend on the effective harvests which depend on the real weather conditions. These so called second stage results are shown in table 3.6.

The optimal solution can be interpreted as that in all scenarios it is more optimal to avoid sales of sugar beet under the unfavourable price, even when it is not used up to the amount of 6000. The land for corn should carry out the feeding requirement when the land yield is on average. The rest of the land is devoted to wheat. This leaves the farmer with an expected average profit of $ 108,390.

	wheat		corn			sugar beets		
scenario	yield	sales	yield	sales	purchases	yield	sales	profit
1 - good years	510	310	288	48	-	6000	6000	$ 167.000
2 - normal years	425	225	240	-	-	5000	5000	$ 109.350
3 - bad years	340	140	192	-	48	4000	4000	$ 48.820

Table 3.6.: Results for different scenarios after stochastic optimization

Looking at the resulting profit for normal years it can clearly be seen that optimum for a single scenario is worse when using stochastic optimization compared to using normal linear optimization for exactly this scenario. This is because stochastic optimization takes into account, that this scenario only occurs with a certain probability and also respects the situation in other scenarios with their given probability of occurrence.

3.2.5. Dynamic programming

Dynamic programming can be used to decrease the runtime needed to optimize large problems. Often a formulated problem can have a big number of variables and constraints, so it can become hard to find a solution if the objective function and all constraints are taken into account at the same moment. If the problem consists of overlapping subproblems with optimal substructures it can be split into a number of smaller optimization problems, which can be solved one after another. Optimal substructures means, that the optimal solutions of all subproblems can be used to find the optimal solution of the master problem. [32, p.5-7]

So the way of finding a solution in dynamic programming can be interpreted as a tree step problem:

- divide the main problem into several subproblems
- solve the subproblems using an appropriate algorithm, either in forward direction or backwards
- use these optimal solutions to construct an optimal solution for the original problem

Example - production optimization

This example describes a production problem, in which the demand for the goods as well as the different kinds of costs incurred for production and storage are changing over different periods. [34]

The example goes over four periods, whereas the inventory is empty at the beginning of the first period and also has to be empty at the end of the last period. Taking items from one period to the next creates fixed costs of $ 1 per unit, the costs for production (a fixed fee for production startup plus production costs per unit) and the demand in each period are given in table 3.7.

At any stage the beginning inventory plus the amount of produced units during a period must be equal to the demand plus the size of the output inventory.

The problem can be solved by splitting it into small subproblems, one for each period, and using backward recursion for optimization. However, in some other cases forward recursion may be more convenient.

period	production setup costs	production costs per unit	demand
period 1	$ 5	$ 1	5
period 2	$ 7	$ 1	7
period 3	$ 9	$ 2	11
period 4	$ 7	$ 2	3

Table 3.7.: Production costs and demands

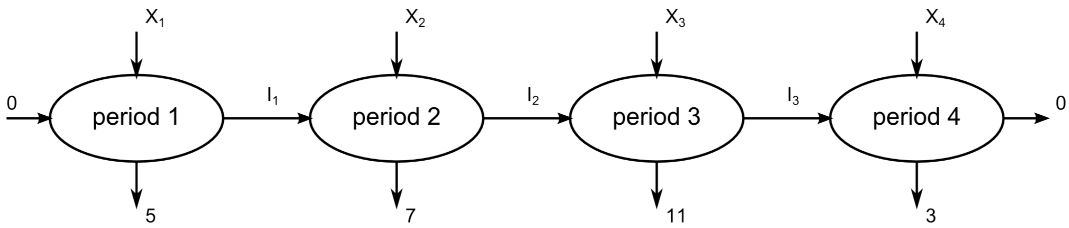

Figure 3.7.: Production problem with its four subproblems (X_n are the productions in each period, I_n the output inventories) [34]

An initial consideration before numerically optimizing the problem is, that each period should either start with an empty beginning inventory, in which case a production should take place, or, in case of a non-empty beginning inventory, no production should be done. This is based on the idea, that if it is profitable to take items to the next period, this must be true for one item up to at least the whole demand in the next period.

In a first step the optimization for period 4 is done. Based on the initial consideration, that a production only should take place when no items were received from the previous period, there are only two possible solutions, either receiving all 3 needed items as input inventory or producing all of them in this period, both with their associated costs.

	beginning inventory	production	total costs
solution 1	0	3	$ 13
solution 2	3	0	$ 0

Table 3.8.: Possible solutions for the period 4 subproblem

Based on this information the optimization can be continued with period 3. As the possible beginning inventories for period 4 can be 3 or 0, all possible scenarios delivering these output inventory values have to be taken into consideration. Again, only allowing either production or a non-zero input inventory, four solutions would be possible.

As solutions 1 and 3 have the same input inventory, the costs created are the only difference when seen from the preceding period. This means, that solution 1, creating the higher costs, can be ignored during further optimization steps.

For period 2 all solutions with an output inventory of 0, 11 or 14 could be of relevance. As for period 3 for each different size of the input inventory only the best solution survives, solution 1 and 3 therefore can be discarded.

Finally when continuing optimization with period 1, no additional solutions can be created, as the input inventory for period 1 is fixed to zero.

	output inventory	beginning inventory	production	total costs (periods 3 to 4)
solution 1	0	0	11	$ 44
solution 2	0	11	0	$ 13
solution 3	3	0	14	$ 40
solution 4	3	14	0	$ 3

Table 3.9.: Possible solutions for the period 3 subproblem

	output inventory	beginning inventory	production	total costs (periods 2 to 4)
solution 1	0	0	7	$ 54
solution 2	0	7	0	$ 40
solution 3	11	0	18	$ 49
solution 4	11	18	0	$ 24
solution 5	14	0	21	$ 45
solution 6	14	21	0	$ 17

Table 3.10.: Possible solutions for the period 2 subproblem

So the optimal solution for period 1 is to produce exactly the demand of 5 items an pass no inventory to the next period. Under this constraint of an empty beginning inventory the optimal solution for period 2 is a production of 21 in combination with an output inventory of 14 items. Continuing through all periods the optimal solution for the master problem is a production of 5 an 21 items in periods 1 and 2 without any production in the last two periods, generating total costs of $ 55.

3.2.6. Nonlinear and Quadratic Programming

The two fields of nonlinear and quadratic programming should just be mentioned here. Both of them are not used in this thesis, as all problems were linearized before optimization.

Nonlinear programming is part of the mathematical optimization which has to be done to find the best solution for continuous nonlinear problems. Objective function and constraints are nonlinear in this case. [29, p.268]

Quadratic programming is just a special case of non-linear programming. [29, p.192]

The general form of an quadratic programming problem is:
[29, p.192]:

$$min: \frac{1}{2} \cdot x^T \cdot G \cdot x + g^T \cdot x \tag{3.61}$$

subject to linear equations and inequalities:

$$A^T \cdot x \geq b \tag{3.62}$$
$$x \geq 0 \tag{3.63}$$

	output inventory	beginning inventory	production	total costs (periods 1 to 4)
solution 2	0	0	5	\$ 55
solution 4	7	0	12	\$ 64
solution 5	18	0	23	\$ 70
solution 6	21	0	26	\$ 69

Table 3.11.: Possible solutions for the period 1 subproblem

3.3. Techniques

3.3.1. Simplex method

The simplex method is used to solve linear problems by analysing the vertexes of the feasible region. [29, p.269]

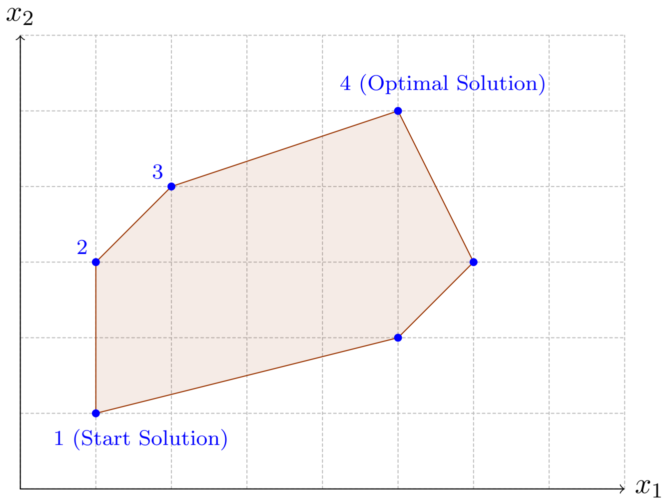

Figure 3.8.: Simplex optimization walking along the vertexes of the feasible region

An optimization using the simplex method involves two phases:
[26, p.45]

- first it is necessary to find any feasible solution
- afterwards the optimal solution can be determined iteratively

If the linear programming problem has one optimal solution it is exactly on one of the vertices of the polyhedron which is formed by the constraints, like mentioned in the section about linear programming. If there is more than one solution, at least one is on one of the vertices. The vertex algorithm is based on iteratively continuing searching on neighboring vertices of the polyhedron, so that the solution is getting better or at least not worse in each step, until the optimal solution is found. An important technique for moving from one vertex to the next is the manipulation of the problem using Gauss-elimination. [26, p.46]

If a problem has k variables and m constraints, with $k > m$, there are more variables than equations, so more than one solution is possible. Instead if $k = m$, exactly one solution exists. [26, p.47]

The principle of the simplex method now is to select m column vectors from this coefficient-matrix, which allow the remaining $k-m$ column vectors to be expressed. The set of m variables is called basic variables, the rest of the variables are called non-basic variables. Moving from one vertex to a neighboring one now conforms to an exchange, whereby exactly one column vector from the selected m column vectors is exchanged with one column vector from the remaining $k-m$ vectors. [26, p.47-49]

For using the simplex method the system is transformed to its standard form. This means that there are just equalities and that there are more columns than rows (more variables than constraints). This transformation can be done by introducing slack variables, which are non-negative variables. For a problem with n variables and m constraints m slack variables have to be used. The slack variables display the absolute difference between the left hand side and the right hand side. [26, p.47-48]

Finding a initial solution is done by setting the not-basic variables to zero. If all values on the right hand side are > 0, it is an feasible solution, otherwise a feasible solution needs to be found by other means. [26, p.49-50]

Example

It is now be explained using an example:
[28, p.147-156][30, p.26-32]

$$max : 4 \cdot x_1 + 3 \cdot x_2 \tag{3.64}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 \leq 6 \tag{3.65}$$
$$3 \cdot x_1 - 2 \cdot x_2 \geq -3 \tag{3.66}$$
$$2 \cdot x_2 \leq 5 \tag{3.67}$$
$$2 \cdot x_1 + x_2 \leq 4 \tag{3.68}$$
$$x_1, x_2 \geq 0 \tag{3.69}$$

By introducing the slack variables the inequalities are converted into equalities:

$$max : 4 \cdot x_1 + 3 \cdot x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 + 0 \cdot s_4 \tag{3.70}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 + s_1 = 6 \tag{3.71}$$
$$-3 \cdot x_1 + 2 \cdot x_2 + s_2 = 3 \tag{3.72}$$
$$2 \cdot x_2 + s_3 = 5 \tag{3.73}$$
$$2 \cdot x_1 + x_2 + s_4 = 4 \tag{3.74}$$
$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0 \tag{3.75}$$

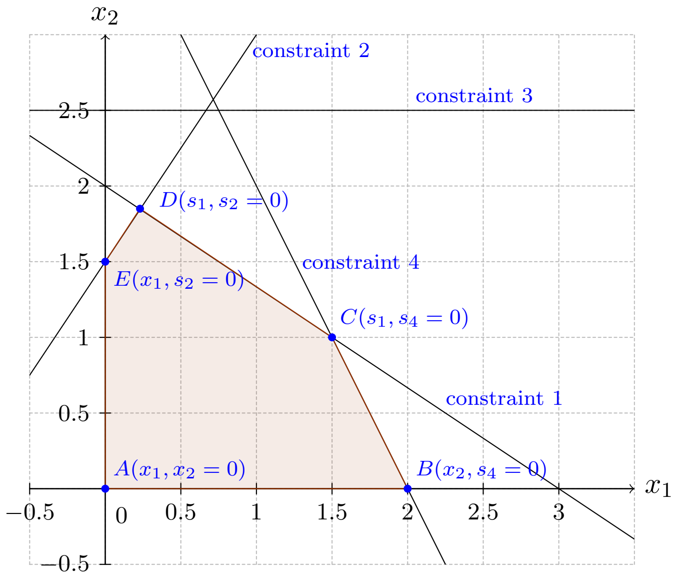

Figure 3.9.: Graphical illustration for the given example

Now the problem has $k = n + m$ variables. For this problem the number of equalities is 4, the total number of of variables including the slack and the surplus variables is 6 and the number of slack variables is 4.

For example for a point being exactly on the line of the first constraint, $2 \cdot x_1 + 3 \cdot x_2 + s_1 = 6$, s_1 has to be zero. As a comparable condition has to be true for all lines representing constraints, and all vertices are located on intersections between such lines, two of the six variables have to be zero for each vertex. The variables which are set to zero are the non-basic variables. This is shown in table 3.12.

vertex	non-basic variables
A	$x_1 = x_2 = 0$
B	$x_2 = s_4 = 0$
C	$s_1 = s_4 = 0$
D	$s_1 = s_2 = 0$
E	$x_1 = s_2 = 0$

Table 3.12.: Vertices with their associated non-basic variables

The problem now can be solved for example by using a simplex tableau:

The objective function is transformed to

$$f(x) = z - 4 \cdot x_1 - 3 \cdot x_2 - 0 \cdot s_1 - 0 \cdot s_2 - 0 \cdot s_3 - 0 \cdot s_4 \tag{3.76}$$

For a feasible basic solution x_1 and x_2 are set to zero. So the starting solution for the basic variables is $s_1 = 6$, $s_2 = 3$, $s_3 = 5$ and $s_4 = 4$.

As a first step in each iteration the so called *pivot-element* has to be identified. This is done using following steps:

z	x_1	x_2	s_1	s_2	s_3	s_4	solution
0	2	3	1	0	0	0	6
0	-3	2	0	1	0	0	3
0	0	2	0	0	1	0	5
0	2	1	0	0	0	1	4
1	-4	-3	0	0	0	0	0

Table 3.13.: Initial simplex tableau for the given example

1. The *pivot-column* is the column in which the coefficient of the objective function has the most negative value for maximization problems or the most positive value for minimization problems.
2. The *pivot-row* is the row with the lowest non-negative quotient, which is formed as

$$\text{quotient} = \frac{\text{solution}}{\text{value of the pivot-column}} \tag{3.77}$$

3. The *pivot-element* is the element at the cross-section of the pivot-column and the pivot-line and is shown in bold.

z	x_1	x_2	s_1	s_2	s_3	s_4	solution	quotient
0	2	3	1	0	0	0	6	$^6/_2$
0	-3	2	0	1	0	0	3	$^3/_{-3}$
0	0	2	0	0	1	0	5	$^5/_0$
0	**2**	1	0	0	0	1	4	$^4/_2$
1	-4	-3	0	0	0	0	0	

Table 3.14.: Simplex tableau with calculated quotients and the pivot-element shown in bold

In this example the pivot-column is the column for x_1, the pivot-row is the fourth row, because the minimum of the not-negative values is $^4/_2$.

Next the pivot row is divided by the pivot element. So the pivot-row now is:

z	x_1	x_2	s_1	s_2	s_3	s_4	solution
0	1	$^1/_2$	0	0	0	$^1/_2$	$^4/_2 = 2$

Table 3.15.: Pivot row after dividing by the pivot element

Now, for each other row, the pivot row is multiplied by a coefficient and added to the row, so that the factor in the pivot column always results to 0. This is called the Gauss-elimination method.

Looking at the objective function itself this would look like:

$$\begin{array}{ll}
\text{original } f(x): & 1 \cdot z - 4 \cdot x_1 - 3 \cdot x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 + 0 \cdot s_4 = 0 \\
\text{plus } 4 \times \text{pivot-row}: & 0 \cdot z + 4 \cdot x_1 + 2 \cdot x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 + 2 \cdot s_4 = 8 \\
\text{new } f(x): & 1 \cdot z + 0 \cdot x_1 - 1 \cdot x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 + 2 \cdot s_4 = 8
\end{array}$$

The same step is done with an multiplicator of -2 for the first row and $+3$ for the second row. The third row does not need to be changed because the coefficient of x_1 is already 0.

This results in the simplex tableau shown in table 3.16 after the first optimization step.

z	x_1	x_2	s_1	s_2	s_3	s_4	solution
0	0	2	1	0	0	-1	2
0	0	$7/2$	0	1	0	$3/2$	9
0	0	2	0	0	1	0	5
0	1	$1/2$	0	0	0	$1/2$	2
1	0	-1	0	0	0	2	8

Table 3.16.: Simplex tableau after the first optimization step

Now x_2 and s_4 are the non-basic variables, because they influence the objective function. The new basic solution is at $x_1 = 2$, $s_1 = 2$, $s_2 = 9$, $s_3 = 5$ and $z = 8$. Looking at the graphical illustration in figure 3.9 it has moved from vertex A to vertex B.

This process has to be repeated as long as the optimal solution is not reached. The optimum is reached when there are no negative coefficients left in the objective function, as no improvement can occur anymore in this case.

In the next step x_2 is the pivot column and the first row is the pivot row, the element at the intersection, shown in bold, is the pivot element.

z	x_1	x_2	s_1	s_2	s_3	s_4	solution	quotient
0	0	**2**	1	0	0	-1	2	$2/2$
0	0	$7/2$	0	1	0	$3/2$	9	$18/7$
0	0	2	0	0	1	0	5	$5/2$
0	2	1	0	0	0	1	4	$4/1$
1	0	-1	0	0	0	2	8	

Table 3.17.: Simplex tableau after the second optimization step

Again the pivot-row is divided by the pivot element, which is 2 in this case. Then the pivot equation is multiplied by $+1$ and added to the objective function. For the second row it is multiplied with $-7/2$, for the third row with $+2$ and for the fourth with -2.

This results in a simplex tableau as shown in table 3.18, which in this case already represents the optimum solution.

z	x_1	x_2	s_1	s_2	s_3	s_4	solution
0	0	2	1	0	0	1	2
0	0	0	$-7/4$	1	0	$1\,3/4$	$1\,1/2$
0	0	0	-1	0	1	1	3
0	2	0	$-1/2$	0	0	$1\,1/2$	3
1	0	0	$1/2$	0	0	$1\,1/2$	9

Table 3.18.: Simplex tableau after reaching the optimum

Now s_1 and s_4 are the non-basic variables and have to be set to zero. The final solution is $x_1 = 1.5$ and $x_2 = 1$ with $f(x) = 9$, which is vertex C in figure 3.9.

3.3.2. Branch and Bound

The branch and bound method is used to solve integer and mix-integer optimization problems. [29, p.266]

The first step when using this method is to find an optimal solution, not restricted to integer conditions, which can be interpreted as the upper-bound for maximization problems or the lower bound for minimization problems. Additionally a feasible integer solution has to be known, which then already represents one solution and can be seen as the lower-bound for maximization problems or the upper-bound for minimization problems. Finding such a feasible integer solution can often be done by setting all variables to zero. [28, p.167]

Afterwords the algorithm consits of two parts: the branching and the bounding.

Branching

In the branching step, the problem is divided into two or more subproblems, so the original problem is simplified. Since this procedure is repeated recursively a tree structure is developed.

This tree is growing dynamically during the search. There are different algorithms for choosing the order in which the nodes of the tree structure should be processed and extended.

The three most used algorithms for this purpose are:

- **Depth first search**
  The process is going fast into the deep of the tree. Always the last entered node is chosen. The advantage is that just a few nodes have to be saved and the sub-problems can be solved fast because they are similar to the preceding problem. The drawback is that also nodes which do not give an optimal solution have to be processed very deep. [33, p.73]

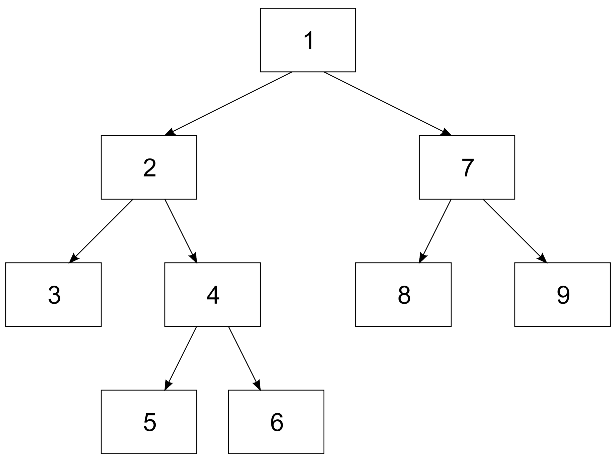

Figure 3.10.: Example for a node processing order when using depth first search [8, p.17]

- **Best first search**
  Among all sub-problems that were not already processed, the one which gives the best result is chosen. The tree is rising breadthwise in this case. As the criterion for the choice is quality-based, good solutions are found very fast. The disadvantage is, that the problems which are solved consecutively typically are not similar. [33, p.73]

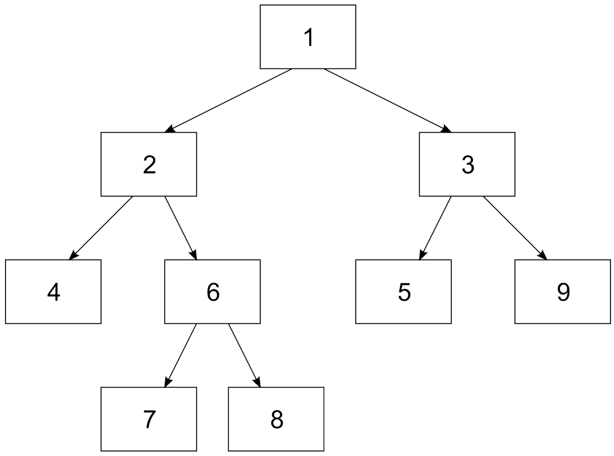

Figure 3.11.: Example for a node processing order when using best first search [8, p.17]

- **Breadth first search**
  All nodes on the same level of the tree are processed before the algorithm is going into the deep. The processing time is usually longer in this case, since the number of nodes is rising exponentially with the depth level. [8, p.18]

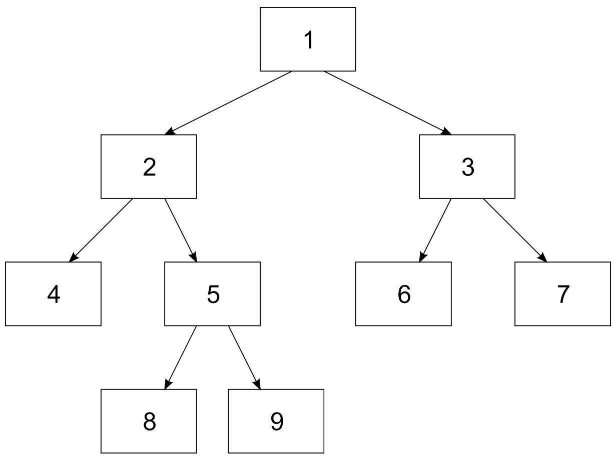

Figure 3.12.: Example for a node processing order when using breadth first search [8, p.17]

Bounding

Each branching step is immediately followed by a bounding step, in which all available solutions are compared, and candidate nodes which do not deliver a better or at least the same solution than the already known optimum are eliminated. This drastically minimizes the necessary effort to find the optimum as the decision tree is reduced. [28, p.175]

If the solution of a sub-problem is a non-integer solution which is worse than the best known integer solution, the node does not need to be branched. If the solution is better, it has to be branched. If the solution is an integer solution, branching the node is not necessary, as it already represents the optimum of the branch. If it is better than the already known best result, it is taken as the new optimum. [28, p.167]

Summarized it has not to be branched if

- either the sub-problem has an infeasible solution
- or the solution is an integer solution, as the branch optimum is reached in this case
- or it is a not integer-solution which is worse than the best already known integer solution

3.3.3. Other optimization techniques

The following optimization techniques are just mentioned, since they are not used in the XA optimization library and also have no direct relation to the thesis.

Interior Point Method

The Interior Point Method dose not analyse the vertices. After selecting a starting point within the feasibly region this point is adjusted during the search, which is continued until the optimum is reached. During the search this point always stays within the interior of feasibly region. [30, p.14]

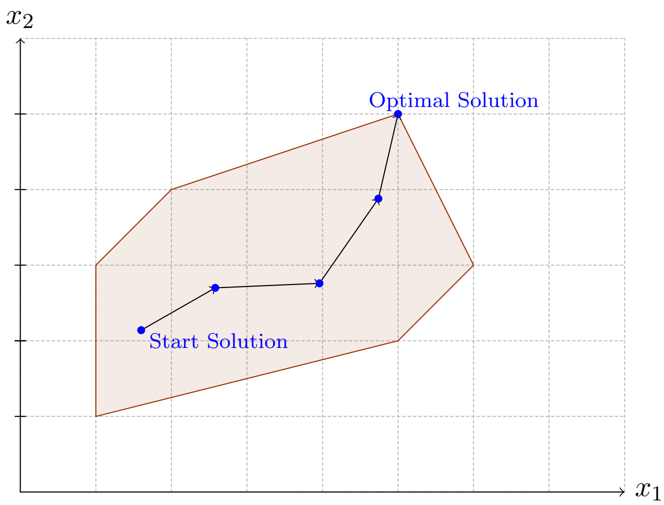

Figure 3.13.: Interior point method

Ellipsoid Method

In this method an ellipse is used which includes the solution space. A straight line is drawn which separates the centre of the ellipse from the solution region. Afterwards a new ellipse is selected which also includes the feasible region and points where the ellipse intersects the straight line. If the center point of the ellipse then is in the solution space, it is the optimum, if not, the process is continued until the optimum is reached. [30, p.15]

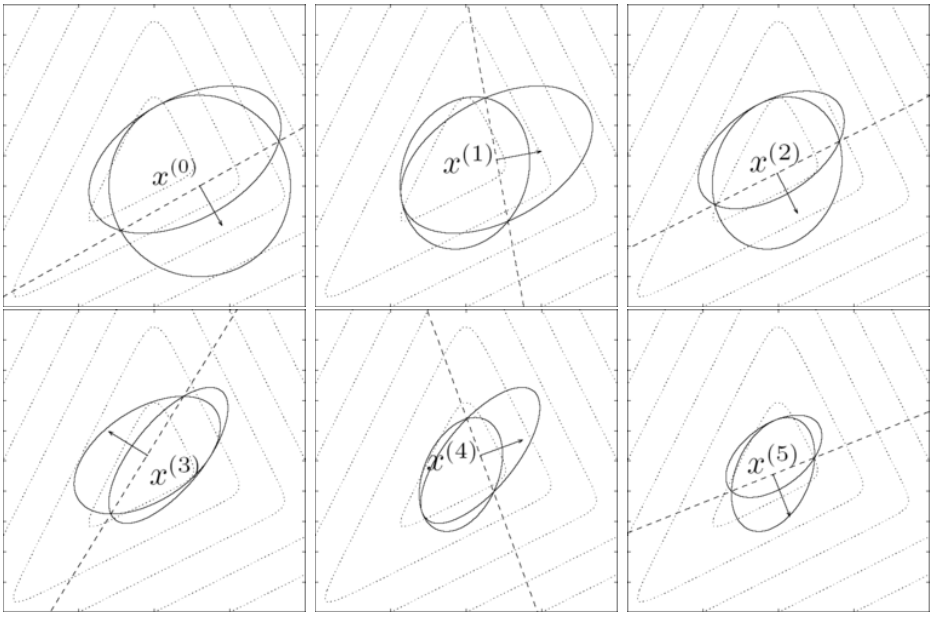

Figure 3.14.: Ellipsoid Method, example iterations

Monte Carlo

The Monte Carlo method is a way to find an extremum of a function with a certain probability by taking random samples. As it is based on statistically analyzing repeatedly taken random samples it never can never find the exact extremum of a function, or at least this is very unlikely, but with a growing number of samples taken the probability of finding a solution near the optimum is rising.

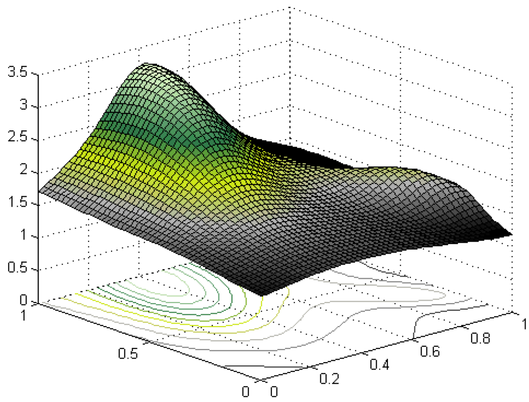

Figure 3.15.: Statistically sampling points of a function the Monte Carlo method can find an optimum with a certain probability [23, p.14]

Steepest descent

A gradient descent is used to solve nonlinear problems by approaching the local minimum or maximum of a function. The algorithm is based on the derivative of the function and a search for the steepest slope. First a starting variable value is selected and the descent of this point is determined. The maximum has to be in the direction of the gradient, so the variable value is changed towards this direction and the descent is determined again. With iterative steps it is moved in the direction of the optimum, where the decent is zero.

An important point for this optimization method is the selection of the step size, as a small step size would lead to a lot of iterations, whereas the possibility of missing the maximum exists for large step sizes [28, p.137]

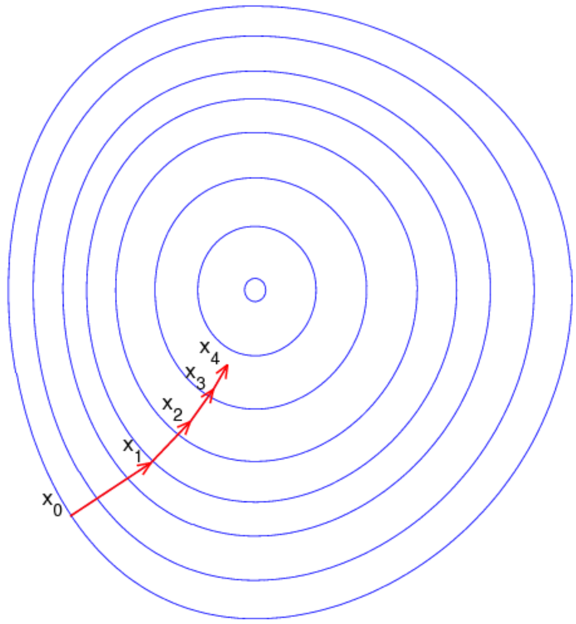

Figure 3.16.: Steepest Descent Method

4. Implementation

4.1. Structure of the System and Description of Linear Implementation

In the hydro thermal scheduling process, the purpose is to find the optimal commitment of thermal and hydro power plants to generate the lowest costs according to the constraint to cover the required load with optimal use of hydro power plants.

The problem can be formulated as a linear problem with linearized constraints.

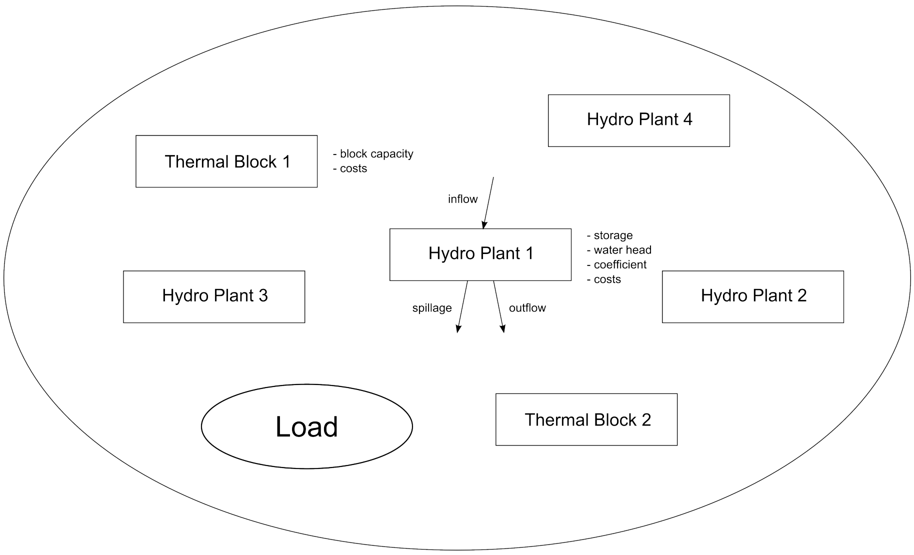

Figure 4.1.: Illustration of a single bubble with thermal and hydro power plants

The thermal power plants are described with the following values:

data	unit
block capacity	MWh
costs	€/MWh

Table 4.1.: Describing data for thermal power plants

For the optimization system the thermal power plants are represented as blocks, where each block represents the maximum generation of all power plants in one region for certain costs. If the thermal power plants are producing more energy, the costs also increase.

The hydro power plants are described with the following values:

Depending on the type of a hydro power plant (storage power plant or run-of-river power plant), the values which are used to describe the plant are different. For run-of-river power plants the storage is set to zero, because there is no storage at all.

data	unit
inflow	m^3/s
storage	m^3
spillage	m^3/s
outflow	m^3/s
water head	m
coefficient	N/m^3
costs	€/MWh

Table 4.2.: Describing data for hydroelectric power plants

The maximum generation of a hydro power plant depends not only on the amount of outflow, but also on the water head and the efficiency of the turbine.

The output of a hydro power plant is defined by the formula:

$$\text{hydro power} = \text{outflow} \times \text{water head} \times \text{coefficient} \tag{4.1}$$

So the generation depends on several factors. The water head can be increased by using long pipes to make the height difference between the source and the water outflow as large as possible. The coefficient represents the gravitational acceleration, the water density and a turbine factor.

The inflow is the natural inflow in the system during the considered period. This can be rain, water from upstream rivers or melt water from the mountains. The inflow can be quite different over the years.

Different trading zones or control areas are modeled as so called bubbles. The system operator should cover the stage load in the system by using the power plants which are contained in the considered zone, or by importing energy from other regions. Energy produced, which exceeds the load in the region, is exported to other trading zones.

The trading zones or control areas are described with the following values:

data	unit
stage loads	MWh
flow between bubbles	MWh
wheeling costs	€/MWh

Table 4.3.: Describing data for the different bubbles

The objective function represents the costs of the system, where the objective is to minimize the costs for all power plants in the system over the total time horizon:

$$\begin{aligned} min : \sum_{t=1}^{T} \Big(\sum_{i=1}^{m} (\text{thermal costs}_i \times \text{thermal generation}_{i,t}) + \\ + \sum_{j=1}^{n} (\text{hydro costs}_j \times \text{hydro generation}_{j,t}) \Big) \end{aligned} \tag{4.2}$$

In this formula $i = 1..m$ stands for the number of thermal power plants, $j = 1..n$ for the number of hydro power plants and $t = 1..T$ for the number of stages over the whole time horizon.

Going more into detail results into:

$$min : \sum_{t=1}^{T} \Big(\sum_{i=1}^{m} (\text{thermal costs}_i \times \text{thermal generation}_{i,t}) + \\ + \sum_{j=1}^{n} (\text{hydro costs}_j \times \text{outflow}_{j,t} \times \text{water head}_{j,t} \times \text{coefficient}_{j,t} \times \Delta t) \Big) \tag{4.3}$$

subject to the lower-bound constraints:

$$\text{thermal generation}_{i,t} \geq 0 \tag{4.4}$$
$$\text{hydro generation}_{j,t} \geq 0 \tag{4.5}$$
$$\text{hydro spillage}_{j,t} \geq \text{minimum spillage}_{j,t} \tag{4.6}$$
$$\text{hydro storage}_{j,t} \geq \text{minimum storage}_{j,t} \tag{4.7}$$

subject to the upper-bound constraints:

$$\text{thermal generation}_{i,t} \leq \text{maximum thermal generation}_{i,t} \tag{4.8}$$
$$\text{hydro generation}_{j,t} \leq \text{maximum hydro generation}_{j,t} \tag{4.9}$$
$$\text{hydro spillage}_{j,t} \leq \text{maximum spillage}_{j,t} \tag{4.10}$$
$$\text{hydro storage}_{j,t} \leq \text{maximum storage}_{j,t} \tag{4.11}$$

subject to the load constraints:

$$\sum_{i=1}^{m} \text{thermal generation}_{i,t} + \sum_{j=1}^{n} \text{hydro generation}_{j,t} = \text{load}_t \tag{4.12}$$

subject to the water continuity constraints:

$$\text{outflow volume}_{j,t} + \text{spillage volume}_{j,t} + \text{storage}_{j,t} = \text{inflow volume}_{j,t} + \text{storage}_{j,(t-1)} \tag{4.13}$$

All the variables which are used in the formulation have upper and lower bounds. The upper and lower bounds represent the physical bandwidth in which the values can be naturally set.

The lower bound of the electrical output for thermal and hydro power plants is zero, because it would not make sense to have negative electrical output, which would mean electrical energy being consumed by the power plant. The power plant is producing the needed electrical energy by itself. Only in turn-on time periods the plant is receiving energy from the grid.

If the power plants have to operate with at least a certain minimum capacity, it is a user input. This can for example be a demand resulting from long turn-on and turn-off times of thermal power plants, which are given because of the thermal components and the thermodynamic demands. A very fast temperature difference can create cracks and breakages in the turbine wheels, so temperature changes can only be done slowly. Therefore the power plants might have to run with at least a minimum output to keep it prepared for faster power boosts.

Run-of-river power plants usually are used to cover the base load in the power system. The flow rate of the river should be used, and the power plant does not store the water. So the water can only be used in the moment when it is passing the power plant. Also for the run-of-river power plants there might be a minimum load set.

In comparison, storage power plants can be turned on and off easily, so they are used to cover the peak load. The water can be stored and used every time when it is needed, especially for the peak load periods.

The minimum spillage is a hydro power plant specification. The obligatory spillage can depend on several reasons. Thus, the system operator can force a minimum amount for the course of the river or the use of downstream power plants. Mostly the generator tries to use the whole amount of water in the storage for producing electrical energy. Reasons for natural spillage can be seepage.

Also the minimum storage can be forced by several reasons. For example if a natural reservoir contains some stored water, because of environmental concerns it may not be allowed to completely empty it. A purpose is also to store the water for periods when more energy is needed and the price for the energy is higher. The price for energy to cover the peak load is much higher than the price for the base load.

The upper bounds are also user-specified and express the physical upper bounds for the maximum electrical capacity which can be produced. Power plants are constructed for a maximum output. The generator is dimensioned for a maximum rated output, for short periods it can be used over this capacity, but this stresses the generator components. The maximum storage is the storage capacity of the reservoir. There can also be reasons for a maximum spillage, for example caused by the construction of the dam.

The equation for the load in the whole system at stage t can be formulated as:

$$\sum_{i=1}^{m} \text{thermal generation}_{i,t} + \sum_{j=1}^{n} (\text{outflow}_{j,t} \cdot \text{water head}_{j,t} \cdot \text{coefficient}_{j,t} \cdot \Delta t) = \text{load}_t \tag{4.14}$$

The total load in the system has to be covered with the production of electrical generation using the contained power plants.

At stage t the water continuity for each power plant can be formulated as:

$$\text{outflow}_{j,t} \cdot \Delta t + \text{spillage}_{j,t} \cdot \Delta t + \text{storage}_{j,t} = \text{inflow}_{j,t} \cdot \Delta t + \text{storage}_{j,(t-1)} \tag{4.15}$$

In the water continuity equations inflow, stored and used water have to be balanced. The storage at the end of a period plus the spillage and the outflow must be equal to the storage at the beginning of the stage plus the natural inflow and the water from the upstream plants.

The transfer between the trading zones or control areas (bubbles) is restricted by the maximum capacity of the transmissions lines available in the grid. This maximum capacity is affected by the voltage level, the material and the diameter of the lines. The maximum capacity can also be limited by a rental agreement between the energy company and the network operator or market restrictions.

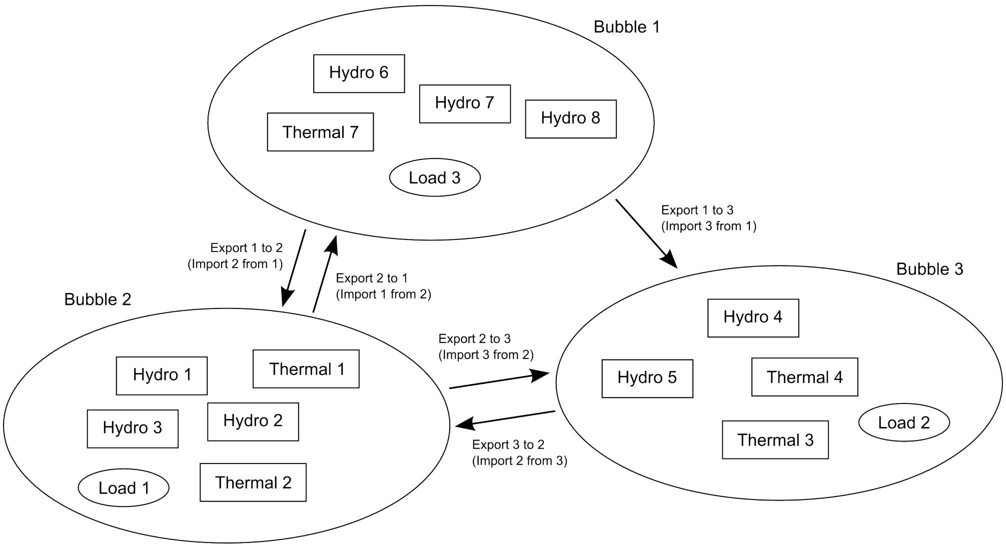

Figure 4.2.: Illustration of the interaction between the single bubbles

Considering the costs for energy exports and imports of each bubble the objective function has to be extended:

$$
\begin{aligned}
min: \sum_{t=1}^{T} \Big(&\sum_{i=1}^{m} (\text{thermal costs}_i \times \text{thermal generation}_{i,t}) + \\
&+ \sum_{j=1}^{n} (\text{hydro costs}_j \times \text{hydro generation}_{j,t}) + \\
&+ \sum_{k=1}^{b} (\text{wheeling costs}_k \times \text{export}_{k,t}) + \\
&+ \sum_{k=1}^{b} (\text{wheeling costs}_k \times \text{import}_{k,t}) \Big)
\end{aligned} \tag{4.16}
$$

with $k = 1..b$ representing all possible interconnections which can be used for energy transfers,

subject to the lower-bound constraints:

$$\text{thermal generation}_{i,t} \geq 0 \tag{4.17}$$
$$\text{hydro generation}_{j,t} \geq 0 \tag{4.18}$$
$$\text{hydro spillage}_{j,t} \geq \text{minimum spillage}_{j,t} \tag{4.19}$$
$$\text{hydro storage}_{j,t} \geq \text{minimum storage}_{j,t} \tag{4.20}$$
$$\text{export}_{k,t} \geq 0 \tag{4.21}$$
$$\text{import}_{k,t} \geq 0 \tag{4.22}$$

subject to the upper-bound constraints:

$$\text{thermal generation}_{i,t} \leq \text{maximum thermal generation}_{i,t} \tag{4.23}$$
$$\text{hydro generation}_{j,t} \leq \text{maximum hydro generation}_{j,t} \tag{4.24}$$
$$\text{hydro spillage}_{j,t} \leq \text{maximum spillage}_{j,t} \tag{4.25}$$
$$\text{hydro storage}_{j,t} \leq \text{maximum storage}_{j,t} \tag{4.26}$$
$$\text{export}_{k,t} \leq \text{maximum wheeling capacity}_{k,t} \tag{4.27}$$
$$\text{import}_{k,t} \leq \text{maximum wheeling capacity}_{k,t} \tag{4.28}$$

subject to the load constraints:

$$\begin{aligned} \sum_{i=m_{bs}}^{m_{be}} \text{thermal generation}_{i,t} + \sum_{j=n_{bs}}^{n_{be}} \text{hydro generation}_{j,t} + \sum_{k=b_{bs}}^{b_{be}} \text{import}_{l,t} = \\ = \text{bubble load}_t + \sum_{k=b_{bs}}^{b_{be}} \text{export}_{l,t} \end{aligned} \tag{4.29}$$

with $i = m_{bs}..m_{be}$ enumerating the thermal power plants in the bubble, $j = n_{bs}..n_{be}$ enumerating the hydro plants and $k = b_{bs}..b_{be}$ the zone interconnections influencing the current bubble, subject to the water continuity constraints, which did not change:

$$\text{outflow volume}_{j,t} + \text{spillage volume}_{j,t} + \text{storage}_{j,t} = \text{inflow volume}_{j,t} + \text{storage}_{j,(t-1)} \tag{4.30}$$

In many systems along rivers or other natural connections, more than one hydro power plant exist on the same watercourse, so they are connected in cascades and influence each other. The relationship between the outflow and the spillage of an upstream power plant, representing an additional inflow to a downstream power plant is described with the so called upstream relation and is included in the system.

In long term scheduling the time horizon of one year is divided in monthly or weekly periods. During this periods, most of the water has already run down the considered system, so the outflow of an upstream hydro power plant can be added to the natural inflow of the downstream power plant in the same period.

Additional factors are used for modelling different situations:

- the outflow water of several power plants is added to one downstream plant
- the outflow of one power plant is split over several power plants
- if some water is blocked on the way down.

The water of several power plants is added together may for example occur if two rivers merge together. The water of one power plant can be split because the river separates into a main stream and an anabranch, blocking may for example occur due to a dam.

This leads to an extended formulation of the water continuity constraint:

$$\text{outflow volume}_{j,t} + \text{spillage volume}_{j,t} + \text{storage}_{j,t} = \tag{4.31}$$

$$= \text{natural inflow volume}_{j,t} + \text{storage}_{j,(t-1)} + \sum_{h=1}^{p} (\text{factor}_h \cdot (\text{outflow volume}_{h,t} + \text{spillage volume}_{h,t}))$$

with $h = 1..p$ representing the upstream power plants influencing the considered hydro power plant.

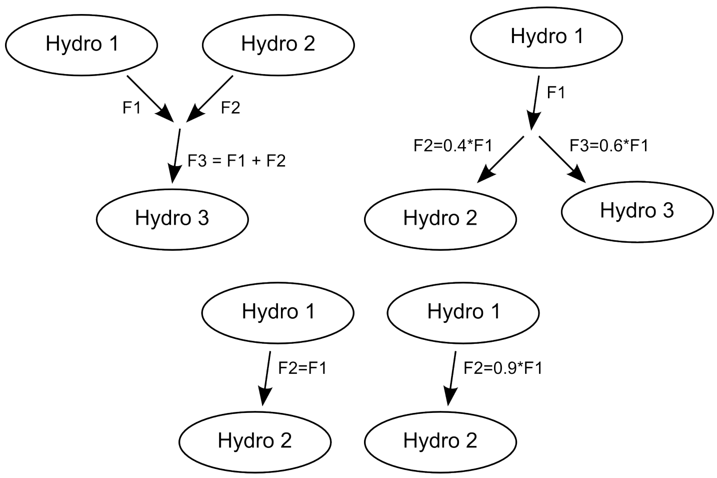

Figure 4.3.: Illustration of different upstream relations for hydro power plants

For the optimization system not only the constraints for the upper- and lower-bounds, the load and the water continuity, but also the upstream relations, the imports and exports between the operator zones, and the influence of the non-constant electrical output of the hydro power plants, depending on the variating head, have to be taken into account.

Sometimes the equations are difficult to solve to solve and it can require a lot of iterations. In certain cases it is not even possible to find a solution as it is not feasible for all the constraints. To avoid this problem, soft constraints are used in the system. That means, that additional upper unbounded variables are added to the constraints.

To avoid solving the problem by only using this unbounded variables (referred to as epsilons), high costs are set for these variables. The solving algorithm tries to solve the problem with using the epsilons only in cases where the constraints are infeasible. In some cases the algorithm needs the epsilons at the beginning, to find an initial solution for the problem. During the optimization the algorithm tries to reduce the values for these variables as much as possible and to work only with the already described variables.

The objective function is extended to:

$$
\begin{aligned}
min: \sum_{t=1}^{T} \Big(\sum_{i=1}^{m} & (\text{thermal costs}_i \times \text{thermal generation}_{i,t}) + \\
& + \sum_{j=1}^{n} (\text{hydro costs}_j \times \text{hydro generation}_{j,t}) + \\
& + \sum_{k=1}^{b} (\text{wheeling costs}_k \times \text{export}_{k,t}) + \\
& + \sum_{k=1}^{b} (\text{wheeling costs}_k \times \text{import}_{k,t}) \Big) \\
& + \sum_{e=1}^{f} (\text{penalty costs}_e \times \epsilon_{e,t})
\end{aligned}
\tag{4.32}
$$

with $e = 1..f$ representing the number of epsilons used in the equations for the load and the water continuity.

With respect to the epsilons the constraints for load and water continuity are extended to:

$$
\sum_{i=1}^{m} \text{thermal generation}_{i,t} + \sum_{j=1}^{n} \text{hydro generation}_{j,t} + \varepsilon_1 - \varepsilon_2 = \text{load}_t \tag{4.33}
$$

$$
\begin{aligned}
\text{outflow volume}_{j,t} + \text{spillage volume}_{j,t} + \text{storage}_{j,t} + \varepsilon_3 - \varepsilon_4 = \\
= \text{inflow volume}_{j,t} + \text{storage}_{j,(t-1)}
\end{aligned}
\tag{4.34}
$$

The epsilons have to be greater than or equal to 0, but they do not have upper bounds, so they can resolve all the infeasibilities in the system.

4.2. Decomposition Approach

As optimizing the scheduling of thermal und hydro power plants is very complex, the following method, called decomposition approach, can be used to reduce this complexity by seperating the whole optimization problem into smaller subproblems, thus making it easier and faster to solve. [11] [12]

The objective of the whole optimization is to minimize the costs over all stages. This problem becomes complex, because the decisions in each stage influence all the following stages and the amount of water available over all stages is limited. So if the storages are empty, the load has to be covered with expensive thermal energy or it may even come to problems to cover the peak load, because not all thermal power plants can follow the load peaks fast enough. On the other hand if too much water is stored and the inflow is increasing, the spillage has to be used without producing energy.

Because a reliable forecast for the natural inflow, the load curve, the fuel price, possible blackouts and problems in the grid, as well as the price for electrical energy on the market is not possible, the problem is a stochastic problem.

Depending on the solver used and the computer configuration available it may happen that it takes a long time to solve the problem or even that the size of the problem is too large to be solved, because there are too many variables.

For example a big system with several hundred reservoirs and scenarios can result in a linear programming problem with several million variables and a number of constraints in the same magnitude.

One part of the implementation of the system is based on Stochastic Dual Dynamic Programming (SDDP) as described by Pereira and Pinto. The related description is an excerpt of their papers. [11] [12]

The approach is based on the approximation of the future costs by piecewise linear functions. This approximated functions are obtained from the solutions of the dual form of the problem as described later.

4.2.1. Deterministic Dual Dynamic Programming

When using Deterministic Dual Dynamic Programming (DDDP) the problem is decomposed into multiple single stage sub-problems. Since each of them is smaller than the original problem, it is much faster to solve.

This algorithm can be interpreted as a Benders decomposition. The information between the different stages is given by the Benders cuts. The accepted future cost curve is approximated by a piecewise linear function from the Benders cuts.

A two stage deterministic hydrothermal optimization problem can be formulated as:

$$min : c_1 \cdot x_1 + c_2 \cdot x_2 \tag{4.35}$$

where x_1 already represents all variables for thermal and hydro generation as in the objective function shown in equation 4.3 for stage one, x_2 all variables for stage two respectively, subject to all constraints, including load balance and water continuity:
(see also equations 4.4 to 4.13)

$$A_1 \cdot x_1 \geq b_1 \tag{4.36}$$
$$E_1 \cdot x_1 + A_2 \cdot x_2 \geq b_2 \tag{4.37}$$

with c as the cost vector, x as the variable vector and b are the known values of the bounds, the load and the inflow. A is the matrix for the variable combination of the current stage, E is the matrix for the variable combination of the previous stage (i.e. for the second stage it is the end-storage of the first stage).

The problem can be interpreted as a two-stage decision process. In this case a stage is a time segment. In long term scheduling the time horizon is one year, and separated in monthly or weekly periods, which means 12 or 52 stages.

If a trial value $\hat{x}_1$ of the first stage solution (i.e. such that $A_1 \cdot x_1 > b_1$) is found, the solution for the second stage function can be calculated with:

$$\begin{aligned} min &: c_2 \cdot x_2 \\ s.t. &: A_2 \cdot x_2 \geq b_2 - E_1 \cdot \hat{x}_1 \end{aligned} \tag{4.38}$$

where $\hat{x}_1$ is a known value.

Since the value $\hat{x}_1$ is a known value it is moving to the right hand side. The objective is the minimization of the sum of costs from the first and second stage.

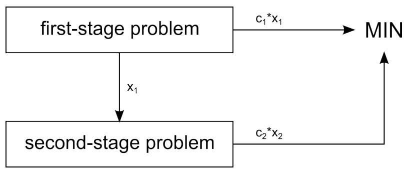

Figure 4.4.: Illustration of the two-stage decision process [11, p.360]

Using decomposition the problem is separated in two sub-problems. The fist stage problem can be formulated as:

$$\begin{aligned} min &: c_1 \cdot x_1 + \alpha_1(x_1) \\ s.t. &: A_1 \cdot x_1 \geq b_1 \end{aligned} \tag{4.39}$$

where $c_1 \cdot x_1$ are the immediate costs and $\alpha_1(x_1)$ are the future costs which depend on decision of x_1.

To keep the notation consistent b_1 can also be written as

$$b_1 = b'_1 - E_0 \cdot x_0 \tag{4.40}$$

with x_0 being the initial state.

Using the future cost function the costs of the second stage are described as a function of the first stage decision x_1 (state variables):

$$\begin{aligned} \alpha_1(x_1) = min : c_2 \cdot x_2 \\ s.t. : A_2 \cdot x_2 \geq b_2 - E_1 \cdot x_1 \end{aligned} \tag{4.41}$$

If the future costs are know, the two stage problem can be solved as a one stage problem.

With the decomposition algorithm the future cost function is constructed by discretization of x_1 into a set of trial values and solving the problem stated in equation 4.41 for each value. After the construction of the future cost function the first-stage problem can be solved. The first stage decision is still dependent on the second stage variables.

The advantage of the decomposition approach is that it can be easily extended to a multi-stage problem or to a stochastic case. The main drawback is the necessary discretization of the state variables to construct the future cost function. So a very large number of combinations are needed for just a few variables. For example for ten components of x, discretized into four values, x_1 has $4^{10} = 1048574$ possible values.

To avoid this problem the future cost function can be approximated by analytical functions. For this the future costs $\alpha_1(x_1)$ are calculated for a few states and the solutions are approximated with a polynomial function. This polynomial function can be used in the first stage to approximate the future costs.

In the paper of Pereira and Pinto a way to solve this is shown, whereby the future cost function is exactly represented by a piecewise linear function. [11] [12]

The dual problem of the future cost function is

$$\begin{aligned} \alpha_1(x_1) = max : \lambda \cdot (b_2 - E_1 \cdot x_1) \\ s.t. : \lambda \cdot A_2 \leq c_2 \end{aligned} \tag{4.42}$$

with λ being the vector of the dual variables. As illustrated in section 3.2.1 the solutions of the objective function of the primal and the dual problem are the same.

The variable x_1 is now contained in the objective function instead of still being on the right hand side of the constraints. So the possible solutions are independent of decision x_1. The constraints of the dual problem form a polyhedron and the possible solutions correspond to the vertices of the polyhedron. $\mathit{\Lambda} = \lambda^1 ... \lambda^v$ is the set of all vertices of the constraints.

The problem can then be rewritten as a linear programming problem:

$$\alpha_1(x_1) = min : \alpha \tag{4.43}$$

subject to:

$$\begin{aligned} &\alpha \geq \lambda^1(b_2 - E_1 \cdot x_1) \\ &... \\ &\alpha \geq \lambda^v(b_2 - E_1 \cdot x_1) \end{aligned} \tag{4.44}$$

where $\alpha_1(x_1)$ is a piecewise linear function of the decision variable x_1 and α is a scalar variable.

It can be complicated to calculate all vertices, another approach by approximating the future cost function based on calculating a subset of the vertices is used. The vertices can be calculated using the following equation:

$$\begin{aligned} \alpha_1(\hat{x}_{1i}) &= min : c_2 \cdot x_2 \\ s.t. : A_2 \cdot x_2 &\geq b_2 - E_1 \cdot \hat{x}_{1i} \\ dual : var\lambda^i & \end{aligned} \tag{4.45}$$

With $\hat{x}_{1i}$ as a trial value and λ^i as the corresponding simplex multiplier vector of the dual problem for each trial i, this vector represent one vertex of all possible solutions of the set of Λ. This vertex can be used to construct a supporting hyperplane of the future cost function.

The approximated future cost function is:

$$\begin{aligned} \hat{\alpha}_1(x_1) &= min : \alpha \\ s.t. : \alpha &\geq \lambda_i(b_2 - E_1 \cdot x_1), i = 1..n \end{aligned} \tag{4.46}$$

This function $\hat{\alpha}_1(x_1)$ is the lower bound of the future cost function $\alpha(x_1)$, because in the problem given by equation 4.46 only some restrictions are included. With help of the approximated future cost function the first stage problem can be solved.

$$\begin{aligned} z &= min : c_1 \cdot x_1 + \hat{\alpha}_1(x_1) \\ s.t. : A_1 \cdot x_1 &\geq b_1 \end{aligned} \tag{4.47}$$

This is an LP problem and can be rewritten as:

$$\begin{aligned} & z = min : c_1 \cdot x_1 + \alpha \\ \textit{subject to:} & \\ & A_1 \cdot x_1 \geq b_1 \\ & \alpha - \lambda^i(b_2 - E_1 \cdot x_1) \geq 0, i = 1..n \end{aligned} \tag{4.48}$$

It can not be guaranteed that the solution is an optimal solution, because $\alpha_1(x_1)$ is an approximated function. Therefore the lower bound $\underline{z}$ of the problem is the sum of the immediate costs and the approximated future costs:

$$\underline{z} = c_1 \cdot \hat{x}_1 + \hat{\alpha} \tag{4.49}$$

The upper bound $\overline{z}$ can be described as the total cost over all stages, which can be calculated by solving the second stage problem for $\hat{x}_1$:

$$\overline{z} = c_1 \cdot \hat{x}_1 + \hat{\alpha}(\hat{x}_1) \tag{4.50}$$

With an increasing number of iterations the future cost function is described in a more detailed way.

Because of the approximation of the future cost function, it can take a lot of iterations before the optimal solution can be found. Thus, a convergence criterion is implemented.

Convergence criterion:

$$\overline{z} - \underline{z} < \text{tolerance} \tag{4.51}$$

The convergence criterion is the difference between the upper and the lower bound, it measures the difference between the expected future costs and the actual future costs of the trial solution. It can be used to verify the accuracy of the approximated future costs. The tolerance is user-defined and has to be a compromise between accuracy and computation time.

The lower bound is rising with the number of iterations, because the future cost function is getting better approximated. The upper bound declines with the number of iterations, because the solution is converging to the optimal solution.

Summarized the process can be described as:
[11, p.364]

- Step 1 - Initialization:
  set the approximate future cost function $\hat{\alpha}(x_1)$ to zero, the upper bound $\overline{z}$ to ∞ and the number of vertices to zero
- Step 2:
  solve the approximate first stage problem (equation 4.47), use $\hat{x}_1$ as the current optimal solution
- Step 3:
  calculate the lower bound $\underline{z}$ (equation 4.49) and stop if the convergence criterion is reached ($\overline{z} - \underline{z} \leq$ tolerance)
- Step 4:
  solve the second stage problem (equation 4.45): calculate $\alpha(\hat{x}_1)$ and update $\overline{z}$ (equation 4.50)

- Step 5:
  increment the number of vertices n by one, let the multiplier associated to the optimal solution of step 4 be λ^n and construct the approximate cost function $\hat{\alpha}_1(x_1)$ using the n vertices (equation 4.46)

- continue with step 2

This algorithm is equivalent to a Benders decomposition algorithm. Also the multi-stage and stochastic cases can be interpreted as a Benders decomposition scheme.

4.2.2. Multi-Stage Dual Dynamic Programming

If the DDDP algorithm should be used for a longer planing horizon, it can be extended to a multi-stage problem:

The steps for doing this are:
[11, p.365]

- Step 1 - Initialization:
  set the approximate future cost function $\hat{\alpha}_t(x_t)$ to zero and the upper bound $\overline{z}$ to ∞, where $t = 1..T$ represents the single stages of the planning horizon

- Step 2:
  solve the approximate first stage problem (equation 4.47), use $\hat{x}_1$ as the current optimal solution

- Step 3:
  calculate the lower bound $\underline{z}$ (equation 4.49) and stop if the convergence criterion is reached ($\overline{z} - \underline{z} \leq$ tolerance)

- Step 4 - Forward Simulation:
  iteratively solve the optimization problem for all further stages $t = 2..T$ based on the trial decision $\hat{x}_{t-1}$ and store the optimal solutions as $\hat{x}_t$:

$$\begin{aligned} min : & \; c_t \cdot x_t + \hat{\alpha}_t(x_t) \\ s.t. : & \; A_t \cdot x_t \geq b_t - E_{t-1} \cdot \hat{x}_{t-1} \end{aligned} \tag{4.52}$$

- Step 5:
  calculate the upper bound:

$$\overline{z} = \sum_{t=1}^{T} c_t \cdot \hat{x}_t \tag{4.53}$$

- Step 6 - Backward Recursion:
  iteratively backwards solve the optimization problem for all stages $t = T, T-1, ..2$ based on the trial decision $\hat{x}_{t-1}$:

$$\begin{aligned} min : & \; c_t \cdot x_t + \hat{\alpha}_t(x_t) \\ s.t. : & \; A_t \cdot x_t \geq b_t - E_{t-1} \cdot \hat{x}_{t-1} \end{aligned} \tag{4.54}$$

Construct an additional supporting hyperplane for the approximate future cost function in the previous stage $\hat{\alpha}_{t-1}(x_{t-1})$ using λ_{t-1} as the multiplier associated to the constraints of the problem given in equation 4.54 at the optimal solution.

- continue with step 2

4.2.3. Stochastic Dual Dynamic Programming

In stochastic programming the different scenarios for wet and dry years, for different load situations, or for cold or warm years are taken into account. The algorithm mentioned before can be used and extended for this purpose.

The main drawback of the original linear formulation is that the size of the problem grows exponentially with the number of stages and states. One solution to avoid the dimensionality is to decompose the problem into multiple stages and to discretize the scenarios into multiple states.

The following is a two-stage stochastic problem formulation:

$$min : c_1 \cdot x_1 + \rho_1 \cdot c_2 \cdot x_{21} + \rho_2 \cdot c_2 \cdot x_{22} + ... + \rho_m \cdot c_2 \cdot x_{2m} \tag{4.55}$$

subject to:

$$A_1 \cdot x_1 \geq b_1 \tag{4.56}$$
$$E_1 \cdot x_1 + A_2 \cdot x_{21} \geq b_{21} \tag{4.57}$$
$$E_1 \cdot x_1 + A_2 \cdot x_{22} \geq b_{22} \tag{4.58}$$
$$... \tag{4.59}$$
$$E_1 \cdot x_1 + A_2 \cdot x_{2m} \geq b_{2m} \tag{4.60}$$

The probability of an associated scenario is represented by ρ, where the sum of all probabilities has to be one.

A more detailed two stage problem with two scenarios at the second stage looks like:

$$min : c_1 \cdot x_1 + \rho_1 \cdot c_2 \cdot x_{21} + \rho_2 \cdot c_2 \cdot x_{22} \tag{4.61}$$

subject to:

$$A_1 \cdot x_1 \geq b_1 \tag{4.62}$$
$$E_1 \cdot x_1 + A_2 \cdot x_{21} \geq b_{21} \tag{4.63}$$
$$E_1 \cdot x_1 + A_2 \cdot x_{22} \geq b_{22} \tag{4.64}$$

In the first stage a decision for x_1 is made. Depending on this decision there are m sub-problems for the second stage to solve. The future costs for each scenario can be formulated as it was done for the deterministic case:

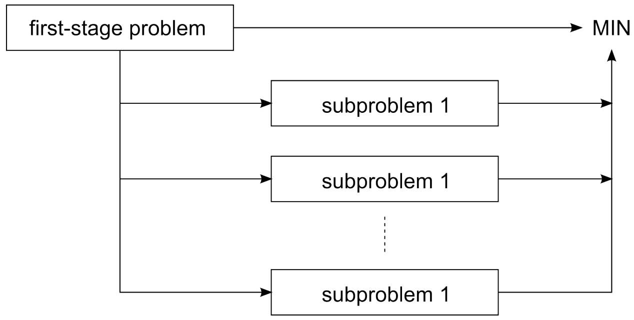

Figure 4.5.: Two-stage decision process for a stochastic case [11, p.366]

$$
\begin{aligned}
\alpha_{1j}(x_1) = min : c_2 \cdot x_{2j} \\
s.t. : A_2 \cdot x_{2j} \leq b_{2j} - E_1 \cdot x_1
\end{aligned} \tag{4.65}
$$

The objective is to minimize the sum of the costs of the first and the second stage, where the second stage costs are:

$$\sum_{j=1}^{m} \rho_j \cdot c_2 \cdot x_{2j} \tag{4.66}$$

The expected future cost function can be represented as the weighted average of the benders cuts:

$$\overline{\alpha}(x_1) = \sum_{j=1}^{m} \rho_j \cdot \alpha_{1j}(x_1) \tag{4.67}$$

The first stage problem is the same as in deterministic dual dynamic programming and can be formulated as following:

$$
\begin{aligned}
min : c_1 \cdot x_1 + \overline{\alpha}_1(x_1) \\
s.t. : A_1 \cdot x_1 \geq b_1
\end{aligned} \tag{4.68}
$$

The future cost function is again a piecewise linear function. It is assumed that the variables of the right hand side are independent random variables, whereby b_t is discretized into m values. This values represent different scenarios with different probabilities. Independent means that the vector b_t does not depend on other realizations in the previous stages.

The algorithm is implemented as following:
[11, p.367]

- Step 1 - Trial Decision Definition

  - define a set $\{\hat{x}_{ti}\}$ of trial decisions, where $i = 1..n$ enumerates the decisions and $t = 1..T$ enumerates the stages

- Step 2 - Backward Recursion:
  - backwards process the stages $t = T, T-1, ..., 2$
  - within each stage process each trial decision $\hat{x}_{t,i}$ (for $i = 1..n$)
  - within each trial decision process each scenario $b_{t,j}$ (for $j = 1..m$)
  - within each scenario solve the optimization problem based on t, $\hat{x}_{(t-1),i}$ and $b_{t,j}$

$$\begin{aligned} min: c_t \cdot x_t + \overline{\alpha}_t(x_t) \\ s.t.: A_t \cdot x_t \geq b_{t,j} - E_{t-1} \cdot \hat{x}_{(t-1),i} \end{aligned} \tag{4.69}$$

    and construct one supporting hyperplane for the approximate future cost function for stage $t-1, \hat{\alpha}_{t-1}(x_{t-1})$ using $\lambda_{(t-1),i,j}$ as the multiplier associated to the constraints of the problem given in equation 4.69 at the optimal solution.

- Step 3:
  - continue with step 1

The final question is: "how to find the trial solutions". In an ideal case a forward simulation is done for all combinations of scenarios. The main drawback of this method is that the number of combinations grows exponentially with the number of stages and states. Using a Monte-Carlo forward simulation for just some of the scenarios can help in this case:
[11, p.368]

- Step 1:
  - solve the first stage problem (equation 4.68), use $\hat{x}_1$ as the optimal solution and initialize $x_{1i} = \hat{x}_1$ for $i = 1..n$
- Step 2:
  - repeat for $t = 2..T$
  - repeat for $i = 1..n$
  - sample a vector $b_{t,i}$ from the set $\{b_{t,j}, j = 1..m\}$, solve the optimization problem for stage t and sample i

$$\begin{aligned} min: c_t \cdot x_t + \overline{\alpha}_t(x_t) \\ s.t.: A_t \cdot x_t \geq b_{t,i} - Et - 1 \cdot \hat{x}_{(t-1),i} \end{aligned} \tag{4.70}$$

    and store the optimal solution as $\hat{x}_{t,i}$

The aim is to find good trial decisions, around which the future cost function is approximated.

As in the deterministic case the lower bound is calculated based on the solution of the first stage problem. The upper bound is an estimation resulting from the Monte Carlo simulation results:

$$\overline{z} = c_1 \cdot x_1 + \frac{1}{n}\sum_{i=1}^{n} z_i \tag{4.71}$$

with z_i representing the total costs resulting from one Monte Carlo run:

$$z_i = \sum_{t=1}^{T} \cdot c_t \cdot \hat{x}_{t,i} \tag{4.72}$$

The uncertainty of this results is given by the standard deviation:

$$\sigma_z = \sqrt{\frac{1}{n^2} \cdot \sum_{i=1}^{n} (z_{upper} - z_i)^2} \tag{4.73}$$

Based on the standard deviation the 95% confidence interval $[z_{upper} - 2\sigma_z, z_{upper} + 2\sigma_z]$, as any other confidence interval, can be used as the convergence criteria, stopping the algorithm when the lower bound reaches this interval.

4.3. Benders decomposition

4.3.1. General Description [10, p.1-2]

The algorithm mentioned in the last section can be interpreted as a Benders decomposition. Originally Benders decomposition was used for solving mixed integer programming problems, where the problem was decomposed into a master problem with all the integer variables and sub-problems containing the continuous variables. In hydro thermal scheduling integer variables can be on/off decision variables for thermal power plants.

As in hydrothermal systems the water continuity equations include variables from the actual stage and the stage before, Benders decomposition is a way to solve these problems using nested sub-problems.

With a given problem definition like:

$$min : z = c_1 \cdot x_1 + c_2 \cdot x_2 \tag{4.74}$$

subject to:

$$A_1 \cdot x_1 = b_1 \tag{4.75}$$
$$E_1 \cdot x_1 + A_2 \cdot x_2 = b_2 \tag{4.76}$$
$$x_1, x_2 \geq 0 \tag{4.77}$$
$$x \in \Re^n \tag{4.78}$$

using Benders decomposition it can be split into a master problem and a subproblem, with the master problem being defined as:

$$min : z = c_1 \cdot x_1 + \alpha \tag{4.79}$$

subject to:

$$A_1 \cdot x_1 = b_1 \tag{4.80}$$
$$x_1 \geq 0 \tag{4.81}$$
$$x \in V \tag{4.82}$$

and a the related subproblem being defined as:

$$\alpha = min : c_2 \cdot x_2 \tag{4.83}$$
$$\tag{4.84}$$

subject to:

$$A_2 \cdot x_2 = b_2 - E_1 \cdot x_1 \tag{4.85}$$
$$x_2 \geq 0 \tag{4.86}$$

with V representing a restricted set of values for the result of stage one, which allows stage two to be solved. Finding V is one of the main tasks when applying Benders decomposition.

Using the strong duality principle the problem can be rewritten as:

$$\alpha = max : \lambda \cdot (b_2 - E_1 \cdot x_1) \tag{4.87}$$

subject to:

$$\lambda \cdot A_2 \leq c_2 \tag{4.88}$$

To find V, Farkas lemma, a mathematical stating about the solvability of inequations, is applied to the second stage problem:

$$V = \{x \in \Re^n \mid \sigma \cdot (b_2 - E_1 \cdot x_1) \leq 0, \ \forall \sigma \mid \sigma \cdot A_2 \leq 0\} \tag{4.89}$$

V and α are just known by their implicit definitions. So the relaxed master problem is solved and additional constraints are added if they are necessary, they are formed by solving the updated subproblem. The constraints are the so called optimal cuts or feasibly cuts.

Finally the master problem can be expressed as:

$$min: z = c_1 \cdot x_1 + \alpha \tag{4.90}$$

subject to:

$$A_1 \cdot x_1 = b_1 \tag{4.91}$$
$$\sigma_k \cdot A_2 \cdot x_1 \geq \sigma_k \cdot b_2 \quad k = 1..K \tag{4.92}$$
$$\alpha \geq \lambda_j \cdot (b_2 - E_1 \cdot x_1) \quad j = 1..J \tag{4.93}$$
$$x_1 \geq 0 \tag{4.94}$$

Summarized the steps of a Benders decomposition for a two stage problem are:

- Step 1 - Initialization:
  define a tolerance > 0 being used for the stopping rule, set the lower bound $\underline{z}$ to $-\infty$ and the upper bound $\overline{z}$ to $+\infty$
- Step 2:
  solve the relaxed master problem, determine x_{1n} and α_n, and set the lower bound to $\underline{z} = c_1 \cdot x_n + \alpha_n$
- Step 3:
  form and solve the restricted subproblem, obtain y_n
  - if the subproblem is infeasible:
    use feasibility cut, obtain σ_{k+1} and update $K = K \cup \{k+1\}$
  - if the subproblem is feasible:
    use optimal cut, obtain α_{j+1}, update $J = J \cup \{j+1\}$ and set upper bound to $\overline{z} = c_1 \cdot x_{1n} + c_2 \cdot x_{2n}$
- Step 4 - Stopping Rule:
  stop if defined tolerance is reached ($\overline{z} - \underline{z} \leq \overline{z} *$ tolerance)
- Step 5:
  continue with step 1

4.3.2. Example for a Benders Decomposition using Optimal Cuts

This example for a Benders decomposition with a feasible subproblem uses only continuous variables.

Given is a problem:

$$min: 7 \cdot x_1 + 28 \cdot x_2 \tag{4.95}$$

subject to:

$$7 \cdot x_1 \geq 13 \tag{4.96}$$
$$2 \cdot x_1 + 2 \cdot x_2 \geq 8 \tag{4.97}$$

First set a tolerance for the stopping rule and initial values for the bounds:

$$\text{tolerance} = 0, \; \underline{z} = -\infty, \; \overline{z} = \infty \tag{4.98}$$

Then construct the master problem (MP):

$$min : 7 \cdot x_1 + \alpha \tag{4.99}$$

subject to:

$$7 \cdot x_1 \geq 13 \tag{4.100}$$
$$x \geq 0 \tag{4.101}$$

This gives a solution of

$$x_1 = {}^{13}/_{7}, \; \alpha = 0 \tag{4.102}$$

which means that the lower bound has to be set to

$$\underline{z} = c_1 \cdot x_1 + \alpha = 7 \cdot {}^{13}/_{7} + 0 = 13 \tag{4.103}$$

This value then is used to solve the relaxed subproblem:

$$min : 28 \cdot x_2 \tag{4.104}$$

subject to:

$$2 \cdot x_2 \geq 8 - 2 \cdot {}^{13}/_{7} \tag{4.105}$$
$$x_2 \geq 0 \tag{4.106}$$

This problem is feasible with $x_2 = {}^{15}/_{7}$, so the dual problem has to be solved:

$$\alpha = max : \lambda \cdot (8 - 2 \cdot x_1) \tag{4.107}$$
$$\lambda \cdot 2 \leq 28 \tag{4.108}$$

This gives a solution of $\lambda = 14$,
the new upper bound is set to $\overline{z} = c_1 \cdot x_1 + c_2 \cdot x_2 = 7 \cdot {}^{13}/_{7} + 28 \cdot {}^{15}/_{7} = 73$.

Now the stopping rule is checked:

$$\overline{z} - \underline{z} \leq \overline{z} \cdot \text{tolerance} \tag{4.109}$$
$$73 - 13 \leq 73 \cdot 0 \tag{4.110}$$
$$60 \leq 0 \tag{4.111}$$

As the stopping rule is not satisfied the relaxed master problem is solved again, now with respect to the optimal cut:

$$min : 7 \cdot x_1 + \alpha \tag{4.112}$$
$$\tag{4.113}$$

subject to:

$$7 \cdot x_1 \geq 13 \tag{4.114}$$

$$\alpha > 14 \cdot (8 - 2 \cdot x_1) \tag{4.115}$$

$$x_1 \geq 0 \tag{4.116}$$

The solution now is $x_1 = 4, \alpha = 0$,
so the new lower bound is $\underline{z} = c_1 \cdot x_1 + \alpha = 7 \cdot 4 + 0 = 28$.

Now the second stage subproblem has to be solved again, using $x_1 = 4$:

$$min : 28 \cdot x_2 \tag{4.117}$$

subject to:

$$2 \cdot x_2 \geq 8 - 2 \cdot x_1 \tag{4.118}$$

$$x_2 \geq 0 \tag{4.119}$$

This results in a new solution of $x_2 = 0$,
the new upper bound is $\overline{z} = c_1 \cdot x_1 + c_2 \cdot x_2 = 7 \cdot 4 + 28 \cdot 0 = 28$.

Now the stopping rule

$$\overline{z} - \underline{z} \leq \overline{z} * \text{tolerance} \tag{4.120}$$

$$28 - 28 \leq 28 * 0 \tag{4.121}$$

$$0 \leq 0 \tag{4.122}$$

is satisfied and the optimization stops with resulting values of $x_1 = 4$ and $x_2 = 0$.

4.3.3. Example for a Benders Decomposition using Feasibility Cuts

This example for a Benders Decomposition has a infeasible subproblem, so feasibility cuts are used to solve the subproblem.

Given is a problem:

$$min : 7 \cdot x_1 + 8 \cdot x_2 \tag{4.123}$$

subject to:

$$7 \cdot x_1 \geq 6 \tag{4.124}$$

$$2 \cdot x_1 - 2 \cdot x_2 \geq 3 \tag{4.125}$$

First a tolerance for the stopping rule as well as the initial values for the bounds are set:

$$\text{tolerance} = 0,\ \underline{z} = -\infty,\ \overline{z} = \infty \tag{4.126}$$

Then the master problem (MP) is constructed:

$$min : 7 \cdot x_1 + \alpha \tag{4.127}$$

subject to:

$$7 \cdot x_1 \geq 6 \tag{4.128}$$

$$x \geq 0 \tag{4.129}$$

Solving the master problem results in $x_1 = {}^6\!/_7,\ \alpha = 0$,
so the lower bounds are set to $\underline{z} = c_1 \cdot x_1 + \alpha = 7 \cdot {}^6\!/_7 + 0 = 6$.

Now the relaxed subproblem has to be solved using $x_1 = {}^6\!/_7$:

$$min : 8 \cdot x_2 \tag{4.130}$$

subject to:

$$-2 \cdot x_2 \geq 3 - 2 \cdot {}^6\!/_7 \tag{4.131}$$

$$x_2 \geq 0 \tag{4.132}$$

This problem is not feasible because of the two following constraints:

$$-2 \cdot x_2 \geq {}^9\!/_7 \tag{4.133}$$

$$x_2 \geq 0 \tag{4.134}$$

This means that σ has to be set to a value fulfilling the requirement

$$\sigma \cdot -2 \leq 0 \tag{4.135}$$

So it is set to $\sigma = 1$, which fulfills this requirement,
the upper bound stays unchanged at $\overline{z} = \infty$.

As the upper bound is still ∞ the stopping rule is not satisfied, so the relaxed master problem is solved again:

$$min : 7 \cdot x_1 + \alpha \tag{4.136}$$

subject to:

$$7 \cdot x_1 \geq 6 \tag{4.137}$$

$$1 \cdot 2 \cdot x_1 \geq 1 \cdot 3 \tag{4.138}$$

$$x_1 \geq 0 \tag{4.139}$$

This gives an solution of $x_1 = {}^3\!/_2$ and $\alpha = 0$,
the new lower bound is $\underline{z} = c_1 \cdot x_1 + \alpha = 7 \cdot {}^3\!/_2 + 0 = {}^{21}\!/_2$.

This value then is used to solve the second stage subproblem again:

$$min : 8 \cdot x_2 \tag{4.140}$$

subject to:

$$-2 \cdot x_2 \geq 3 - 2 \cdot {}^3\!/_2 \tag{4.141}$$

$$x_2 \geq 0 \tag{4.142}$$

This now results in a solution of $x_2 = 0$
with a new upper bound of $\overline{z} = c_1 \cdot x_1 + c_2 \cdot x_2 = 7 \cdot {}^{3}\!/_{2} + 8 \cdot 0 = {}^{21}\!/_{2}$.

Now the stopping condition is met:

$$\overline{z} - \underline{z} \leq \overline{z} * tol \tag{4.143}$$

$$ {}^{21}\!/_{2} - {}^{21}\!/_{2} \leq {}^{21}\!/_{2} * 0 \tag{4.144}$$

$$0 \leq 0 \tag{4.145}$$

This means that the optimum value is found (within the specified tolerance), the result is $x_1 = {}^{3}\!/_{2}$ and $x_2 = 0$.

4.4. Flexible Array Class

For storing big amounts of data in composite structures such as multi-dimensional arrays, it is difficult to find a clear-structured solution, which is also less dependent on memory resources. This problem is amplified by the fact that both the number of necessary dimensions and their expanses cannot be determined before the program is executed in run time, making the use of classical array structures virtually impossible.

On the other hand, the use of dynamic array structures also entails with certain problems. Not only the functions that construct and access these data are more complex, it is also less efficient to manage a large amount of these structures if they are small in size, significantly compromising the performance of the operating system.

The situation could be improved by using nested array objects, which already exist in the Microsoft Foundation Classes (MFC). However, this alternative would have the same problem of having to use complex functions for creating array trees and accessing several elements, again an inefficient approach.

The solution was the implementation of a special class, which can be initialized at runtime and saves the necessary memory capacity for one block. Inside objects of this class all data elements are mapped to a linear array. Additionally, the number of dimensions and the associated expanses are saved. Based on this information, the position of the data in the linear array can be calculated for each access to the required element.

During the construction of the object no storage space is reserved. Following the initialization via the function *Create*, which also specifies the number of dimensions and their respective expanse, the required storage space is demanded. Afterwards it is possible to have access to the array.

For accessing elements of the multi dimensional array in detail, the functions *Element* and *ElementPtr* exist.

Sequentially to the call the indices for all dimensions are passed like parameters. The function then calculates the position of the required data and returns a pointer or a reference to the wanted position. This can be used for writing or reading accesses. It is a good strategy to limit the consequences of the use of invalid indices. This is achieved by returning pointers or references to a prepared data element. An access to that has no effect or returns invalid values, but it avoids the program from breaking.

Supporting functions such as *Discard* and *GetSize* exist.

The *Discard* method clears the contents of the array and releases the used memory capacity. Afterwards a new initialization with *Create* again is possible. An automatic call to the *Discard* function occurs in the wake of deleting the CFlexArray-object or when the program end is reached. With the function *GetSize* it is possible to request the size of the reserved data space for the array. This information can then be useful during program development and for the unavoidable debugging process.

4.5. XA Optimisation Library [9]

XA is an optimisation software from Sunset Software Technology. It can solve linear programming problems, mixed integer programming problems and quadratic programming problems.

For linear programming problems XA uses primal and dual simplex and for mixed integer programming the branch and bound approach. It also has an interior-point implementation, but a license for this was not available during the implementation of the system. For quadratic programming the barrier algorithm is used.

For this thesis the XA Callable Library was used. This is a software library, providing a set of functions to solve linear, binary, integer and semi-continuous linear programming problems and allowing to use these functions in programs written in several programming languages, including C, Visual Basic, Fortan and Pascal.

Two different techniques can be used to define models to be solved by the XA optimisation library:

- RCC (Row, Column and Coefficient) style
- Classic style, using arrays

While the classic style problem definition provides compatibility with as well older versions of XA as many other solver libraries, the RCC style problem definition method was used for the project, because it has advantages with regards to memory requirements and implementation complexity.

4.5.1. RCC Style Problem Definition

When defining problems in RCC style all data is provided in three parallel arrays for row names, column names and the respective coefficients. The use of special row and column names as *OBJ* for elements of the objective function or *MIN*, *MAX* and *FIX* for constraints allow to fully define a model, while the order in which the elements are passed is not relevant. XA internally completes the model. This simplifies the use of the interface used for problem definition.

The main advantages of using the RCC style problem definition method are:

- easier model maintenance, as adding new dimensions is possible
- scalability within dimensions without reloading the whole model
- simplified debugging by using speaking names for rows and columns

- use of a simple data structure, only needing three arrays, reducing memory requirements
- changes can be applied to a model after it is already loaded in the optimizer

As an example, given a problem definition as:

$$min : 10 \cdot x_1 + 20 \cdot x_2 \tag{4.146}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 \leq 10 \tag{4.147}$$
$$5 \cdot x_1 - 2 \cdot x_2 \geq 2 \tag{4.148}$$
$$5 \cdot x_1 - 2 \cdot x_2 \leq 20 \tag{4.149}$$
$$x_1, x_2 \geq 0 \tag{4.150}$$

would lead to an RCC style problem definition presented to XA as:

RowName	ColName	Coef
OBJ	x_1	10
OBJ	x_2	20
C1	x_1	2
C1	x_2	3
C2	x_1	5
C2	x_2	-2
C1	MAX	10
C2	MIN	2
C2	MAX	20

Table 4.4.: RCC style problem definition

If the problem has to be modified after already being loaded this can be done by just adding elements of a further function or deleting elements of already defined functions.

The example following shows how such a modification can work. A new column x_3, also containing an integer condition, is added and the x_2 for the second constraint is changed, to get the new problem definition:

$$min : 10 \cdot x_1 + 20 \cdot x_2 + 30 \cdot x_3 \tag{4.151}$$

subject to:

$$2 \cdot x_1 + 3 \cdot x_2 + 4 \cdot x_3 \leq 10 \tag{4.152}$$
$$5 \cdot x_1 - 6 \cdot x_3 \geq 2 \text{and} \leq 20 \tag{4.153}$$
$$x_1, x_2 \geq 0 \tag{4.154}$$
$$1 \leq x_3 \leq 100 \tag{4.155}$$
$$x_3 : \text{integer} \tag{4.156}$$

To update the originally loaded problem definition to this new one following element definitions have to be passed to XA:

RowName	ColName	Coef
C1	x_3	4
C2	x_3	-6
MIN	x_3	1
C2	x_2	0
OBJ	x_3	30
STATUS	x_3	4
MAX	x_3	100

Table 4.5.: RCC style problem definition update

4.5.2. Typical XA Calling Sequence

A typical sequence of calls to the XA library for passing data and and multiple iterations of solving, retrieving data and modifying the model can look like shown below.

XAINIT	initialize XA code
...	load data to RCC arrays
XARCCI	initialize new problem with RCC data
XARCCM	continue XARCCM with new RCC data until model is completed
XARCCM	continue XARCCM
XASOLV	solve the problem
XAACTC	retrieve the results (returns the activity of a column or data)
XADUALC	extract more data (returns the dual activity of a column or row)
XARCCM	update the problem until model is completely redefined
XARCCM	
XASOLV	solve the problem
XAACTC	retrieve the results
XADUALC	extract more data
XARCCI	initialize a new problem with RCC data
XARCCM	continue XARCCM with new RCC data
XARCCM	
XASOLV	solve the problem
XAACTC	retrieve the results
XADUALC	extract more data
XADONE	close XA files and release memory

Table 4.6.: Typical XA calling sequence

5. Results

5.1. Input Data

The following system is implemented as described in section 4.1.

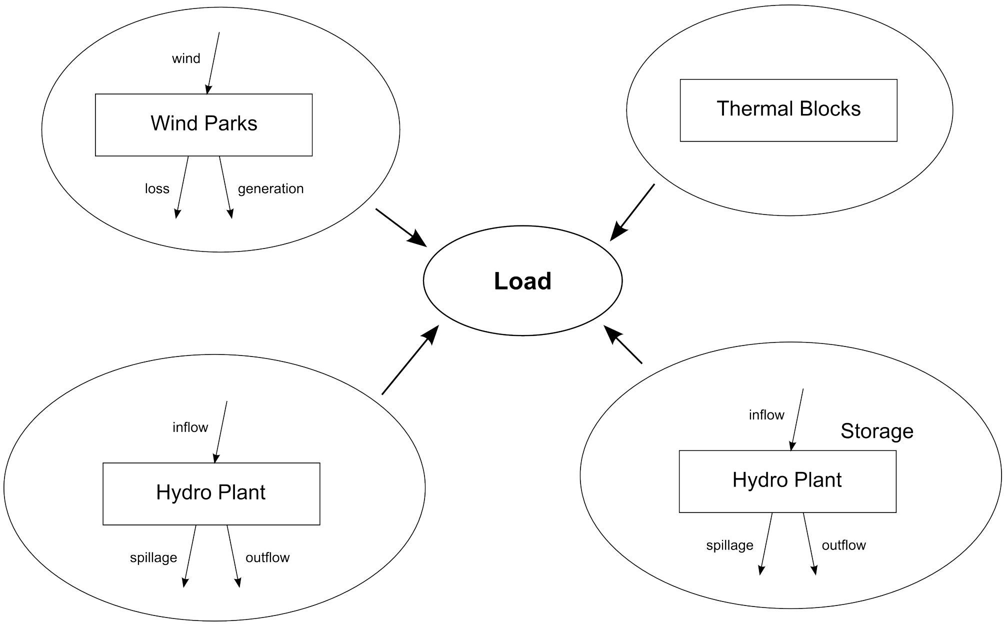

Figure 5.1.: Structure of the test scenarios

All input data are given in MWh. The considered input variables for hydro power plants, thermal blocks and wind parks are shown in tables 5.1 to 5.4. As energy values are used, water head, coefficient and time duration are already considered in the input data.

data	unit
thermal generation	MWh
costs	€/MWh

Table 5.1.: Describing data for thermal power plants

data	unit
hydro generation	MWh
inflow	MWh
storage	MWh
spillage	MWh
costs	€/MWh

Table 5.2.: Describing data for hydroelectric power plants

data	unit
wind generation	MWh
wind strength	MW
costs	€/MWh

Table 5.3.: Describing data for wind power plants

data	unit
solar generation	MWh
solar radiation	MWh
costs	€/MWh

Table 5.4.: Describing data for solar power plants

5.2. General Test Results

The test scenarios 1 to 11 are used to verify the basic functionality of the software.

Afterwards in section 5.3 linear programming and deterministic dual dynamic programming are compared, while the difference between deterministic and stochastic optimization is shown in section 5.4. Very simple scenarios are used for this cases.

Section 5.6 shows the optimization of several simplified scenarios including renewable energy together with the related production costs and CO_2-emissions.

Finally a comparison of problem size, runtime and results between the usage of linear and stochastic dual dynamic programming for a complex scenario is given in section 5.7.

5.2.1. Test 1 - Constant load

Test 1 consists of only two thermal power plants and one storage power plant. The overall load as well as the inflow are set to constant values over all periods, the volume of the water in the storage is not allowed to change. To make the problem feasible the production has to be the same as the load in this case and is therefore set to the average load over the year.

Being forced to hold the volume in the storage constant, the storage power plant works the same way as a run-of-river power plant in this case, the inflow is used in the moment it gets available. This results in an optimization output with all variables being constant over all stages.

Using hydro power generates the lowest production costs, therefore its capacity is fully used. The thermal power plants are used based on their production costs, which means the plant with the lesser costs runs at its full capacity and the second thermal power plant is then used to cover the rest of the load.

The total costs over the year in this scenario are € 301,793,568.

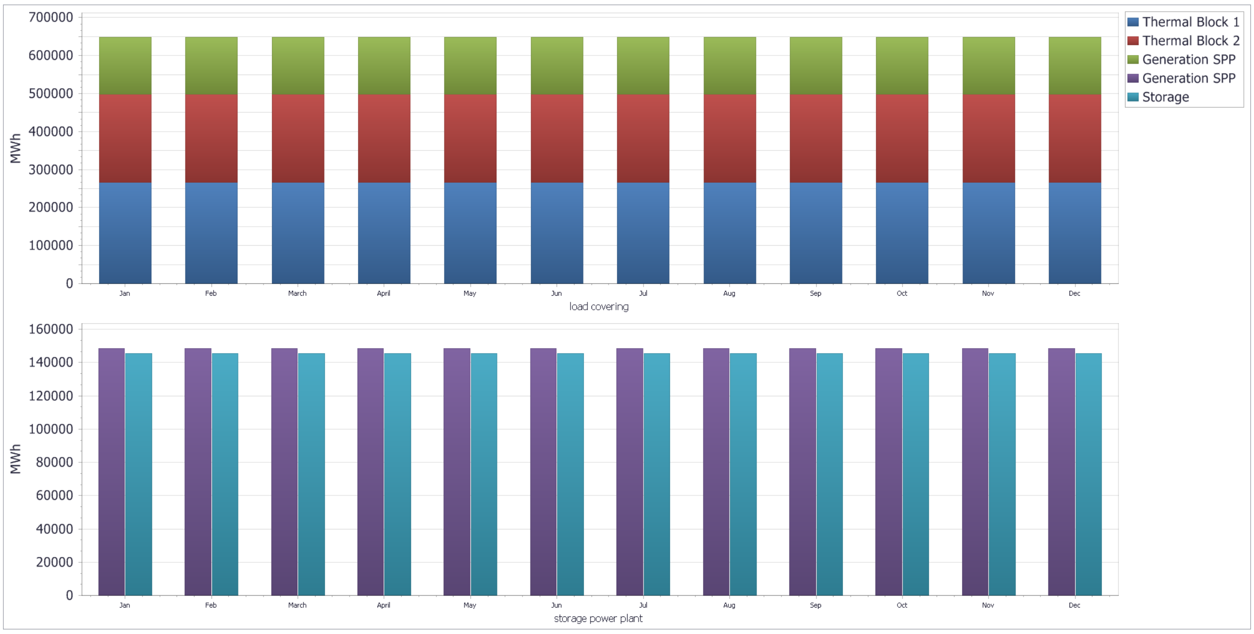

Figure 5.2.: Optimization results for constant load

5.2.2. Test 2 - Changing load and constant inflow

In test scenario 2 the load changes over the year, with a higher consumption during the winter months. The number of thermal power plants is increased to four, while the storage power plant still is not allowed to change the volume held back in the storage, which of course are unrealistic conditions.

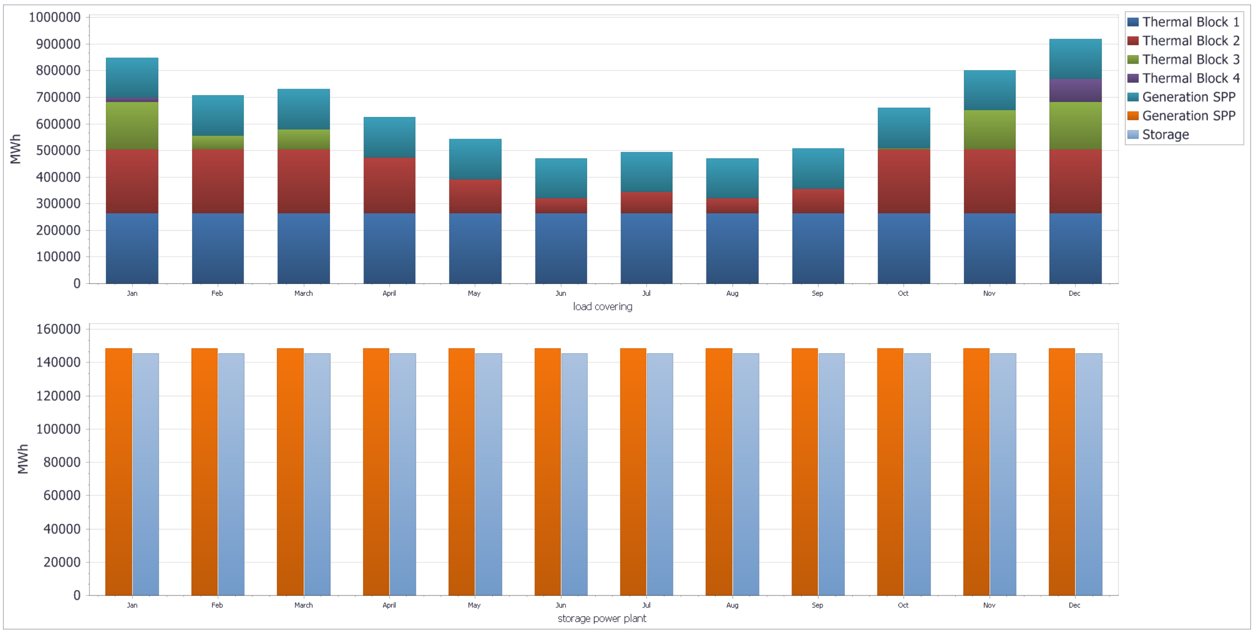

Figure 5.3.: Optimization results for changing load

As in test 1 the inflow to the storage power plant is still used up constantly. However, as the condition of fully covering the load in each period has to be met, the difference between the load and the power generated from the hydro power plant is covered by the production of the four thermal power plants. The scheduling of the thermal power plants is based on the costs they incur during production. This means, that the plants being more expensive are only used if necessary, while the cheaper ones contribute their production during all stages.

The total costs over the year are €310,738,144. Although the average load over the year is the same, the costs are higher than in test scenario 1. This is because the thermal blocks 3 and 4 have to be used in the months with higher loads, generating higher costs. The lower production during the summer months can not compensate this additional costs, as the costs resulting from the thermal blocks 1 and 2 are less.

5.2.3. Test 3 - Constant load, variable storage

The test scenario 3 is simplified again, only containing two thermal and one storage power plant with a constant overall load over the year as in test scenario 1. However the volume in the storage is allowed to change between 50 % and 100 % of its maximum capacity now, initially containing the minimum volume of 50 %. The maximum storage volume is equivalent to a production capacity of 247,680 MWh.

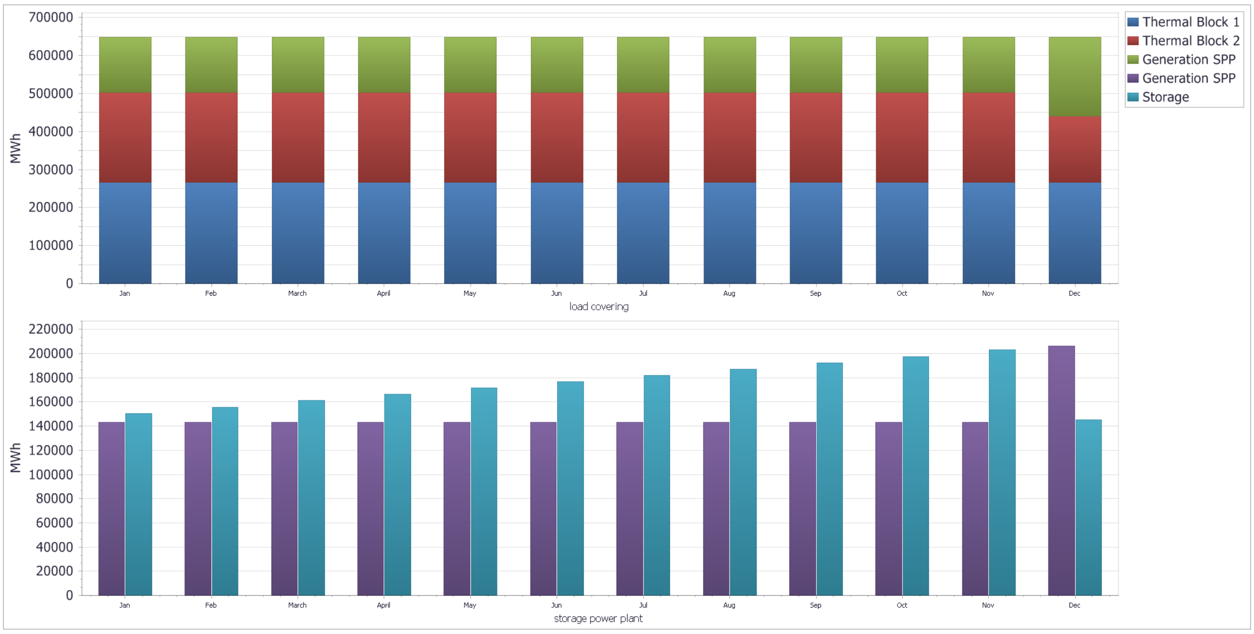

Figure 5.4.: Optimization results for constant load with variable storage

The storage of the hydro power plant is filled up during the months of lower load in this case. However, as the energy produced by the storage power plant is the cheapest one, its production capacity is completely used and the storage is used up to the lower bound of 50 % of its maximum in the last stage. So the whole yearly inflow is used.

This example shows, that the result of the optimization algorithm not necessarily represents the results of a human decision. In this case every scheduling scenario completely making use of the production capacities of the storage power plant and the cheaper thermal block over the year generates the same optimal costs and therefore is equal to the optimizer. Without additional constraints the optimization algorithm returns with the first scenario found meeting this minimal costs requirement.

With respect to costs the scheduling results from test scenarios 1 and 3 are equivalent, as all other schedulings fully taking use of the production capacities of both, the hydro power plant and thermal block 1 over the year. The total costs are €301,783,568, which are the same as in scenario 1.

5.2.4. Test 4 - Changing load, variable storage

In this test scenario the load changes over the year and the volume in the storage is allowed to change under the same restrictions as in test scenario 3. The storage power plant therefore can be used to optimize production costs by reducing the amount of energy which has to be produced by the more expensive thermal blocks 3 and 4. The inflow to the storage is still constant in this scenario.

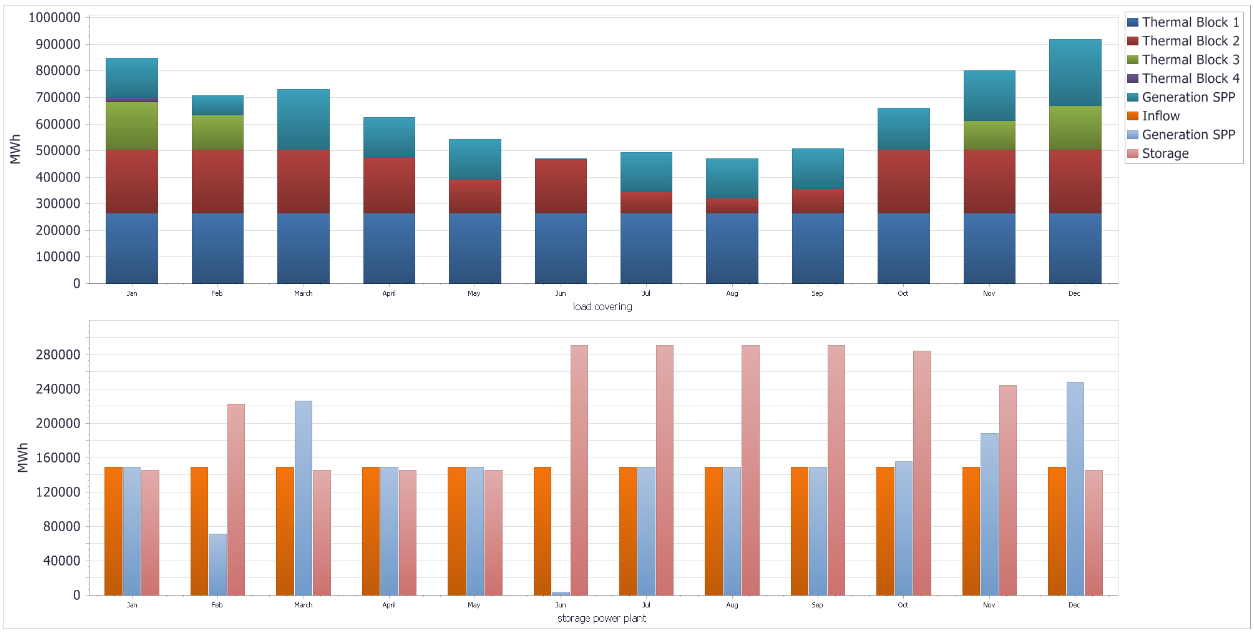

Figure 5.5.: Optimization results for changing load with variable storage

Again there would be an unlimited number of optimal results, all incurring the same total costs. All solutions fully using the production capacities of the storage power plant and the cheaper thermal blocks are equivalent.

With €305,566,624 the total costs are less than in test scenario 2, although the load distribution over the months, the power plants and the inflow being available are the same. The cost reduction results from a better use of the storage power plant by shifting the production to periods of higher load.

5.2.5. Test 5 - Spillage due to constant storage

In this test scenario the volume in the storage has to be constant while the inflow changes over the year. The load is the same in all stages. (unrealistic scenario)

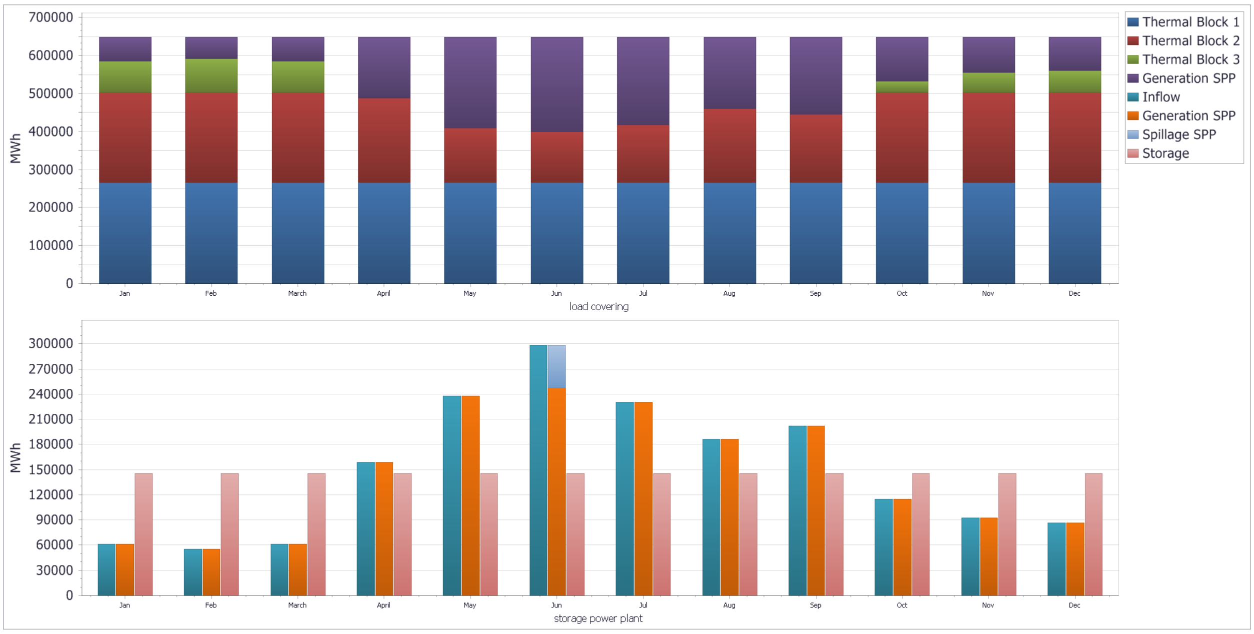

Figure 5.6.: Optimization results for test scenario 5, with necessary spillage

As the volume in the storage is not allowed to change, the whole inflow has to be used immediately - for power generation and/or spillage. If the inflow exceeds the generation capacity of the storage power plant, as this happens in June, the remaining water has to be spilled, reducing the overall performance of the system.

The total costs are €304,443,936.

5.2.6. Test 6 - Storage used to prevent spillage

Compared to test scenario 5 the volume in the storage is allowed to vary between 50 % and 100 % of the maximum in this case. The initial level is at the allowed minimum.

With the volume in the storage being at its minimum at the beginning of the first stage, no water from the storage can be used for production at this moment. The inflow can either be used for production or stored in the reservoir. This allows for shifting the production of the storage power plant from periods with high inflow and low demand to periods with low inflow and high load, optimizing the costs by preventing spillage and minimizing the use of the expensive thermal blocks.

This additional optimizations lead to reduced total costs of €303,057,024 compared to test scenario 5.

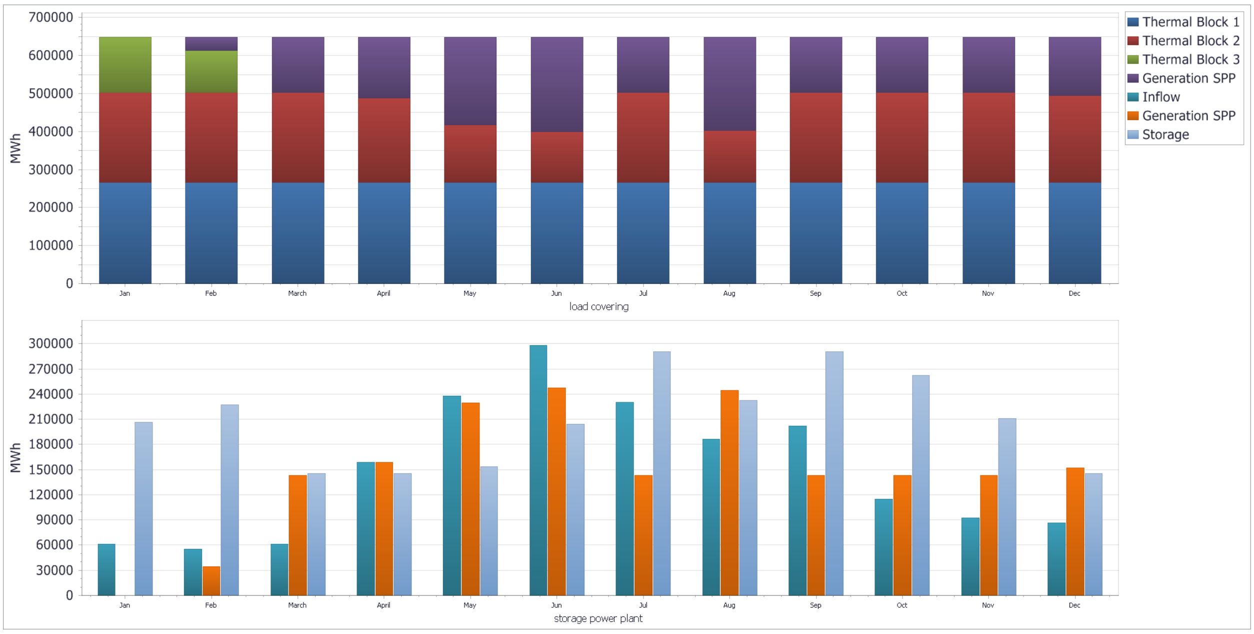

Figure 5.7.: Optimization results for test scenario 6, with storage preventing spillage

5.2.7. Test 7 - Scenario with 24 stages

In this test scenario the number of stages is increased to 24, representing two full years. The input data of the first year is repeated for the second year.

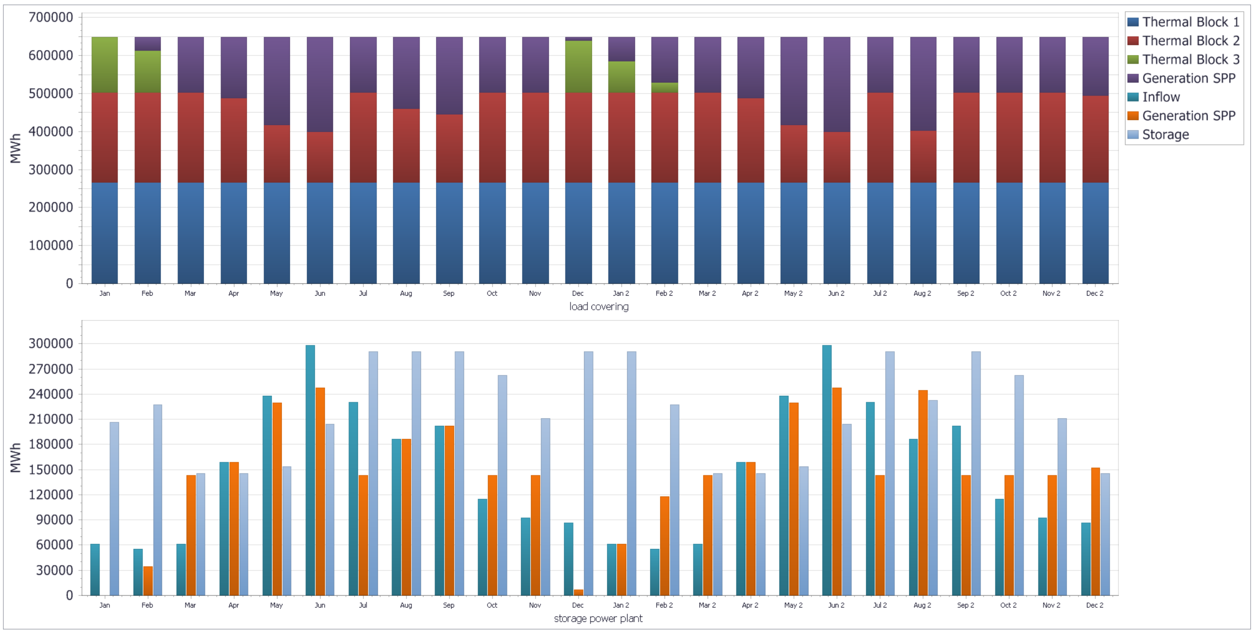

Figure 5.8.: Optimization results for a scenario with 24 stages (2 years)

If a scenario with the possibility to change the storage volume has no restrictions for the filling level of the storage at the end of the scenario, the water in the storage will always result in being used up to its minimum limit. This occurs because no costs are connected with the production of energy from stored water. Extending a scenario with a second year significantly changes the optimal scheduling of the storage power plant.

The total costs for this scenario are €606,070,336, being €303,035,168 per year. This is slightly better then in test scenario 6, as the optimization of the scheduling can be done over more stages.

5.2.8. Test 8 - Run-of-River Power Plant

For this test scenario a run-of-river power plant is added to the system.

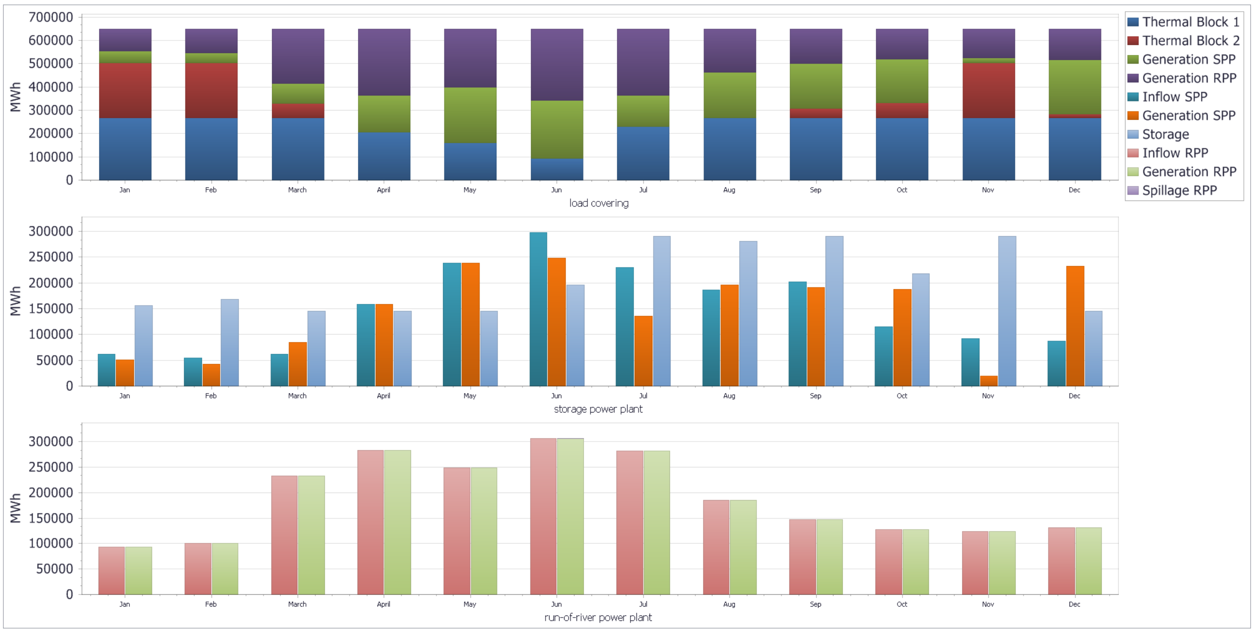

Figure 5.9.: Optimization results for test scenario 8, including a run-of-river power plant

It can be seen that the inflow of the run-of-river power plant is always used for production up to the plants maximal generation capacity, the rest of the water is spillage. The scheduling for the production for the remaining load is optimized as in the test scenarios before.

The total costs for this scenario are €274,641,888.

5.2.9. Test 9 - Changing inflow

To be more realistic the inflows to the run-of-river and the storage power plants are changing in this test scenario.

As in the last test scenario the inflow of the run-of-river power plant is always used for production, but the changing inflow leads to a less optimal use of the plant. During the dry months the inflow is much less than the generation capacity of the plant, while during the wet months the inflow reaches the generation capacity.

The scheduling of the storage power plant and the thermal blocks is optimized in such a way, that the use of the more expensive thermal blocks is reduced. During the first months the whole inflow

Figure 5.10.: Optimization results for test scenario 9, with changing inflows

to the storage power plant is used for production, to minimize the use of the thermal blocks, which however are needed to cover the load. During the summer months water is held back in the storage while the cheapest thermal block is still used to produce energy. This water is used to produce additional energy in the last months, reducing the need for the more expensive thermal blocks 2, 3 and 4. As the scenario only contains 12 stages, the water in the storage is used up to its lower limit in the last stage.

The total costs incurred for production over the year are €285,809,664.

5.2.10. Test 10 - Enlarged storage

This test scenario is the same as scenario 9, except for the storage volume available for the storage power plant, which is ten times larger. Nearly the whole yearly inflow can be stored therefore.

The enlarged storage capacity makes no difference during the first months of the year, as the whole inflow is used during this stages. However, more water is stored during the summer months, which is used to completely work without thermal blocks 3 and 4 during the last stages. The total costs so can be reduced to €281,667,232.

5.2.11. Test 11 - Wind Plant

In test scenario 11 a wind power plant is added to the system.

This leads to the effect, that the full load can be covered using only the wind and the run-of-river plants between April and August, in June not even using the full generation capacity of the run-of-river plant. While more spillage than necessary technically is used in this example, in reality the excess energy can be sold on the market. As no water has to be used from the storage power

Figure 5.11.: Optimization results for test scenario 10, with enlarged storage

Figure 5.12.: Optimization results for test scenario 11, with wind power plant

plant during the summer months, even more production capacity from this plant can be shifted to the last stages. Together with the additional wind power this allows to totally work without the thermal blocks 3 and 4 during this months. The total costs are €360,955,698.

5.3. Comparison of Linear Programming and Deterministic Dual Dynamic Programming

5.3.1. Linear Programming

The scenario for this test (test scenario 12) consists of four thermal power plants and one storage power plant. The lower bound for the amount of water in the storage is now set to 30 % (145,286 MWh), its initial filling level to around 35 % (174,344 MWh) of the total storage capacity (484,286 MWh).

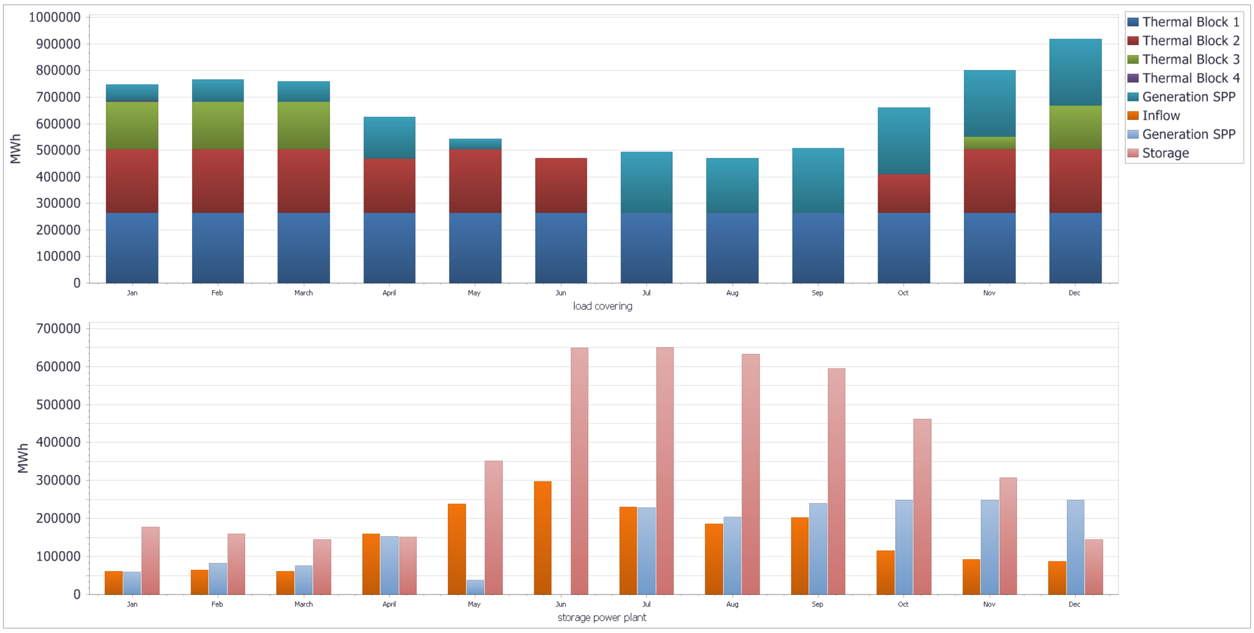

Figure 5.13.: Optimization results for test scenario 12, solved using linear programming

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	4967	58516
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	0	75777
April	623841	158849	266400	204038	0	0	153403
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	204424	0	0	0
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	0	0	0	239735
October	659153	115028	266400	145073	0	0	247680
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1979135	755146	4967	1822543

Table 5.5.: Optimization results for test scenario 12 (LP)

As no further stages are considered, the whole inflow to the storage as well as its usable initial content (29,058 MWh) are used. Similar to the tests before more water is used for production during stages with higher load, to reduce the use of the more expensive thermal power plants.

The total costs for production are €304,835,424.

5.3.2. Deterministic Dual Dynamic Programming

This test (test scenario 13) is based on the same scenario (test scenario 12), but the problem is solved using deterministic dual dynamic programming.

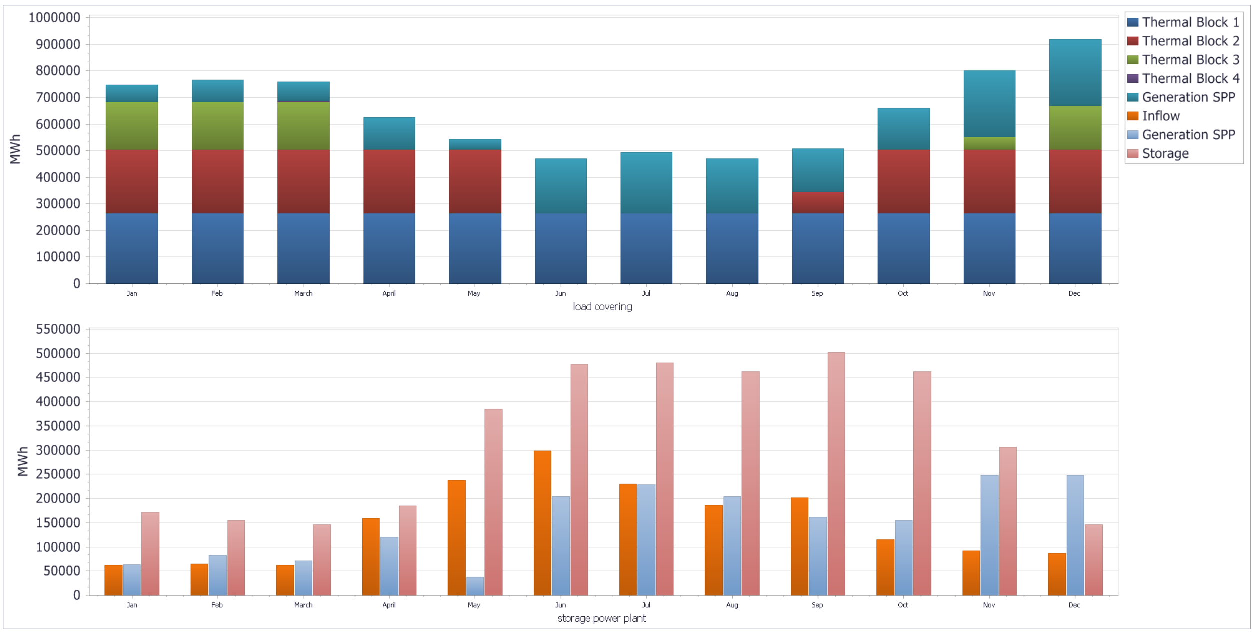

Figure 5.14.: Optimization results for test scenario 13, solved using deterministic dual dynamic programming

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	4967	70810
April	623841	158849	266400	237600	0	0	119841
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	0	0	0	204424
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	78334	0	0	161401
October	659153	115028	266400	237600	0	0	155153
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166427	0	247678
sum	7758591	1793485	3196800	1979134	755147	4967	1822543

Table 5.6.: Optimization results for test scenario 13 (DDDP)

The optimization stops with a value of €304,835,424 for the lower as well as the upper bound, being identical to the result when solving the same problem using linear programming. The lower bound represents the sum of the immediate and the approximated future costs, the upper bound the total costs over all stages. Although both values are the same, this may not be the case for other scenarios.

The solutions for both, using linear and deterministic dual dynamic programming, look quite similar. There are only minor differences in the scheduling of thermal block 4 and the storage power plant. This is because both algorithms return one optimal solution out of an indetermined number of existing optimal solutions. As DDDP uses a convergence criterion to stop the optimization it may not return the exact optimum for more complex scenarios, but a solution being within a certain tolerance defined by this convergence criterion.

5.4. Comparison of Deterministic and Stochastic Programming (scenarios with different inflows and loads)

To compare the results achieved when using deterministic programming for different scenarios to the results when using stochastic programming, three years with different inflow and load conditions are simulated.

Again the scenarios for this tests consist of four thermal power plants and one storage power plant. The lower bound for the amount of water in the storage is now set to 30 % (145,286 MWh), its initial filling level to around 35 % (174,344 MWh) of the total storage capacity (484,286 MWh). So the total inflow plus 29,058 MWh of the storage can be used.

In the first scenario the inflow also the load are set to the average. For the second scenario the load is higher in the following stages, the inflow is lesser compared to scenario one. The third scenario represent a higher load and more inflow compared to scenario one.

scenario	conditions	costs in €
scenario 1	average load, average inflow	304,835,424
scenario 2	higher load, lower inflow	337,207,904
scenario 3	higher load, higher inflow	314,153,696

Table 5.7.: Production costs for the different scenarios

5.4.1. Deterministic Optimization

When using deterministic optimization each scenario is processed separately, so an optimal result for the considered year can be achieved.

The total costs for each scenario are:

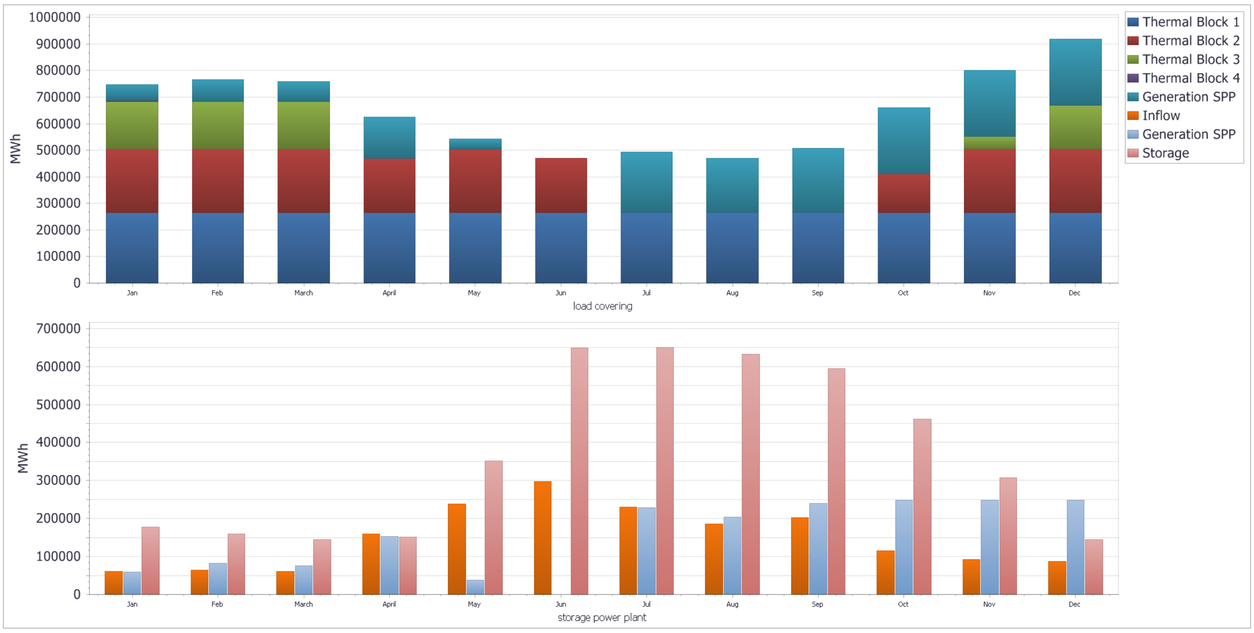

Figure 5.15.: Deterministic optimization for scenario 1 (average load, average inflow)

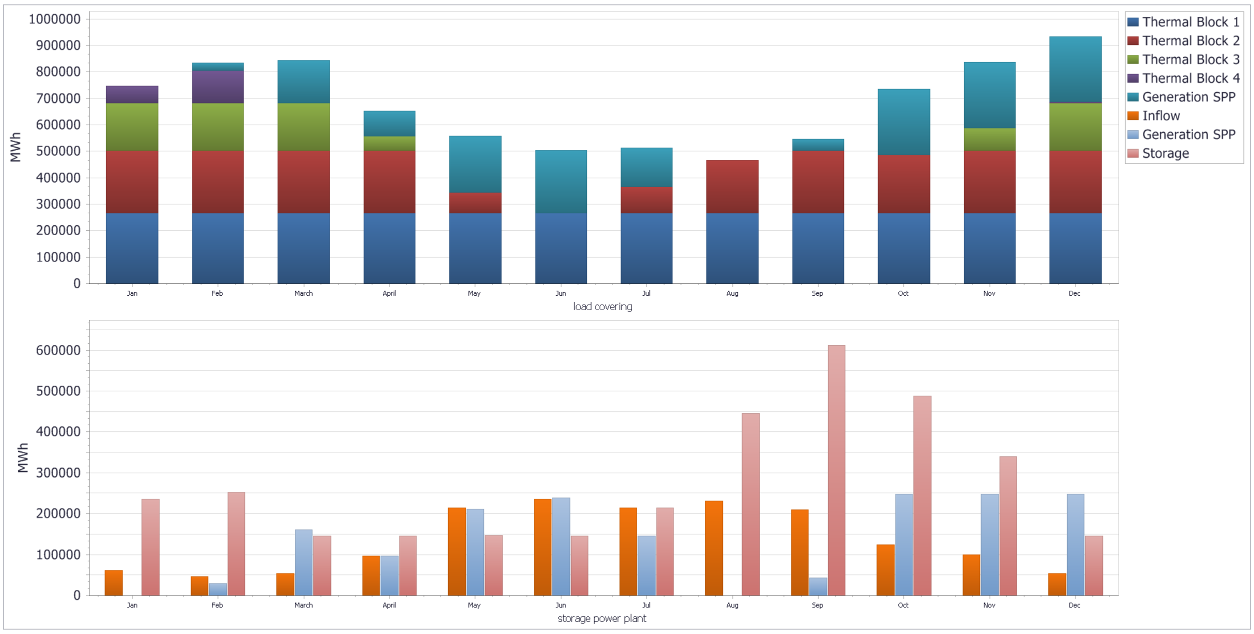

Figure 5.16.: Deterministic optimization for scenario 2 (higher load, lower inflow)

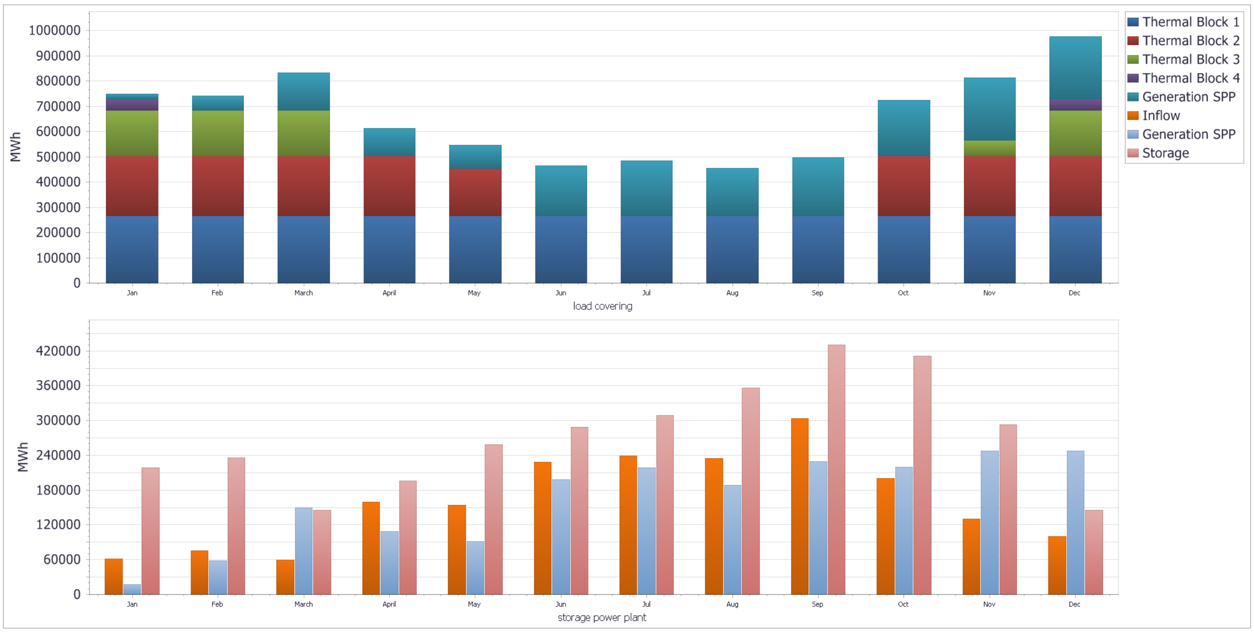

Figure 5.17.: Deterministic optimization for scenario 3 (higher load, higher inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	4967	58516
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	0	75777
April	623841	158849	266400	204038	0	0	153403
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	204424	0	0	0
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	0	0	0	239735
October	659153	115028	266400	145073	0	0	247680
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1979135	755146	4967	1822543

Table 5.8.: Deterministic optimization for scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	8168215	61348	266400	237600	180000	63483	0
February	7420732	45352	266400	237600	180000	120987	28358
March	6587387	52952	266400	237600	180000	0	160352
April	5743035	95801	266400	237600	53133	0	95801
May	5090101	213282	266400	79280	0	0	211252
June	4533169	235895	266400	0	0	0	237925
July	4028844	213282	266400	100813	0	0	145091
August	3516540	230638	266400	198132	0	0	0
September	3052008	210125	266400	237600	0	0	42774
October	2505234	123189	266400	220154	0	0	247680
November	1771000	99674	266400	237600	84774	0	247680
December	934546	53997	266400	237600	180000	2866	247680
sum	8168215	1635535	3196800	2261579	857907	187336	1664593

Table 5.9.: Deterministic optimization for scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	46075	17408
February	742345	75571	266400	237600	180000	0	58345
March	833535	59311	266400	237600	180000	0	149535
April	612435	159385	266400	237600	0	0	108435
May	545824	153889	266400	188020	0	0	91404
June	464584	228086	266400	0	0	0	198184
July	484952	239078	266400	0	0	0	218552
August	454694	234956	266400	0	0	0	188294
September	495455	303656	266400	0	0	0	229055
October	723594	200373	266400	237600	0	0	219594
November	812349	129688	266400	237600	60669	0	247680
December	977106	99767	266400	237600	180000	45426	247680
sum	7894356	1945108	3196800	1851220	780669	91501	1974166

Table 5.10.: Deterministic optimization for scenario 3 (higher load, higher inflow)

5.4.2. Stochastic Optimization

Performing a stochastic optimization, taking into account all scenarios together with their probability of occurrence, delivers one result for the first stage, together with one solution for each scenario for the following stages. This solution is not cost optimal with respect to each single scenario, but optimal with respect to the stochastic occurrence of the scenarios. The three solutions for the following stages represent the optimal solutions for the different scenarios based on the conditions at the end of the first stage.

The average costs for all years are €318,732,352

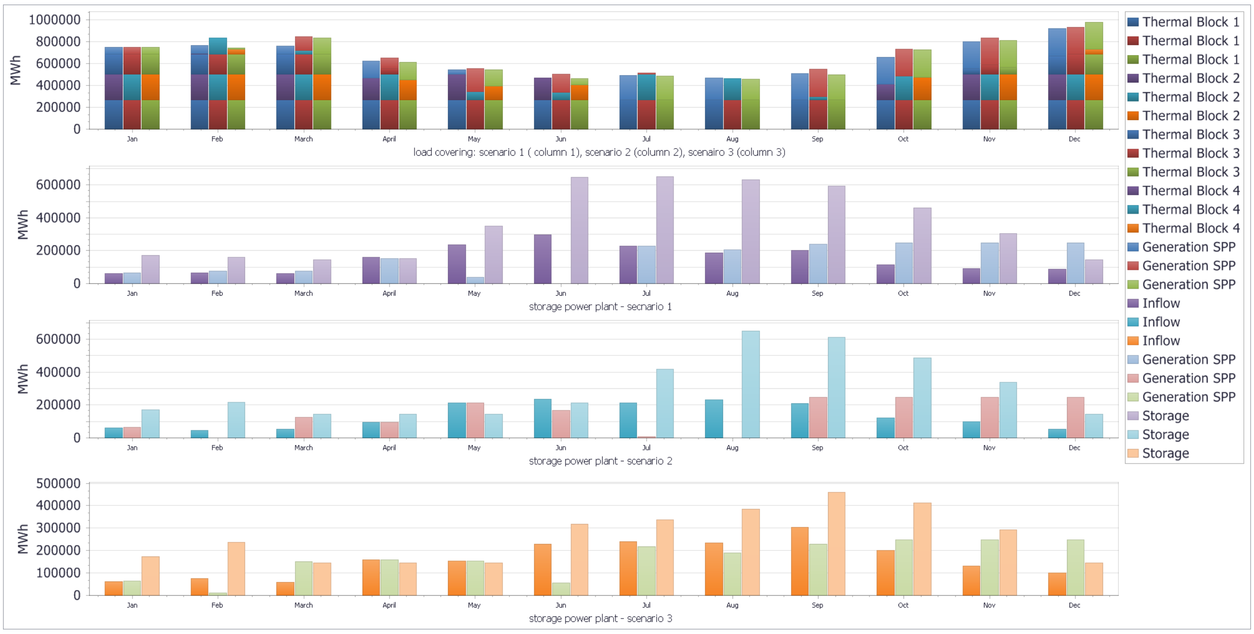

Figure 5.18.: Stochastic optimization, taking into account all scenarios

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	766236	64775	266400	237600	180000	4967	77269
March	759777	61348	266400	237600	180000	0	75777
April	623841	158849	266400	204038	0	0	153403
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	204424	0	0	0
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	0	0	0	239735
October	659153	115028	266400	145073	0	0	247680
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1979135	755146	4967	1822543

Table 5.11.: Stochastic optimization for scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	833345	45352	266400	237600	180000	149345	0
March	844352	52952	266400	237600	180000	35125	125227
April	652934	95801	266400	237600	53133	0	95801
May	556932	213282	266400	77250	0	0	213282
June	504325	235895	266400	70149	0	0	167776
July	512304	213282	266400	237600	0	0	8304
August	464532	230638	266400	198132	0	0	0
September	546774	210125	266400	32694	0	0	247680
October	734234	123189	266400	220154	0	0	247680
November	836454	99674	266400	237600	84774	0	247680
December	934546	53997	266400	237600	180000	2866	247680
sum	8168215	1635535	3196800	2261579	857907	187336	1664593

Table 5.12.: Stochastic optimization for scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	742345	75571	266400	237600	180000	46075	12270
March	833535	59311	266400	237600	180000	0	149535
April	612435	159385	266400	186650	0	0	159385
May	545824	153889	266400	125535	0	0	153889
June	464584	228086	266400	141521	0	0	56663
July	484952	239078	266400	0	0	0	218552
August	454694	234956	266400	0	0	0	188294
September	495455	303656	266400	0	0	0	229055
October	723594	200373	266400	209514	0	0	247680
November	812349	129688	266400	237600	60669	0	247680
December	977106	99767	266400	237600	180000	45426	247680
sum	7894356	1945108	3196800	1851220	780669	91501	1974166

Table 5.13.: Stochastic optimization for scenario 3 (higher load, higher inflow)

5.4.3. Stochastic Optimization using Dual Dynamic Programming

Compared to the solution of the stochastic optimisation using linear programming, for the first and the third scenario, not the howl inflow and usable storage capacity is used. Because of the approximation and the break-up criterion the total optimum is not reached.

The lower bound is €316,703,808, the upper bound is €319,854,810. The upper bound represents the costs over all stages. This costs are higher than when using linear formulation (€318,732,352). This shows that the real optimum is not reached.

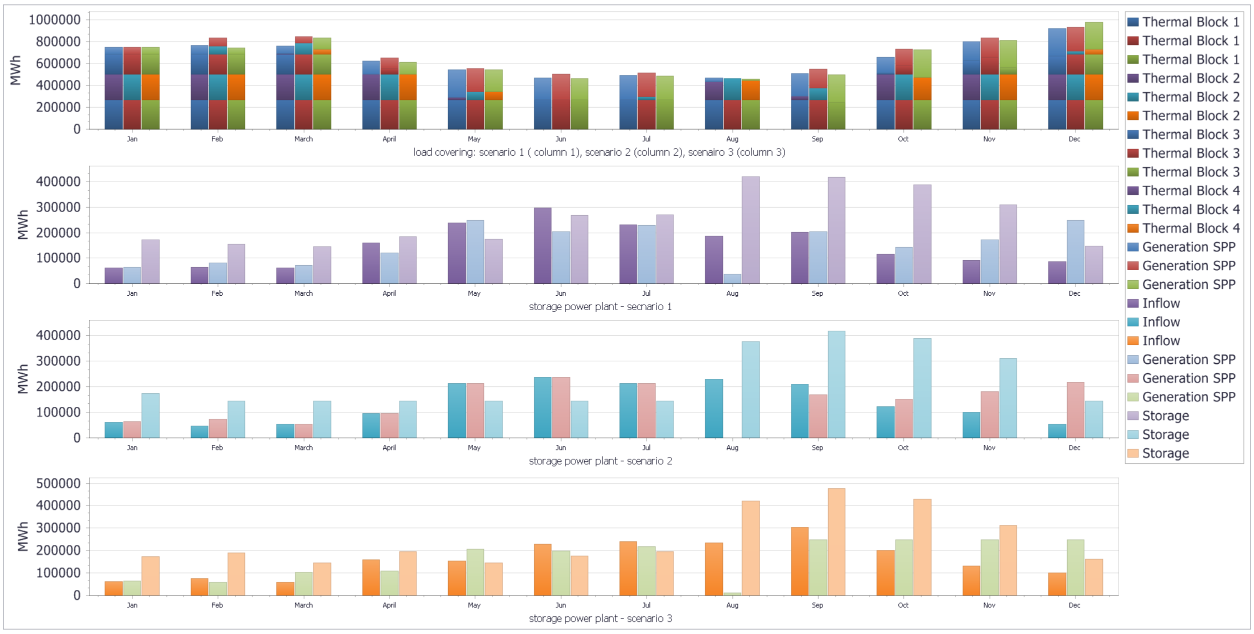

Figure 5.19.: Stochastic optimization, using dual dynamic programming

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	4967	70810
April	623841	158849	266400	237600	0	0	119841
May	541447	237725	266400	27367	0	0	247680
June	470824	297978	266400	0	0	0	204424
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	168437	0	0	35987
September	506135	201573	266400	35363	0	0	204372
October	659153	115028	266400	237600	12200	0	142953
November	800400	92023	266400	237600	124415	0	171985
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1894367	843042	4967	1819415

Table 5.14.: SDDP optimization for scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	833345	45352	266400	237600	180000	77070	72275
March	844352	52952	266400	237600	180000	107400	52952
April	652934	95801	266400	237600	53133	0	95801
May	556932	213282	266400	77250	0	0	213282
June	504325	235895	266400	2030	0	0	235895
July	512304	213282	266400	32622	0	0	213282
August	464532	230638	266400	198132	0	0	0
September	546774	210125	266400	111760	0	0	168614
October	734234	123189	266400	237600	79120	0	151114
November	836454	99674	266400	237600	152818	0	179636
December	934546	53997	266400	237600	180000	32286	218260
sum	8168215	1635535	3196800	2084994	1005072	216756	1664593

Table 5.15.: SDDP optimization for scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	742345	75571	266400	237600	180000	0	58345
March	833535	59311	266400	237600	180000	46075	103460
April	612435	159385	266400	237600	0	0	108435
May	545824	153889	266400	74585	0	0	204839
June	464584	228086	266400	0	0	0	198184
July	484952	239078	266400	0	0	0	218552
August	454694	234956	266400	177858	0	0	10436
September	495455	303656	247775	0	0	0	247680
October	723594	200373	266400	209514	0	0	247680
November	812349	129688	266400	237600	60669	0	247680
December	977106	99767	266400	237600	180000	45426	247680
sum	7894356	1945108	3178175	1887557	780669	91501	1956454

Table 5.16.: SDDP optimization for scenario 3 (higher load, higher inflow)

5.5. Comparison of Deterministic and Stochastic Programming (scenarios for normal, dry and wet year, covering the same load)

The scenarios for this tests consist of four thermal power plants and one storage power plant. The lower bound for the amount of water in the storage is now set to 50 % (145,286 MWh), its initial filling level also to 50 % (145,286 MWh) of the total storage capacity (290,572 MWh). The maximum capacity for the storage power plant is 247,680 MWh.

The load is the same for all scenarios, the inflow is different for a normal, a dry and a wet year. The problems are solved using linear formulation.

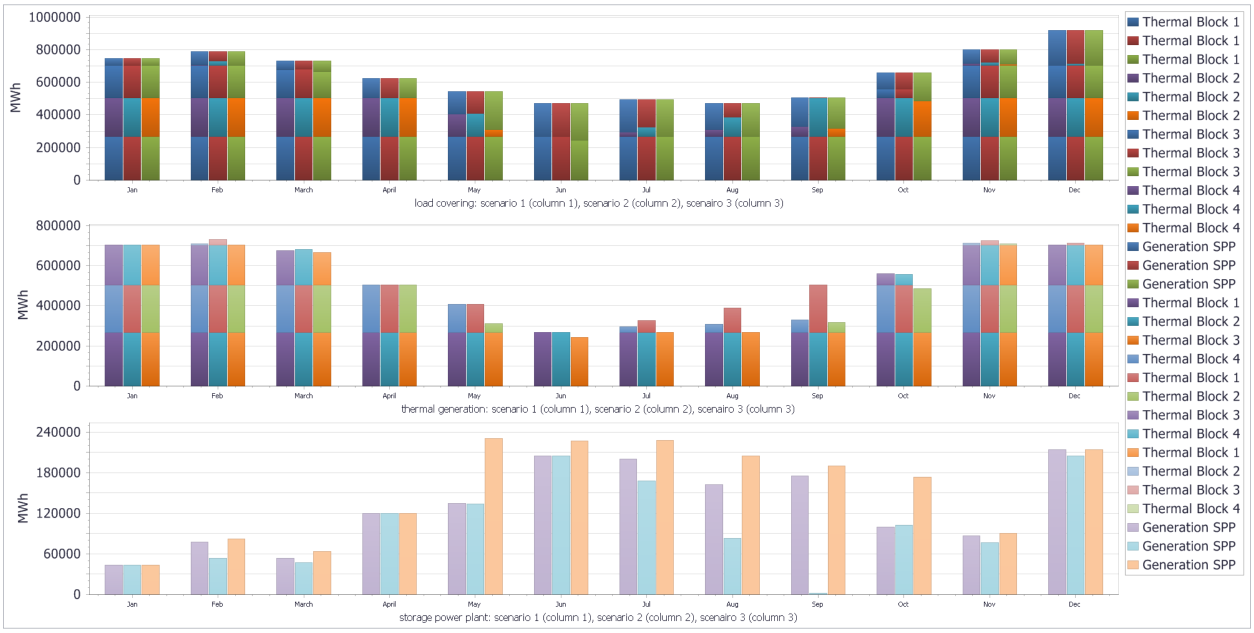

Figure 5.20.: Stochastic optimization, using linear programming, normal-dry-wet year

During dry years the thermal power plants have to be used more, in wet years the production of thermal generation can be reduced compared to the normal years. This leads to higher production costs in dry years and lower costs in wet years.

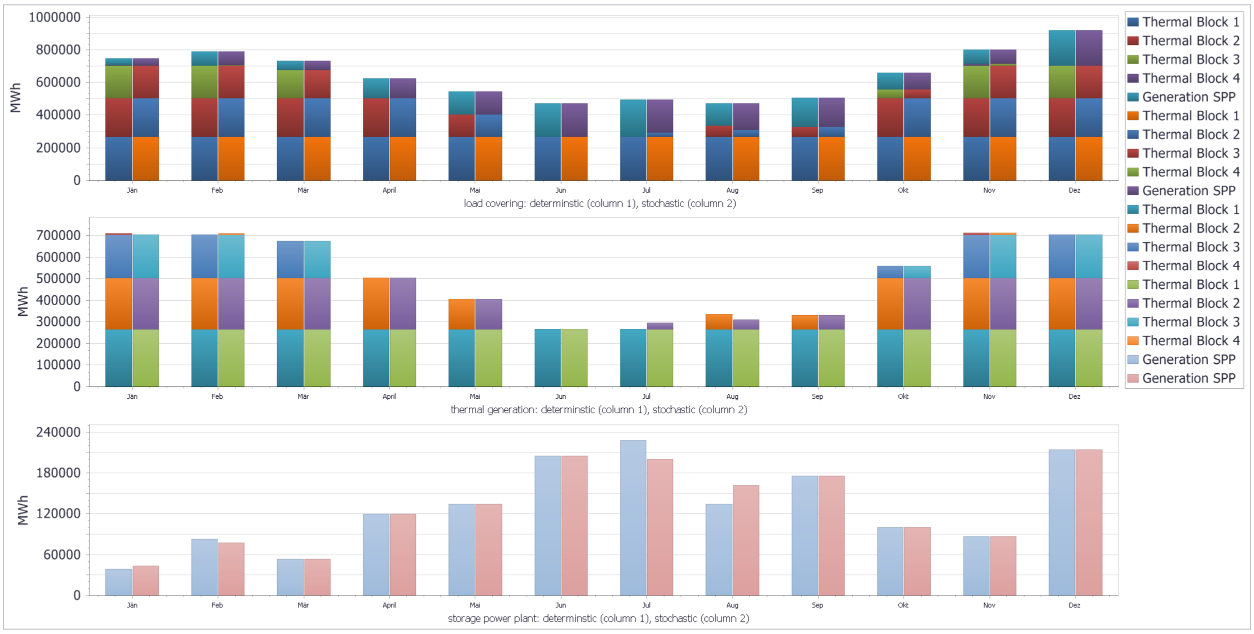

Figure 5.21.: Compare deterministic and stochastic optimization for normal year, using linear programming

As shown in figure 5.21 there is a difference in the use of the storage power plant in the first stage when solving the problem deterministically compared to stochastically. The solution for the normal year solved stochastically is influenced by the wet year, so more capacity of the storage power plant is used in January.

5.6. Scenarios including Wind Power Plants

The scenarios in this section show a typical daily load curve of a combined hydrothermal and wind power system, being oriented on the current situation in Germany.

The diagrams show how the load is covered in half-hour intervals. The scheduling is optimized only according to the marginal costs in the merit order. Because of laws regarding renewable energy (i.e. EEG), the costs for wind, biomass, solar and hydro energy are set to zero. Because of regulatory demands this power plants can feed the whole production into the grid. [6, p.36]

Technical conditions (i.e. start and stop times for thermal power plants) and other economical conditions (i.e. costs for a non-optimal use of thermal power plants) are not taken into account.

5.6.1. Scenario 1 - Normal Situation

This scenario, shown in figure 5.22, represents a basic situation with a daily production of 75 GWh resulting from wind power plants.

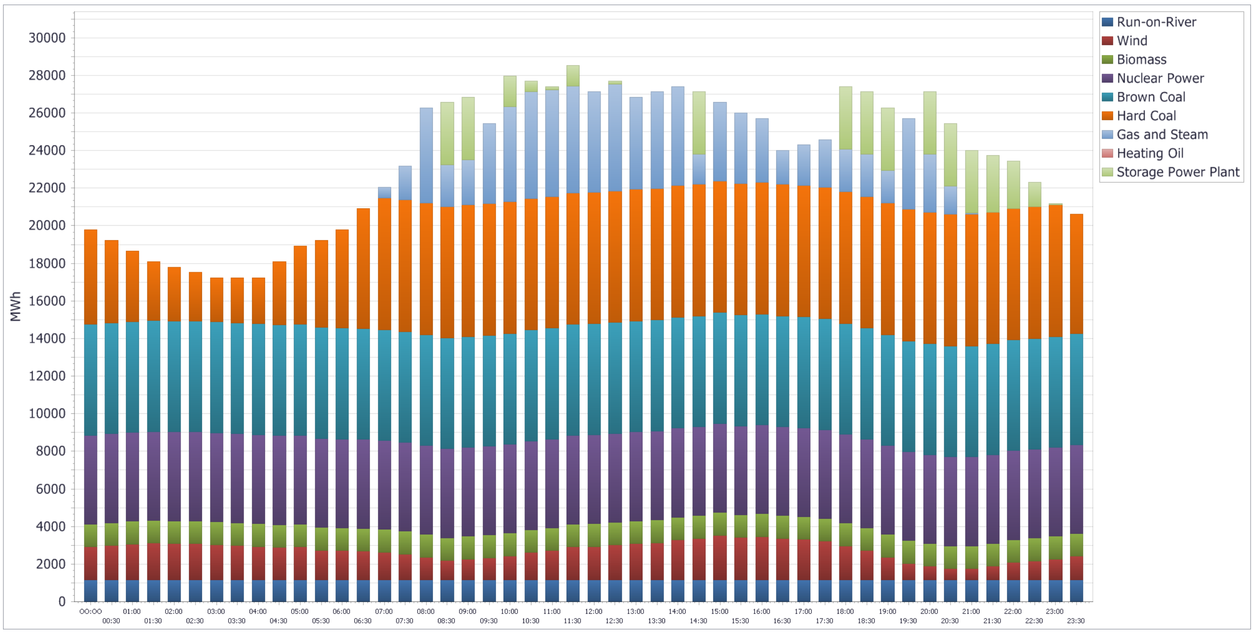

Figure 5.22.: Optimization of a one day scenario including wind power plants, normal situation (7 % wind energy, plus nuclear power)

5.6.2. Scenario 2 - Strong Wind

In this scenario a day with stronger wind is shown. During periods with strong wind it is possible to make full use of the installed generation capacity of the wind parks, which is 29 GW (14.5 GWh per half-hour) in this example. The scheduling is shown in figure 5.23.

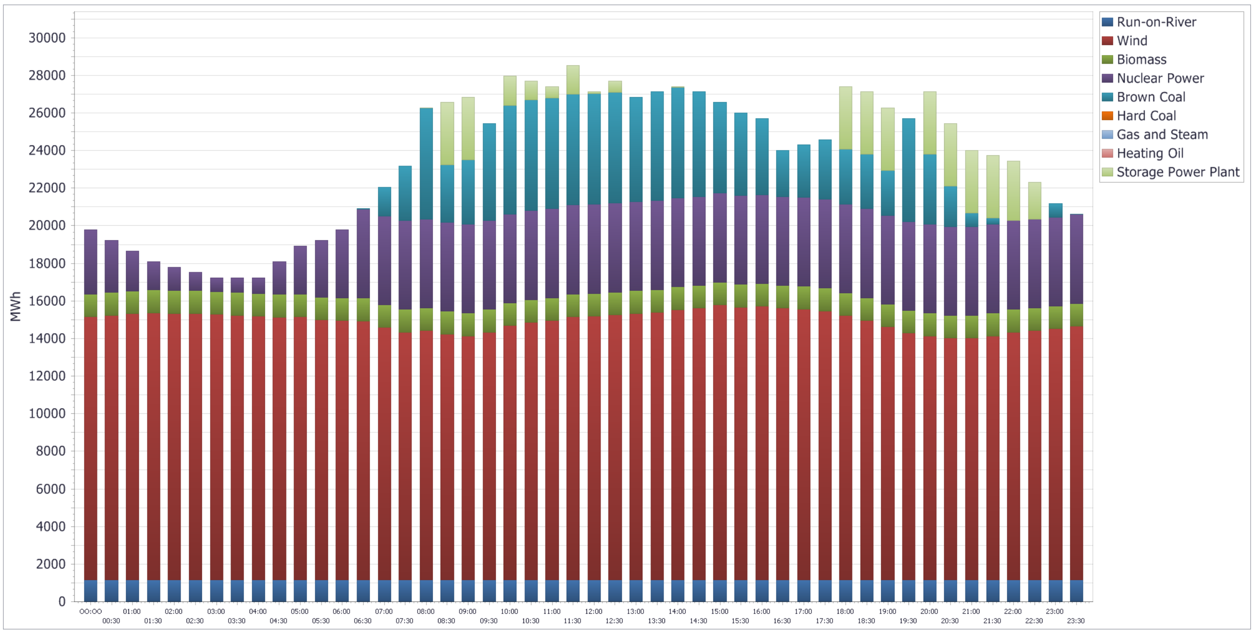

Figure 5.23.: Optimization of a one day scenario including wind power plants, with stronger wind (59 % wind energy, plus nuclear power)

5.6.3. Scenario 3 - Without Nuclear Power Plants

This example shows a situation with all nuclear power plants being turned off. In this case it is necessary to make use of fossil burning power plants up to a higher extent. The optimized scheduling is shown in figure 5.24.

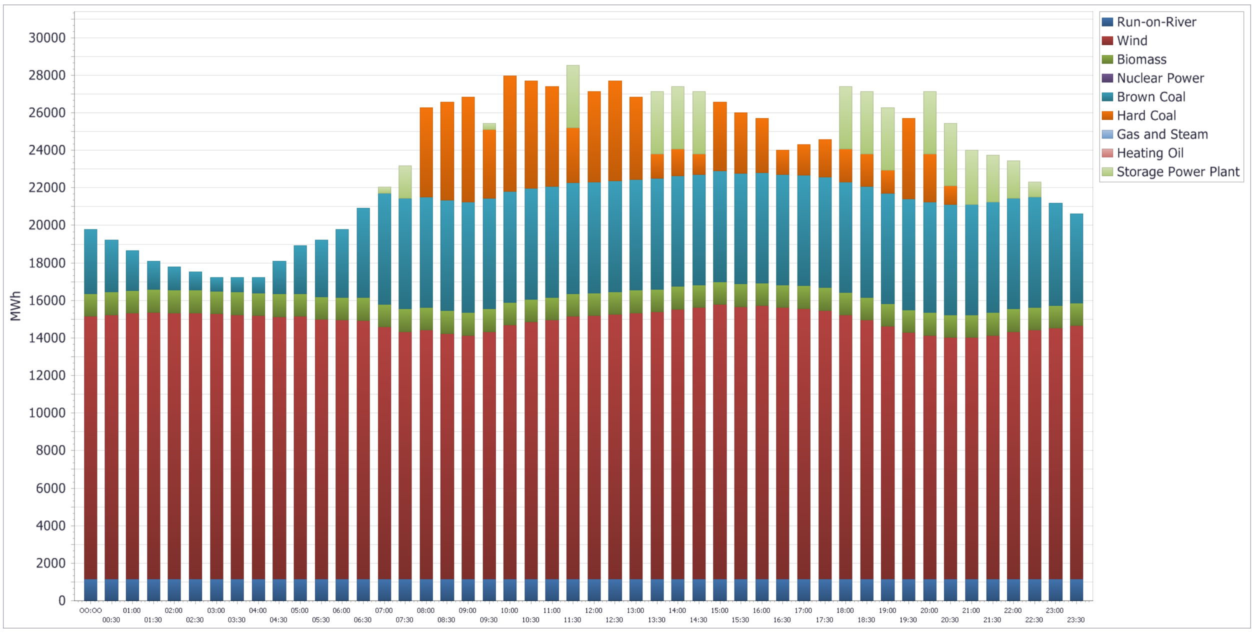

Figure 5.24.: Optimization of a one day scenario including wind power plants, without nuclear power plants (59 % wind energy)

5.6.4. Scenario 4 - Heavily Changing Wind

In a situation with changing wind conditions, which occur frequently, as wind is very stochastic, also generated wind power is strongly changing. So the load has to be covered by other resources, which ideally are storage power plants, but as their capacity is restricted, the rest has to be covered by the fossil fired power plants. This is shown in figure 5.25.

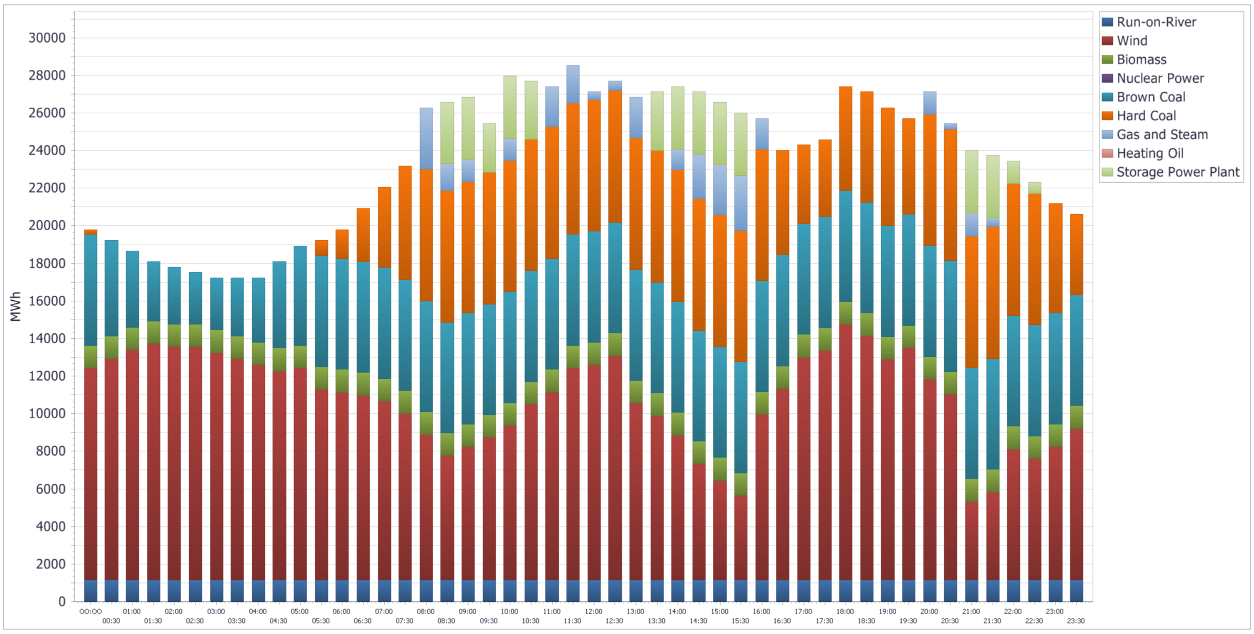

Figure 5.25.: Optimization of a one day scenario including wind power plants, with heavily changing wind (40 % wind energy, without nuclear power)

5.6.5. Scenario 5 - Bottlenecks in the Grid

This example shows a situation where the produced wind power exceeds the transport capacity of the grid and thus cannot be transported to the customer. So other power plants have to cover the load as shown in figure 5.26.

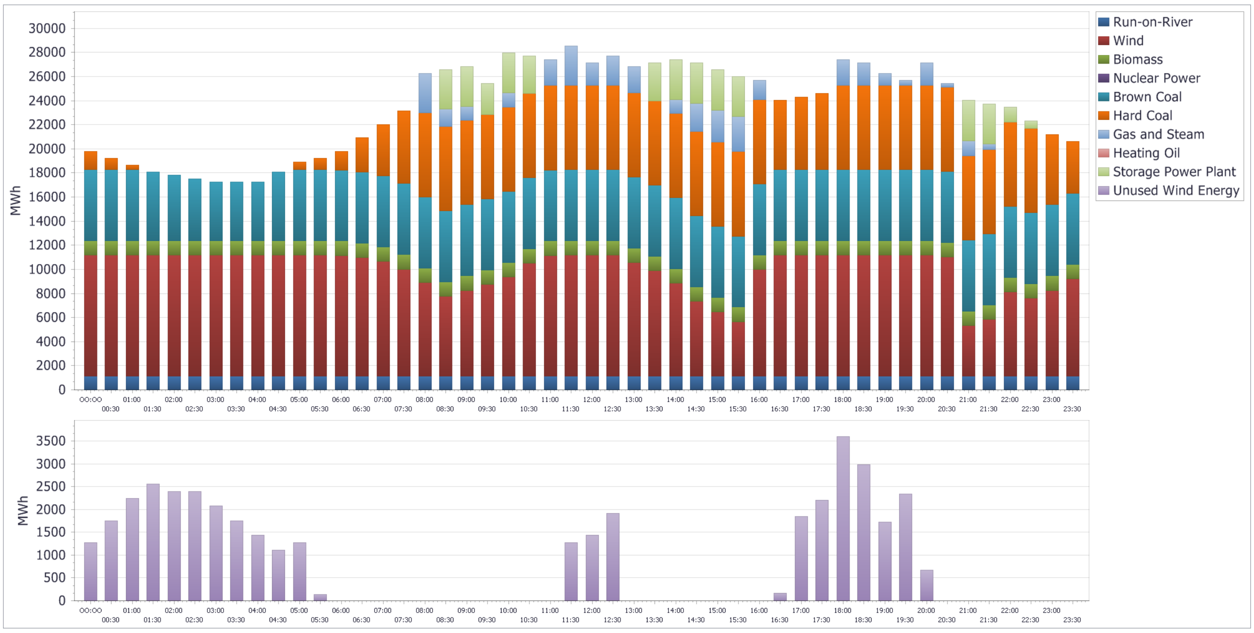

Figure 5.26.: Optimization of a one day scenario including wind power plants, restricted transport (37 % wind energy, without nuclear power, bottleneck in the grid)

5.6.6. Scenario 6 - Heavily Changing Wind including Solar Power Plants

In this scenario also the generation from solar power plants is included. The solar energy can be an important part to cover the load during midday, this is shown in 5.27

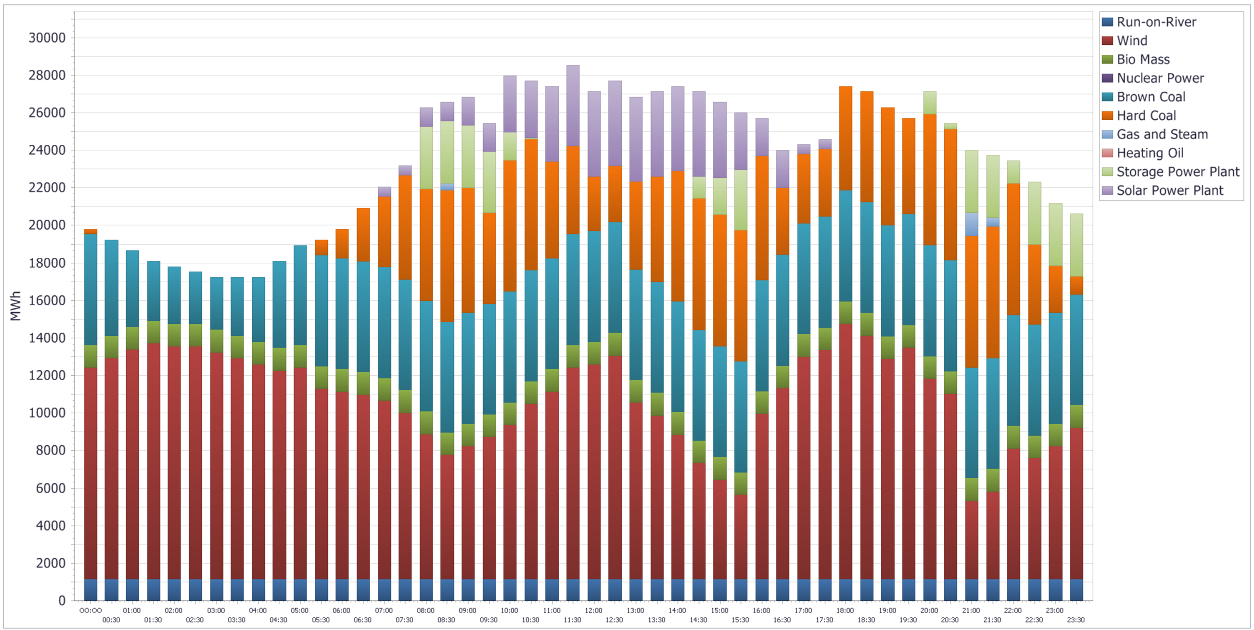

Figure 5.27.: Optimization of a one day scenario including wind and solar power plants (40 % wind energy, 5 % solar energy)

5.6.7. Electrical Production Costs

The real production costs for the electrical energy can be quite different from the costs used in the optimization algorithm. This is especially true for renewable energy, which has a special role, as it has a guaranteed price, which is high compared to other power sources, and there is an obligation to use renewable energy when it is available.

In the literature, the productions costs are given quite different, so a detailed discussion is difficult.

type of power plant	costs per MWh
brown coal	€ 35
hard coal	€ 48
gas and steam	€ 52
nuclear (new)	€ 39
wind	€ 82
biomass	€ 92
hydro power plants	€ 46
solar power plants	€ 130

Table 5.17.: Electrical production costs for different types of power plants [4, p.237,278,248][21, p.429]

This leads to production costs for the former examples as shown in table 5.18. It can be seen, that situations with a lot of wind being available lead to higher costs, although more production capacity is available. The costs would get lower without the existence of regulatory directives.

scenario	costs	
scenario 1	€ 53,868,770	normal situation
scenario 2	€ 75,809,492	strong wind situation
scenario 3	€ 76,149,077	strong wind, without nuclear power plants
scenario 4	€ 69,073,479	heavily changing wind
scenario 5	€ 67,517,006	restricted transport capacity for wind power
scenario 6	€ 73,832,203	heavily changing wind and solar power

Table 5.18.: Total electrical production costs for the example scenarios

5.6.8. CO_2-Emissions

Also the CO_2-emissions occurring during the production of electrical energy can be of interest, especially as there is a huge difference depending on the type of production.

This can be discussed in different manners, depending on which sources of CO_2-emissions are considered. The range starts with only taking into account the emissions resulting from burning the fuel and continues to consider all emissions produced for transport and preparation of the fuel and even construction and planning of the power plants.

Even here the range in the literature can be quiet different.

For biomass the discussion is difficult. In most papers emissions are handled as 0, as all CO_2 contained in biomass material already was available in the environment before and would at least partially also be released without being burned. [22, p.18] Whereby the procurement, the transportation and the preparation are not included.

In the following table, the CO_2-emissions are refereed to the emissions occurred by burning the fuel.

type of power plant	CO_2-emissions in kg/MWh
hydro power plants	0
wind energy	0
biomass	0
nuclear power	0
solar energy	0
gas and steam	420
hard coal	930
brown coal	1200

Table 5.19.: Typical CO_2-emissions for different types of power plants [22, p.20]

In the former examples this would lead to emissions as listed in table 5.20 for the whole daily production.

scenario	tons per day	
scenario 1	651,495 tons	normal situation
scenario 2	147,655 tons	strong wind situation
scenario 3	353,709 tons	strong wind, without nuclear power plants
scenario 4	533,245 tons	heavily changing wind
scenario 5	570,080 tons	restricted transport capacity of wind power
scenario 6	491,339 tons	heavily changing wind and solar power

Table 5.20.: CO_2-emissions for the example scenarios

5.6.9. Differences in costs and emissions

A comparison of the differences in the production costs and the CO_2-emissions is listed in table 5.21.

scenario	cost difference in €	CO_2-emission difference	CO_2-reduction costs
scenario 2 - scenario 1	+21,940,722	-503,840 tons	€ 44/ton
scenario 3 - scenario 2	+339,589	+206,054 tons	
scenario 4 - scenario 3	-7,075,598	+179,536 tons	
scenario 5 - scenario 4	-1,556,473	+36,835 tons	
scenario 4 - scenario 6	+4,758,724	-41,906 tons	€ 114/ton

Table 5.21.: Comparison of CO_2-emissions and electrical production costs between the different scenarios

Scenario 1 (7 % wind energy, plus nuclear power) compared with scenario 2 (59 % wind energy, plus nuclear power) leads to higher costs, as the guaranteed compensation for electricity fed into the grid is taken into account for wind energy, while the thermal power plants can cover the load with lower costs. The CO_2-emissions are reduced dramatically. If the costs are set in relation to the CO_2, the price for reducing one ton of CO_2 is around € 44.

If the nuclear power plants are turned off as in scenario 3, the costs increase a bit more, as the expensive fossil fired power plants are used more. Additionally the CO_2-emissions increase, as brown coal and hard coal fired power plants are used.

If the wind is changing heavily as in scenario 4, the 19 % lower production of wind energy has to be covered by fossil fired power plants. So the CO_2-emissions are increasing while the costs are lower than in scenario 3.

If the transportation through the grid is restricted as in scenario 5, the wind energy used by the customer decreases to 37 %. The CO_2-emissions are increasing, while the costs are decreasing in this case.

If solar energy is added to scenario 4, resulting in a total of 40 % wind and 5 % solar energy, the electrical production costs are increasing even more, while the CO_2-emissions are decreasing slightly. One ton of CO_2-reduction costs € 114 in this scenario.

The costs shown in table 5.17 are of older date. Newer electric production costs are discussed in [36], a deeper discussion on electric production costs and CO_2 certificate costs can be found in [37].

5.7. Comparison of Linear Programming and Stochastic Dual Dynamic Programming

The comparison of linear programming and stochastic dual dynamic programming is based on scenarios with 21 thermal blocks.

Table 5.22 gives an overview of the number of variables and constraints which are related to test cases with a rising number of reservoirs, scenarios and stages. The additional epsilon-terms, used to relax constraints, are already taken into account in this number. As explained in 4.1.

The number of variables is increasing linear with the number of reservoirs, steeper linear with the number of stages and exponentially with the number of scenarios.

Table 5.23 shows the results including the number of iterations and the calculation time for optimizing the same scenario using SDDP and LP in each case.

reservoirs	scenarios	stages	variables	constraints
10	10	12	8103	14985
10	10	52	37303	68985
20	10	12	13653	24975
20	20	12	27183	49725
20	30	12	40713	74475
30	10	12	19203	34965
30	20	12	38233	69615
30	30	12	57263	104265
30	60	12	114353	208215
30	100	12	190473	346815
40	10	12	24753	44955
50	10	12	30303	54945
50	50	12	150423	272745
50	100	12	300573	544995
100	50	12	288173	520695

Table 5.22.: Number of variables and constraints depending on model size

reservoirs/scenarios/stages			LP		SDDP		
res.	scen.	stag.	solution	duration	solution	iterations	duration
10	10	12	1615105152	21 s	1643136256	9	50 s
10	10	52	2038861440	451 s	2155891968	1	31 s
20	10	12	1676020480	75 s	1721686272	11	121 s
20	20	12	1839321728	368 s	1854447104	6	131 s
20	30	12	1851897216	899 s	1868171392	11	362 s
30	10	12	1798478592	155 s	194315040	11	277 s
30	20	12	1753113472	737 s	1772224000	7	292 s
30	30	12	1759147904	1791 s	1831559168	11	676 s
30	60	12	*too big for LP*	-	1802390784	11	1609 s
30	100	12	*too big for LP*	-	18824858624	11	2687 s
40	10	12	1781228544	283 s	1917719296	11	365 s
50	10	12	1930445696	502 s	2148826624	11	535 s
50	50	12	*too big for LP*	-	2104971264	11	2741 s
50	100	12	*too big for LP*	-	2067765376	11	5598 s
100	50	12	*too big for LP*	-	3037334016	31	27877 s

Table 5.23.: Comparison of LP and SDDP algorithms for several test cases

6. Discussion and conclusions

Stochastic optimization compared to deterministic optimization involves much more effort as well for preparing the input data as for calculation time and memory requirements, as the size and the variability of the problems can rise significantly. A stochastic point of view was already important for optimizing the use of hydro power plants. However, the necessity to consider multiple stochastic scenarios will rise in future for the integration of wind and solar power sources.

By linear problem formulation usually the real optimum can be found for the deterministic as well as the stochastic case. However when using deterministic dual dynamic programming or stochastic dual dynamic programming a quicker solution close to the optimum can be reached. This difference occurs because of an approximation of the future cost function at each stage in the case of dynamic programming. This approximation affects the number of iterations. The definition of the stopping criteria allows to find a compromise between the number of iterations and the accuracy of the solution.

If the problem contains a small number of scenarios a LP formulation is faster than a SDDP formulation. With a rising number of scenarios and variables SDDP becomes faster than linear programming, because the problem is divided in multiple nested smaller subproblems, which can be solved faster, except for a slow convergence in some test cases.

The formulation of epsilons helps to find solutions faster. Such additional terms, relaxing the constraints, are used to find a feasible solution during the first iterations. Later their use is reduced due to the high costs associated with them. An additional aspect is, that these terms help to solve infeasible problems, which have to be checked manually afterwards.

Even more detailed problem formulations increase the number of variables and the complexity of the system, which then increases the time needed for the optimization or even makes it impossible to solve them using a linear formulation. Dynamic programming therefore is mandatory for complex problems.

The XA solver library allows an easy definition of the model by constructing arrays in a multi-dimensional form and transfer it to the optimizer step by step. Defined models can even be altered, reducing the effort when redefining models. Intermediate results can be saved and used in following stages and iterations.

Optimization of storage power plant scheduling in a mixed power plant portfolio is a valuable and effective tool to reduce the costs of power production. This was already important in pure hydro-thermal scenarios, but gains even more importance with the increasing use of wind and solar energy. As other plants have to compensate for fluctuations in load and energy being available, the optimal use of storage power plants is getting more important to reduce CO_2-emissions.

Future optimisation tools need to take into account the whole system, including the technical restrictions of the grid and the power plants, which are for example

- for the grid:
  - bottlenecks, voltage level, load flow
- for thermal power plants:
  - startup and shutdown times
  - downtimes and periods of use
  - efficiency, control techniques, technologies to reduce the CO_2-emissions

As during midday the solar energy can take a great part of covering the load, the storage power plants can be used to cover the load during other times. This means the use of storage power plants may change in the future. It can even happen that the production of energy from renewable sources exceeds the load. Because of regulatory demands this power plants are allowed to feed the whole production into the grid. So regulatory demands need to be discussed. Discussions about CO_2 certificate costs are also necessary. More flexible power plants or control techniques are needed to compensate the fluctuating production of renewable energy.

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A. Appendix

Inputs for test 1 to 11, test 5.3, test 5.4 and test 5.5

type of power plant	€ /MWh
Thermal Block 1	16
Thermal Block 2	25
Thermal Block 3	41
Thermal Block 4	75
Generation SPP & RPP	1
Wind Power Station	0.1

Table A.1.: Variable costs used for optimization

type of power plant	€ /MWh
Thermal Block 1	35
Thermal Block 2	47
Thermal Block 3	52
Thermal Block 4	104
Generation SPP & RPP	33
Wind Power Station	82

Table A.2.: Electrical production costs used to calculate the total costs in the system

Variable costs used for optimization in tests 5.22 until 5.27

	variable costs for optimisation € /MWh
wind power stations	0
solar power plants	0
run-of-river power plants	0
storage power plants	0
biomass power plants	0
nuclear power plants	7
brown coal power plants	16
hard coal power plants	25
gas and steam power plants	75
heating oil power plants	120

Table A.3.: Variable costs used for optimization

Optimized variable costs and electrical production costs

	optimized variable costs € per year	electrical production costs € per year
Test 1	122639985	301793568
Test 2	136839614	310738144
Test 3	122639986	301783568
Test 4	131548845	305566624
Test 5	130791096	304443936
Test 6	127435055	303057024
Test 7	127369558	303035168
Test 8	71757421	274641888
Test 9	80997443	285809664
Test 10	76534460	281667232
Test 11	39398848	306955698

Table A.4.: Variable costs and electrical production costs for each scenario

Test 1

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh
January	647383	148624	145286	145286	0	400000	247680	266400	237600
February	647383	148624	145286	145286	0	400000	247680	266400	237600
March	647383	148624	145286	145286	0	400000	247680	266400	237600
April	647383	148624	145286	145286	0	400000	247680	266400	237600
May	647383	148624	145286	145286	0	400000	247680	266400	237600
June	647383	148624	145286	145286	0	400000	247680	266400	237600
July	647383	148624	145286	145286	0	400000	247680	266400	237600
August	647383	148624	145286	145286	0	400000	247680	266400	237600
September	647383	148624	145286	145286	0	400000	247680	266400	237600
October	647383	148624	145286	145286	0	400000	247680	266400	237600
November	647383	148624	145286	145286	0	400000	247680	266400	237600
December	647383	148624	145286	145286	0	400000	247680	266400	237600
sum	7768596	1783488	1743432	1743432	0	4800000	2972160	3196800	2851200

Table A.5.: Input Test 1

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Generation SPP MWh	Storage MWh
January	266400	232359	148624	145286
February	266400	232359	148624	145286
March	266400	232359	148624	145286
April	266400	232359	148624	145286
May	266400	232359	148624	145286
June	266400	232359	148624	145286
July	266400	232359	148624	145286
August	266400	232359	148624	145286
September	266400	232359	148624	145286
October	266400	232359	148624	145286
November	266400	232359	148624	145286
December	266400	232359	148624	145286
sum	3196800	2788308	1783488	1743432

Table A.6.: Output Test 1

Test 2

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	847483	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
February	706236	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
March	729777	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
April	623841	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
May	541447	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
June	470824	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
July	494365	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
August	470824	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
September	506135	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
October	659153	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
November	800400	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
December	918106	148624	145286	145286	0	400000	247680	266400	237600	180000	180000
sum	7768591	1783488	1743432	1743432	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.7.: Input Test 2

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	180000	14859	148624	145286
February	266400	237600	53612	0	148624	145286
March	266400	237600	77153	0	148624	145286
April	266400	208817	0	0	148624	145286
May	266400	126423	0	0	148624	145286
June	266400	55800	0	0	148624	145286
July	266400	79341	0	0	148624	145286
August	266400	55800	0	0	148624	145286
September	266400	91111	0	0	148624	145286
October	266400	237600	6529	0	148624	145286
November	266400	237600	147776	0	148624	145286
December	266400	237600	180000	85482	148624	145286
sum	3196800	2042892	645070	100341	1783488	1743432

Table A.8.: Output Test 2

Test 3

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh
January	647383	148624	145286	290572	0	400000	247680	266400	237600
February	647383	148624	145286	290572	0	400000	247680	266400	237600
March	647383	148624	145286	290572	0	400000	247680	266400	237600
April	647383	148624	145286	290572	0	400000	247680	266400	237600
May	647383	148624	145286	290572	0	400000	247680	266400	237600
June	647383	148624	145286	290572	0	400000	247680	266400	237600
July	647383	148624	145286	290572	0	400000	247680	266400	237600
August	647383	148624	145286	290572	0	400000	247680	266400	237600
September	647383	148624	145286	290572	0	400000	247680	266400	237600
October	647383	148624	145286	290572	0	400000	247680	266400	237600
November	647383	148624	145286	290572	0	400000	247680	266400	237600
December	647383	148624	145286	290572	0	400000	247680	266400	237600
sum	7768596	1783488	1743432	3486864	0	4800000	2972160	3196800	2851200

Table A.9.: Input Test 3

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	143383	150527
February	266400	237600	143383	155768
March	266400	237600	143383	161009
April	266400	237600	143383	166250
May	266400	237600	143383	171491
June	266400	237600	143383	176732
July	266400	237600	143383	181973
August	266400	237600	143383	187214
September	266400	237600	143383	192455
October	266400	237600	143383	197696
November	266400	237600	143383	202937
December	266400	174708	206275	145286
sum	3196800	2788308	1783488	2089338

Table A.10.: Output Test 3

Test 4

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	847483	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
February	706236	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
March	729777	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
April	623841	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
May	541447	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
June	470824	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
July	494365	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
August	470824	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
September	506135	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
October	659153	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
November	800400	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
December	918106	148624	145286	290572	0	400000	247680	266400	237600	180000	180000
sum	7768591	1783488	1743432	3486864	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.11.: Input Test 4

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	180000	14859	148624	145286
February	266400	237600	130765	0	71471	222439
March	266400	237600	0	0	225777	145286
April	266400	208817	0	0	148624	145286
May	266400	126423	0	0	148624	145286
June	266400	201086	0	0	3338	290572
July	266400	79341	0	0	148624	290572
August	266400	55800	0	0	148624	290572
September	266400	91111	0	0	148624	290572
October	266400	237600	0	0	155153	284043
November	266400	237600	108075	0	188325	244342
December	266400	237600	166426	0	247680	145286
sum	3196800	2188178	585266	14859	1783488	2639542

Table A.12.: Output Test 4

Test 5

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	647383	61348	145286	145286	0	400000	247680	266400	237600	180000	180000
February	647383	54775	145286	145286	0	400000	247680	266400	237600	180000	180000
March	647383	61348	145286	145286	0	400000	247680	266400	237600	180000	180000
April	647383	158849	145286	145286	0	400000	247680	266400	237600	180000	180000
May	647383	237725	145286	145286	0	400000	247680	266400	237600	180000	180000
June	647383	297978	145286	145286	0	400000	247680	266400	237600	180000	180000
July	647383	230057	145286	145286	0	400000	247680	266400	237600	180000	180000
August	647383	186236	145286	145286	0	400000	247680	266400	237600	180000	180000
September	647383	201573	145286	145286	0	400000	247680	266400	237600	180000	180000
October	647383	115028	145286	145286	0	400000	247680	266400	237600	180000	180000
November	647383	92023	145286	145286	0	400000	247680	266400	237600	180000	180000
December	647383	86545	145286	145286	0	400000	247680	266400	237600	180000	180000
sum	7768596	1783485	1743432	1743432	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.13.: Input Test 5

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Generation SPP MWh	Spillage SPP MWh	Storage MWh
January	266400	237600	82035	61348	0	145286
February	266400	237600	88608	54775	0	145286
March	266400	237600	82035	61348	0	145286
April	266400	222134	0	158849	0	145286
May	266400	143258	0	237725	0	145286
June	266400	133303	0	247680	50298	145286
July	266400	150926	0	230057	0	145286
August	266400	194747	0	186236	0	145286
September	266400	179410	0	201573	0	145286
October	266400	237600	28355	115028	0	145286
November	266400	237600	51360	92023	0	145286
December	266400	237600	56838	86545	0	145286
sum	3196800	2449378	389231	1733187	50298	1743432

Table A.14.: Output Test 5

Test 6

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	647383	61348	145286	290572	0	400000	247680	266400	237600	180000	180000
February	647383	54775	145286	290572	0	400000	247680	266400	237600	180000	180000
March	647383	61348	145286	290572	0	400000	247680	266400	237600	180000	180000
April	647383	158849	145286	290572	0	400000	247680	266400	237600	180000	180000
May	647383	237725	145286	290572	0	400000	247680	266400	237600	180000	180000
June	647383	297978	145286	290572	0	400000	247680	266400	237600	180000	180000
July	647383	230057	145286	290572	0	400000	247680	266400	237600	180000	180000
August	647383	186236	145286	290572	0	400000	247680	266400	237600	180000	180000
September	647383	201573	145286	290572	0	400000	247680	266400	237600	180000	180000
October	647383	115028	145286	290572	0	400000	247680	266400	237600	180000	180000
November	647383	92023	145286	290572	0	400000	247680	266400	237600	180000	180000
December	647383	86545	145286	290572	0	400000	247680	266400	237600	180000	180000
sum	7768596	1783485	1743432	3486864	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.15.: Input Test 6

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	143383	0	206634
February	266400	237600	109295	34088	227321
March	266400	237600	0	143383	145286
April	266400	222134	0	158849	145286
May	266400	151572	0	229411	153600
June	266400	133303	0	247680	203898
July	266400	237600	0	143383	290572
August	266400	136557	0	244426	232382
September	266400	237600	0	143383	290572
October	266400	237600	0	143383	262217
November	266400	237600	0	143383	210857
December	266400	228867	0	152116	145286
sum	3196800	2535633	252678	1783485	2513911

Table A.16.: Output Test 6

Test 7

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	647383	61348	145286	290572	0	400000	247680	266400	237600	180000	180000
February	647383	54775	145286	290572	0	400000	247680	266400	237600	180000	180000
March	647383	61348	145286	290572	0	400000	247680	266400	237600	180000	180000
April	647383	158849	145286	290572	0	400000	247680	266400	237600	180000	180000
May	647383	237725	145286	290572	0	400000	247680	266400	237600	180000	180000
June	647383	297978	145286	290572	0	400000	247680	266400	237600	180000	180000
July	647383	230057	145286	290572	0	400000	247680	266400	237600	180000	180000
August	647383	186236	145286	290572	0	400000	247680	266400	237600	180000	180000
September	647383	201573	145286	290572	0	400000	247680	266400	237600	180000	180000
October	647383	115028	145286	290572	0	400000	247680	266400	237600	180000	180000
November	647383	92023	145286	290572	0	400000	247680	266400	237600	180000	180000
December	647383	86545	145286	290572	0	400000	247680	266400	237600	180000	180000
January	647383	61348	145286	290572	0	400000	247680	266400	237600	180000	180000
February	647383	54775	145286	290572	0	400000	247680	266400	237600	180000	180000
March	647383	61348	145286	290572	0	400000	247680	266400	237600	180000	180000
April	647383	158849	145286	290572	0	400000	247680	266400	237600	180000	180000
May	647383	237725	145286	290572	0	400000	247680	266400	237600	180000	180000
June	647383	297978	145286	290572	0	400000	247680	266400	237600	180000	180000
July	647383	230057	145286	290572	0	400000	247680	266400	237600	180000	180000
August	647383	186236	145286	290572	0	400000	247680	266400	237600	180000	180000
September	647383	201573	145286	290572	0	400000	247680	266400	237600	180000	180000
October	647383	115028	145286	290572	0	400000	247680	266400	237600	180000	180000
November	647383	92023	145286	290572	0	400000	247680	266400	237600	180000	180000
December	647383	86545	145286	290572	0	400000	247680	266400	237600	180000	180000
sum	15537192	3566970	3486864	6973728	0	9600000	5944320	6393600	5702400	4320000	4320000

Table A.17.: Input Test 7

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	143383	0	206634
February	266400	237600	109295	34088	227321
March	266400	237600	0	143383	145286
April	266400	222134	0	158849	145286
May	266400	151572	0	229411	153600
June	266400	133303	0	247680	203898
July	266400	237600	0	143383	290572
August	266400	194747	0	186236	290572
September	266400	179410	0	201573	290572
October	266400	237600	0	143383	262217
November	266400	237600	0	143383	210857
December	266400	237600	136553	6830	290572
January	266400	237600	82035	61348	290572
February	266400	237600	25357	118026	227321
March	266400	237600	0	143383	145286
April	266400	222134	0	158849	145286
May	266400	151572	0	229411	153600
June	266400	133303	0	247680	203898
July	266400	237600	0	143383	290572
August	266400	136557	0	244426	232382
September	266400	237600	0	143383	290572
October	266400	237600	0	143383	262217
November	266400	237600	0	143383	210857
December	266400	228867	0	152116	145286
sum	6393600	5079999	496623	3566970	5315236

Table A.18.: Output Test 7

Test 8

month	load MWh	inflow SPP MWh	storage LB MWh	storage UB MWh	max.gen.SPP MWh	inflow RPP MWh	mx.gen.RPP	min.spill. MWh	max.spill. MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	647383	61348	145286	290572	247680	92527	306000	0	400000	266400	237600	180000	180000
February	647383	54775	145286	290572	247680	100938	306000	0	400000	266400	237600	180000	180000
March	647383	61348	145286	290572	247680	233187	306000	0	400000	266400	237600	180000	180000
April	647383	158849	145286	290572	247680	283002	306000	0	400000	266400	237600	180000	180000
May	647383	237725	145286	290572	247680	248421	306000	0	400000	266400	237600	180000	180000
June	647383	297978	145286	290572	247680	306180	306000	0	400000	266400	237600	180000	180000
July	647383	230057	145286	290572	247680	281973	306000	0	400000	266400	237600	180000	180000
August	647383	186236	145286	290572	247680	184493	306000	0	400000	266400	237600	180000	180000
September	647383	201573	145286	290572	247680	147576	306000	0	400000	266400	237600	180000	180000
October	647383	115028	145286	290572	247680	127762	306000	0	400000	266400	237600	180000	180000
November	647383	92023	145286	290572	247680	123930	306000	0	400000	266400	237600	180000	180000
December	647383	86545	145286	290572	247680	130846	306000	0	400000	266400	237600	180000	180000
sum	7768596	1783485	1743432	3486864	2972160	2260835	3672000	0	4800000	3196800	2851200	2160000	2160000

Table A.19.: Input Test 8

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Generation SPP MWh	Storage MWh	Generation RPP MWh	Spillage RPP MWh
January	266400	237600	50856	155778	92527	0
February	266400	237600	42445	168108	100938	0
March	266400	63626	84170	145286	233187	0
April	205532	0	158849	145286	283002	0
May	161237	0	237725	145286	248421	0
June	93703	0	247680	195584	306000	180
July	230341	0	135069	290572	281973	0
August	266400	0	196490	280318	184493	0
September	266400	42088	191319	290572	147576	0
October	266400	65623	187598	218002	127762	0
November	266400	237600	19453	290572	123930	0
December	266400	18306	231831	145286	130846	0
sum	2822013	902443	1783485	2470650	2260655	180

Table A.20.: Output Test 8

Test 9

month	load MWh	inflow SPP MWh	storage LB MWh	storage UB MWh	max.gen.SPP MWh	inflow RPP MWh	mx.gen.RPP	min.spill. MWh	max.spill. MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	847483	61348	145286	290572	247680	92527	306000	0	400000	266400	237600	180000	180000
February	706236	54775	145286	290572	247680	100938	306000	0	400000	266400	237600	180000	180000
March	729777	61348	145286	290572	247680	233187	306000	0	400000	266400	237600	180000	180000
April	623841	158849	145286	290572	247680	283002	306000	0	400000	266400	237600	180000	180000
May	541447	237725	145286	290572	247680	248421	306000	0	400000	266400	237600	180000	180000
June	470824	297978	145286	290572	247680	306180	306000	0	400000	266400	237600	180000	180000
July	494365	230057	145286	290572	247680	281973	306000	0	400000	266400	237600	180000	180000
August	470824	186236	145286	290572	247680	184493	306000	0	400000	266400	237600	180000	180000
September	506135	201573	145286	290572	247680	147576	306000	0	400000	266400	237600	180000	180000
October	659153	115028	145286	290572	247680	127762	306000	0	400000	266400	237600	180000	180000
November	800400	92023	145286	290572	247680	123930	306000	0	400000	266400	237600	180000	180000
December	918106	86545	145286	290572	247680	130846	306000	0	400000	266400	237600	180000	180000
sum	7768591	1783485	1743432	3486864	2972160	2260835	3672000	0	4800000	3196800	2851200	2160000	2160000

Table A.21.: Input Test 9

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Spillage SPP MWh	Storage MWh	Generation RPP MWh	Spillage RPP MWh
January	266400	237600	180000	9608	61348	0	145286	92527	0
February	266400	237600	46523	0	54775	0	145286	100938	0
March	266400	168842	0	0	61348	0	145286	233187	0
April	181990	0	0	0	158849	0	145286	283002	0
May	55301	0	0	0	237725	0	145286	248421	0
June	0	0	0	0	164824	5533	272907	306000	180
July	0	0	0	0	212392	0	290572	281973	0
August	38651	0	0	0	247680	0	229128	184493	0
September	218430	0	0	0	140129	0	290572	147576	0
October	266400	149963	0	0	115028	0	290572	127762	0
November	266400	237600	80447	0	92023	0	290572	123930	0
December	266400	237600	51429	0	231831	0	145286	130846	0
sum	2092772	1269205	358399	9608	1777952	5533	2536039	2260655	180

Table A.22.: Output Test 9

Test 10

month	load MWh	inflow SPP MWh	storage LB MWh	storage UB MWh	max.gen.SPP MWh	inflow RPP MWh	mx.gen.RPP	min.spill. MWh	max.spill. MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	847483	61348	145286	2905720	247680	92527	306000	0	400000	266400	237600	180000	180000
February	706236	54775	145286	2905720	247680	100938	306000	0	400000	266400	237600	180000	180000
March	729777	61348	145286	2905720	247680	233187	306000	0	400000	266400	237600	180000	180000
April	623841	158849	145286	2905720	247680	283002	306000	0	400000	266400	237600	180000	180000
May	541447	237725	145286	2905720	247680	248421	306000	0	400000	266400	237600	180000	180000
June	470824	297978	145286	2905720	247680	306180	306000	0	400000	266400	237600	180000	180000
July	494365	230057	145286	2905720	247680	281973	306000	0	400000	266400	237600	180000	180000
August	470824	186236	145286	2905720	247680	184493	306000	0	400000	266400	237600	180000	180000
September	506135	201573	145286	2905720	247680	147576	306000	0	400000	266400	237600	180000	180000
October	659153	115028	145286	2905720	247680	127762	306000	0	400000	266400	237600	180000	180000
November	800400	92023	145286	2905720	247680	123930	306000	0	400000	266400	237600	180000	180000
December	918106	86545	145286	2905720	247680	130846	306000	0	400000	266400	237600	180000	180000
sum	7768591	1783485	1743432	34868640	2972160	2260835	3672000	0	4800000	3196800	2851200	2160000	2160000

Table A.23.: Input Test 10

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh	Generation RPP MWh	Spillage RPP MWh
January	266400	237600	180000	9608	61348	145286	92527	0
February	266400	237600	46523	0	54775	145286	100938	0
March	266400	168842	0	0	61348	145286	233187	0
April	181990	0	0	0	158849	145286	283002	0
May	55301	0	0	0	237725	145286	248421	0
June	0	0	0	0	164824	278440	306000	180
July	22906	0	0	0	189486	319011	281973	0
August	266400	0	0	0	19931	485316	184493	0
September	266400	0	0	0	92159	594730	147576	0
October	266400	17311	0	0	247680	462078	127762	0
November	266400	162390	0	0	247680	306421	123930	0
December	266400	237600	35580	0	247680	145286	130846	0
sum	2391397	1061343	262103	9608	1783485	3317712	2260655	180

Table A.24.: Output Test 10

Test 11

month	load MWh	inflow SPP MWh	storage LB MWh	storage UB MWh	max.gen.SPP MWh	inflow RPP MWh	mx.gen.RPP	min.spill. MWh	max.spill. MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh	wind strength MWh	max.wind.gen
January	847483	61348	145286	2905720	247680	92527	306000	0	400000	266400	237600	180000	180000	197465	300000
February	706236	54775	145286	2905720	247680	100938	306000	0	400000	266400	237600	180000	180000	224357	300000
March	729777	61348	145286	2905720	247680	233187	306000	0	400000	266400	237600	180000	180000	150302	300000
April	623841	158849	145286	2905720	247680	283002	306000	0	400000	266400	237600	180000	180000	210625	300000
May	541447	237725	145286	2905720	247680	248421	306000	0	400000	266400	237600	180000	180000	161295	300000
June	470824	297978	145286	2905720	247680	306180	306000	0	400000	266400	237600	180000	180000	238674	300000
July	494365	230057	145286	2905720	247680	281973	306000	0	400000	266400	237600	180000	180000	150302	300000
August	470824	186236	145286	2905720	247680	184493	306000	0	400000	266400	237600	180000	180000	79736	300000
September	506135	201573	145286	2905720	247680	147576	306000	0	400000	266400	237600	180000	180000	102760	300000
October	659153	115028	145286	2905720	247680	127762	306000	0	400000	266400	237600	180000	180000	161295	300000
November	800400	92023	145286	2905720	247680	123930	306000	0	400000	266400	237600	180000	180000	86980	300000
December	918106	86545	145286	2905720	247680	130846	306000	0	400000	266400	237600	180000	180000	172812	300000
sum	7768591	1783485	1743432	34868640	2972160	2260835	3672000	0	4800000	3196800	2851200	2160000	2160000	1936603	3600000

Table A.25.: Input Test 11

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Generation WPP MWh	Generation SPP MWh	Storage MWh	Generation RPP MWh	Spillage RPP MWh
January	266400	237600	197465	53491	153143	92527	0
February	266400	70449	224357	44092	163826	100938	0
March	266400	0	150302	79888	145286	233187	0
April	0	0	210625	130214	173921	283002	0
May	0	0	161295	131731	279915	248421	0
June	0	0	238674	0	577893	232150	74030
July	0	0	150302	62090	745860	281973	0
August	0	0	79736	206595	725501	184493	0
September	8119	0	102760	247680	679394	147576	0
October	122416	0	161295	247680	546742	127762	0
November	266400	75410	86980	247680	391085	123930	0
December	266400	100368	172812	247680	229950	130846	0
sum	1462535	483827	1936603	1698821	4812516	2186805	74030

Table A.26.: Output Test 11

Test 5.3

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	61348	145286	484286	174344	0	400000	247680	266400	237600	180000	180000
February	766236	64775	145286	484286		0	400000	247680	266400	237600	180000	180000
March	759777	61348	145286	484286		0	400000	247680	266400	237600	180000	180000
April	623841	158849	145286	484286		0	400000	247680	266400	237600	180000	180000
May	541447	237725	145286	484286		0	400000	247680	266400	237600	180000	180000
June	470824	297978	145286	484286		0	400000	247680	266400	237600	180000	180000
July	494365	230057	145286	484286		0	400000	247680	266400	237600	180000	180000
August	470824	186236	145286	484286		0	400000	247680	266400	237600	180000	180000
September	506135	201573	145286	484286		0	400000	247680	266400	237600	180000	180000
October	659153	115028	145286	484286		0	400000	247680	266400	237600	180000	180000
November	800400	92023	145286	484286		0	400000	247680	266400	237600	180000	180000
December	918106	86545	145286	484286		0	400000	247680	266400	237600	180000	180000
sum	7758591	1793485	1743432	5811432	174344	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.27.: Input for test 5.3

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	4967	58516
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	0	75777
April	623841	158849	266400	204038	0	0	153403
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	204424	0	0	0
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	0	0	0	239735
October	659153	115028	266400	145073	0	0	247680
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1979135	755146	4967	1822543

Table A.28.: Optimization results for test 5.3, scenario 12

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	4967	70810
April	623841	158849	266400	237600	0	0	119841
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	0	0	0	204424
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	78334	0	0	161401
October	659153	115028	266400	237600	0	0	155153
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166427	0	247678
sum	7758591	1793485	3196800	1979134	755147	4967	1822543

Table A.29.: Optimization results for test 5.3, scenario 13

Tests 5.4

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	61348	145286	484286	174344	0	400000	247680	266400	237600	180000	180000
February	766236	64775	145286	484286		0	400000	247680	266400	237600	180000	180000
March	759777	61348	145286	484286		0	400000	247680	266400	237600	180000	180000
April	623841	158849	145286	484286		0	400000	247680	266400	237600	180000	180000
May	541447	237725	145286	484286		0	400000	247680	266400	237600	180000	180000
June	470824	297978	145286	484286		0	400000	247680	266400	237600	180000	180000
July	494365	230057	145286	484286		0	400000	247680	266400	237600	180000	180000
August	470824	186236	145286	484286		0	400000	247680	266400	237600	180000	180000
September	506135	201573	145286	484286		0	400000	247680	266400	237600	180000	180000
October	659153	115028	145286	484286		0	400000	247680	266400	237600	180000	180000
November	800400	92023	145286	484286		0	400000	247680	266400	237600	180000	180000
December	918106	86545	145286	484286		0	400000	247680	266400	237600	180000	180000
sum	7758591	1793485	1743432	5811432	174344	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.30.: Input for test 5.4, scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	61348	145286	484286	174344	0	400000	247680	266400	237600	180000	180000
February	833345	45352	145286	484286		0	400000	247680	266400	237600	180000	180000
March	844352	52952	145286	484286		0	400000	247680	266400	237600	180000	180000
April	652934	95801	145286	484286		0	400000	247680	266400	237600	180000	180000
May	556932	213282	145286	484286		0	400000	247680	266400	237600	180000	180000
June	504325	235895	145286	484286		0	400000	247680	266400	237600	180000	180000
July	512304	213282	145286	484286		0	400000	247680	266400	237600	180000	180000
August	464532	230638	145286	484286		0	400000	247680	266400	237600	180000	180000
September	546774	210125	145286	484286		0	400000	247680	266400	237600	180000	180000
October	734234	123189	145286	484286		0	400000	247680	266400	237600	180000	180000
November	836454	99674	145286	484286		0	400000	247680	266400	237600	180000	180000
December	934546	53997	145286	484286		0	400000	247680	266400	237600	180000	180000
sum	8168215	1635535	1743432	5811432	174344	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.31.: Input for test 5.4, scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	61348	145286	484286	174344	0	400000	247680	266400	237600	180000	180000
February	742345	75571	145286	484286		0	400000	247680	266400	237600	180000	180000
March	833535	59311	145286	484286		0	400000	247680	266400	237600	180000	180000
April	612435	159385	145286	484286		0	400000	247680	266400	237600	180000	180000
May	545824	153889	145286	484286		0	400000	247680	266400	237600	180000	180000
June	464584	228086	145286	484286		0	400000	247680	266400	237600	180000	180000
July	484952	239078	145286	484286		0	400000	247680	266400	237600	180000	180000
August	454694	234956	145286	484286		0	400000	247680	266400	237600	180000	180000
September	495455	303656	145286	484286		0	400000	247680	266400	237600	180000	180000
October	723594	200373	145286	484286		0	400000	247680	266400	237600	180000	180000
November	812349	129688	145286	484286		0	400000	247680	266400	237600	180000	180000
December	977106	99767	145286	484286		0	400000	247680	266400	237600	180000	180000
sum	7894356	1945108	1743432	5811432	174344	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.32.: Input for test 5.4, scenario 3 (higher load, higher inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	4967	58516
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	0	75777
April	623841	158849	266400	204038	0	0	153403
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	204424	0	0	0
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	0	0	0	239735
October	659153	115028	266400	145073	0	0	247680
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1979135	755146	4967	1822543

Table A.33.: Deterministic optimization for scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	8168215	61348	266400	237600	180000	63483	0
February	7420732	45352	266400	237600	180000	120987	28358
March	6587387	52952	266400	237600	180000	0	160352
April	5743035	95801	266400	237600	53133	0	95801
May	5090101	213282	266400	79280	0	0	211252
June	4533169	235895	266400	0	0	0	237925
July	4028844	213282	266400	100813	0	0	145091
August	3516540	230638	266400	198132	0	0	0
September	3052008	210125	266400	237600	0	0	42774
October	2505234	123189	266400	220154	0	0	247680
November	1771000	99674	266400	237600	84774	0	247680
December	934546	53997	266400	237600	180000	2866	247680
sum	8168215	1635535	3196800	2261579	857907	187336	1664593

Table A.34.: Deterministic optimization for scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	46075	17408
February	742345	75571	266400	237600	180000	0	58345
March	833535	59311	266400	237600	180000	0	149535
April	612435	159385	266400	237600	0	0	108435
May	545824	153889	266400	188020	0	0	91404
June	464584	228086	266400	0	0	0	198184
July	484952	239078	266400	0	0	0	218552
August	454694	234956	266400	0	0	0	188294
September	495455	303656	266400	0	0	0	229055
October	723594	200373	266400	237600	0	0	219594
November	812349	129688	266400	237600	60669	0	247680
December	977106	99767	266400	237600	180000	45426	247680
sum	7894356	1945108	3196800	1851220	780669	91501	1974166

Table A.35.: Deterministic optimization for scenario 3 (higher load, higher inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	766236	64775	266400	237600	180000	4967	77269
March	759777	61348	266400	237600	180000	0	75777
April	623841	158849	266400	204038	0	0	153403
May	541447	237725	266400	237600	0	0	37447
June	470824	297978	266400	204424	0	0	0
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	0	0	0	204424
September	506135	201573	266400	0	0	0	239735
October	659153	115028	266400	145073	0	0	247680
November	800400	92023	266400	237600	48720	0	247680
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1979135	755146	4967	1822543

Table A.36.: Stochastic optimization for scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	833345	45352	266400	237600	180000	149345	0
March	844352	52952	266400	237600	180000	35125	125227
April	652934	95801	266400	237600	53133	0	95801
May	556932	213282	266400	77250	0	0	213282
June	504325	235895	266400	70149	0	0	167776
July	512304	213282	266400	237600	0	0	8304
August	464532	230638	266400	198132	0	0	0
September	546774	210125	266400	32694	0	0	247680
October	734234	123189	266400	220154	0	0	247680
November	836454	99674	266400	237600	84774	0	247680
December	934546	53997	266400	237600	180000	2866	247680
sum	8168215	1635535	3196800	2261579	857907	187336	1664593

Table A.37.: Stochastic optimization for scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	742345	75571	266400	237600	180000	46075	12270
March	833535	59311	266400	237600	180000	0	149535
April	612435	159385	266400	186650	0	0	159385
May	545824	153889	266400	125535	0	0	153889
June	464584	228086	266400	141521	0	0	56663
July	484952	239078	266400	0	0	0	218552
August	454694	234956	266400	0	0	0	188294
September	495455	303656	266400	0	0	0	229055
October	723594	200373	266400	209514	0	0	247680
November	812349	129688	266400	237600	60669	0	247680
December	977106	99767	266400	237600	180000	45426	247680
sum	7894356	1945108	3196800	1851220	780669	91501	1974166

Table A.38.: Stochastic optimization for scenario 3 (higher load, higher inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	766236	64775	266400	237600	180000	0	82236
March	759777	61348	266400	237600	180000	4967	70810
April	623841	158849	266400	237600	0	0	119841
May	541447	237725	266400	27367	0	0	247680
June	470824	297978	266400	0	0	0	204424
July	494365	230057	266400	0	0	0	227965
August	470824	186236	266400	168437	0	0	35987
September	506135	201573	266400	35363	0	0	204372
October	659153	115028	266400	237600	12200	0	142953
November	800400	92023	266400	237600	124415	0	171985
December	918106	86545	266400	237600	166426	0	247680
sum	7758591	1793485	3196800	1894367	843042	4967	1819415

Table A.39.: SDDP optimization for scenario 1 (average load, average inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	833345	45352	266400	237600	180000	77070	72275
March	844352	52952	266400	237600	180000	107400	52952
April	652934	95801	266400	237600	53133	0	95801
May	556932	213282	266400	77250	0	0	213282
June	504325	235895	266400	2030	0	0	235895
July	512304	213282	266400	32622	0	0	213282
August	464532	230638	266400	198132	0	0	0
September	546774	210125	266400	111760	0	0	168614
October	734234	123189	266400	237600	79120	0	151114
November	836454	99674	266400	237600	152818	0	179636
December	934546	53997	266400	237600	180000	32286	218260
sum	8168215	1635535	3196800	2084994	1005072	216756	1664593

Table A.40.: SDDP optimization for scenario 2 (higher load, lower inflow)

month	load MWh	inflow MWh	thermal 1 MWh	thermal 2 MWh	thermal 3 MWh	thermal 4 MWh	storage power plant MWh
January	747483	61348	266400	237600	180000	0	63483
February	742345	75571	266400	237600	180000	0	58345
March	833535	59311	266400	237600	180000	46075	103460
April	612435	159385	266400	237600	0	0	108435
May	545824	153889	266400	74585	0	0	204839
June	464584	228086	266400	0	0	0	198184
July	484952	239078	266400	0	0	0	218552
August	454694	234956	266400	177858	0	0	10436
September	495455	303656	247775	0	0	0	247680
October	723594	200373	266400	209514	0	0	247680
November	812349	129688	266400	237600	60669	0	247680
December	977106	99767	266400	237600	180000	45426	247680
sum	7894356	1945108	3178175	1887557	780669	91501	1956454

Table A.41.: SDDP optimization for scenario 3 (higher load, higher inflow)

Test 5.5

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	53346	145286	290572	145286	0	400000	247680	266400	237600	180000	180000
February	786236	67631	145286	290572		0	400000	247680	266400	237600	180000	180000
March	729777	53346	145286	290572		0	400000	247680	266400	237600	180000	180000
April	623842	138129	145286	290572		0	400000	247680	266400	237600	180000	180000
May	541448	206718	145286	290572		0	400000	247680	266400	237600	180000	180000
June	470824	259111	145286	290572		0	400000	247680	266400	237600	180000	180000
July	494365	200049	145286	290572		0	400000	247680	266400	237600	180000	180000
August	470824	161945	145286	290572		0	400000	247680	266400	237600	180000	180000
September	506136	175281	145286	290572		0	400000	247680	266400	237600	180000	180000
October	659154	100025	145286	290572		0	400000	247680	266400	237600	180000	180000
November	800401	80020	145286	290572		0	400000	247680	266400	237600	180000	180000
December	918107	75257	145286	290572		0	400000	247680	266400	237600	180000	180000
sum	7748597	1570858	1743432	3486864	145286	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.42.: Input - normal year

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	53346	145286	290572	145286	0	400000	247680	266400	237600	180000	180000
February	786236	43825	145286	290572		0	400000	247680	266400	237600	180000	180000
March	729777	62399	145286	290572		0	400000	247680	266400	237600	180000	180000
April	623842	104788	145286	290572		0	400000	247680	266400	237600	180000	180000
May	541448	167660	145286	290572		0	400000	247680	266400	237600	180000	180000
June	470824	170518	145286	290572		0	400000	247680	266400	237600	180000	180000
July	494365	167660	145286	290572		0	400000	247680	266400	237600	180000	180000
August	470824	142892	145286	290572		0	400000	247680	266400	237600	180000	180000
September	506136	87641	145286	290572		0	400000	247680	266400	237600	180000	180000
October	659154	101930	145286	290572		0	400000	247680	266400	237600	180000	180000
November	800401	76209	145286	290572		0	400000	247680	266400	237600	180000	180000
December	918107	59062	145286	290572		0	400000	247680	266400	237600	180000	180000
sum	7748597	1237930	1743432	3486864	145286	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.43.: Input - dry year

month	load MWh	inflow MWh	storage LB MWh	storage UB MWh	initial storage MWh	min.spill. MWh	max.spill. MWh	max.gen.SPP MWh	max.th.gen.1 MWh	max.th.gen.2 MWh	max.th.gen.3 MWh	max.th.gen.4 MWh
January	747483	53346	145286	290572	145286	0	400000	247680	266400	237600	180000	180000
February	786236	84783	145286	290572		0	400000	247680	266400	237600	180000	180000
March	729777	71446	145286	290572		0	400000	247680	266400	237600	180000	180000
April	623842	100025	145286	290572		0	400000	247680	266400	237600	180000	180000
May	541448	230528	145286	290572		0	400000	247680	266400	237600	180000	180000
June	470824	245291	145286	290572		0	400000	247680	266400	237600	180000	180000
July	494365	284832	145286	290572		0	400000	247680	266400	237600	180000	180000
August	470824	274353	145286	290572		0	400000	247680	266400	237600	180000	180000
September	506136	189570	145286	290572		0	400000	247680	266400	237600	180000	180000
October	659154	173376	145286	290572		0	400000	247680	266400	237600	180000	180000
November	800401	84783	145286	290572		0	400000	247680	266400	237600	180000	180000
December	918107	74304	145286	290572		0	400000	247680	266400	237600	180000	180000
sum	7748597	1866637	1743432	3486864	145286	0	4800000	2972160	3196800	2851200	2160000	2160000

Table A.44.: Input - wet year

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	200000	0	43483	155149
February	266400	237600	200000	4742	77494	145286
March	266400	237600	172431	0	53346	145286
April	266400	237600	0	0	119842	163573
May	266400	140642	0	0	134406	235885
June	266400	0	0	0	204424	290572
July	266400	27916	0	0	200049	290572
August	266400	42479	0	0	161945	290572
September	266400	64455	0	0	175281	290572
October	266400	237600	55129	0	100025	290572
November	266400	237600	200000	9945	86456	284136
December	266400	237600	200000	0	214107	145286
sum	3196800	1938692	1027560	14687	1570858	2727461

Table A.45.: Output - stochastic optimization, normal year

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	200000	0	43483	155149
February	266400	237600	200000	28548	53688	145286
March	266400	237600	178432	0	47345	160340
April	266400	237600	0	0	119842	145286
May	266400	141294	0	0	133754	179192
June	266400	0	0	0	204424	145286
July	266400	60305	0	0	167660	145286
August	266400	121313	0	0	83111	205067
September	266400	237600	0	0	2136	290572
October	266400	237600	53224	0	101930	290572
November	266400	237600	200000	20192	76209	290572
December	266400	237600	200000	9759	204348	145286
sum	3196800	2223712	1031656	58499	1237930	2297894

Table A.46.: Output - stochastic optimization, dry year

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	200000	0	43483	155149
February	266400	237600	200000	0	82236	157696
March	266400	237600	161738	0	64039	165103
April	266400	237600	0	0	119842	145286
May	266400	44520	0	0	230528	145286
June	244023	0	0	0	226801	163776
July	266400	0	0	0	227965	220643
August	266400	0	0	0	204424	290572
September	266400	50166	0	0	189570	290572
October	266400	219378	0	0	173376	290572
November	266400	237600	200000	6135	90266	285089
December	266400	237600	200000	0	214107	145286
sum	3174423	1739664	961738	6135	1866637	2455030

Table A.47.: Output - stochastic optimization, wet year

month	Thermal Block 1 MWh	Thermal Block 2 MWh	Thermal Block 3 MWh	Thermal Block 4 MWh	Generation SPP MWh	Storage MWh
January	266400	237600	200000	4742	38741	159891
February	266400	237600	200000	0	82236	145286
March	266400	237600	172431	0	53346	145286
April	266400	237600	0	0	119842	163573
May	266400	140642	0	0	134406	235885
June	266400	0	0	0	204424	290572
July	266400	0	0	0	227965	262656
August	266400	70395	0	0	134029	290572
September	266400	64455	0	0	175281	290572
October	266400	237600	55129	0	100025	290572
November	266400	237600	200000	9945	86456	284136
December	266400	237600	200000	0	214107	145286
sum	3196800	1938692	1027560	14687	1570858	2704287

Table A.48.: Output - deterministic optimization, normal year

Test 5.22

	load	biomass PP max.gen.	nuclear power PP max.gen.	brown coal PP max.gen.	hard coal PP max.gen.	gas and steam PP max.gen.	heating oil PP max.gen.	run-of-river PP max.gen.	run-of-river PP inflow	wind PP max.gen.	wind PP wind strength	storage PP max.gen.	storage PP volume
	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh
00:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	1750	3327	40480
00:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	1825	3327	
01:00	18646	1210	4730	5900	7000	5680	1510	1300	1165	14500	1900	3327	
01:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	1950	3327	
02:00	17798	1210	4730	5900	7000	5680	1510	1300	1165	14500	1925	3327	
02:30	17516	1210	4730	5900	7000	5680	1510	1300	1165	14500	1925	3327	
03:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	1875	3327	
03:30	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	1825	3327	
04:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	1775	3327	
04:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	1725	3327	
05:00	18928	1210	4730	5900	7000	5680	1510	1300	1165	14500	1750	3327	
05:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	1575	3327	
06:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	1550	3327	
06:30	20906	1210	4730	5900	7000	5680	1510	1300	1165	14500	1525	3327	
07:00	22036	1210	4730	5900	7000	5680	1510	1300	1165	14500	1475	3327	
07:30	23166	1210	4730	5900	7000	5680	1510	1300	1165	14500	1375	3327	
08:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	1200	3327	
08:30	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	1025	3327	
09:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	1100	3327	
09:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	1175	3327	
10:00	27969	1210	4730	5900	7000	5680	1510	1300	1165	14500	1275	3327	
10:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	1450	3327	
11:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	1550	3327	
11:30	28534	1210	4730	5900	7000	5680	1510	1300	1165	14500	1750	3327	
12:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	1775	3327	
12:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	1850	3327	
13:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	1925	3327	
13:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	1975	3327	
14:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	2125	3327	
14:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	2200	3327	
15:00	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	2375	3327	
15:30	25991	1210	4730	5900	7000	5680	1510	1300	1165	14500	2250	3327	
16:00	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	2300	3327	
16:30	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	2200	3327	
17:00	24296	1210	4730	5900	7000	5680	1510	1300	1165	14500	2150	3327	
17:30	24578	1210	4730	5900	7000	5680	1510	1300	1165	14500	2050	3327	
18:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	1800	3327	
18:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	1550	3327	
19:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	1200	3327	
19:30	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	875	3327	
20:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	725	3327	
20:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	600	3327	
21:00	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	600	3327	
21:30	23731	1210	4730	5900	7000	5680	1510	1300	1165	14500	725	3327	
22:00	23448	1210	4730	5900	7000	5680	1510	1300	1165	14500	925	3327	
22:30	22318	1210	4730	5900	7000	5680	1510	1300	1165	14500	1000	3327	
23:00	21188	1210	4730	5900	7000	5680	1510	1300	1165	14500	1100	3327	
23:30	20623	1210	4730	5900	7000	5680	1510	1300	1165	14500	1250	3327	
sum	1130324	58080	227040	283200	336000	272640	72480	62400	55920	696000	75775	159696	40480

Table A.49.: Input - scenario 1

	run-on-river MWh	wind MWh	biomass MWh	nuclear power MWh	brown coal MWh	hard coal MWh	gas and steam MWh	heating oil MWh	storage power plant MWh
00:00	1165	1750	1210	4730	5900	5021	0	0	0
00:30	1165	1825	1210	4730	5900	4381	0	0	0
01:00	1165	1900	1210	4730	5900	3741	0	0	0
01:30	1165	1950	1210	4730	5900	3126	0	0	0
02:00	1165	1925	1210	4730	5900	2868	0	0	0
02:30	1165	1925	1210	4730	5900	2586	0	0	0
03:00	1165	1875	1210	4730	5900	2353	0	0	0
03:30	1165	1825	1210	4730	5900	2403	0	0	0
04:00	1165	1775	1210	4730	5900	2453	0	0	0
04:30	1165	1725	1210	4730	5900	3351	0	0	0
05:00	1165	1750	1210	4730	5900	4173	0	0	0
05:30	1165	1575	1210	4730	5900	4631	0	0	0
06:00	1165	1550	1210	4730	5900	5221	0	0	0
06:30	1165	1525	1210	4730	5900	6376	0	0	0
07:00	1165	1475	1210	4730	5900	7000	556	0	0
07:30	1165	1375	1210	4730	5900	7000	1786	0	0
08:00	1165	1200	1210	4730	5900	7000	5068	0	0
08:30	1165	1025	1210	4730	5900	7000	2199	0	3327
09:00	1165	1100	1210	4730	5900	7000	2406	0	3327
09:30	1165	1175	1210	4730	5900	7000	4246	0	0
10:00	1165	1275	1210	4730	5900	7000	5037	0	1652
10:30	1165	1450	1210	4730	5900	7000	5680	0	551
11:00	1165	1550	1210	4730	5900	7000	5680	0	169
11:30	1165	1750	1210	4730	5900	7000	5680	0	1099
12:00	1165	1775	1210	4730	5900	7000	5341	0	0
12:30	1165	1850	1210	4730	5900	7000	5680	0	151
13:00	1165	1925	1210	4730	5900	7000	4908	0	0
13:30	1165	1975	1210	4730	5900	7000	5141	0	0
14:00	1165	2125	1210	4730	5900	7000	5274	0	0
14:30	1165	2200	1210	4730	5900	7000	1589	0	3327
15:00	1165	2375	1210	4730	5900	7000	4176	0	0
15:30	1165	2250	1210	4730	5900	7000	3736	0	0
16:00	1165	2300	1210	4730	5900	7000	3403	0	0
16:30	1165	2200	1210	4730	5900	7000	1808	0	0
17:00	1165	2150	1210	4730	5900	7000	2141	0	0
17:30	1165	2050	1210	4730	5900	7000	2523	0	0
18:00	1165	1800	1210	4730	5900	7000	2272	0	3327
18:30	1165	1550	1210	4730	5900	7000	2239	0	3327
19:00	1165	1200	1210	4730	5900	7000	1741	0	3327
19:30	1165	875	1210	4730	5900	7000	4828	0	0
20:00	1165	725	1210	4730	5900	7000	3064	0	3327
20:30	1165	600	1210	4730	5900	7000	1494	0	3327
21:00	1165	600	1210	4730	5900	7000	81	0	3327
21:30	1165	725	1210	4730	5900	7000	0	0	3001
22:00	1165	925	1210	4730	5900	7000	0	0	2518
22:30	1165	1000	1210	4730	5900	7000	0	0	1313
23:00	1165	1100	1210	4730	5900	7000	0	0	83
23:30	1165	1250	1210	4730	5900	6368	0	0	0
sum	55920	75775	58080	227040	283200	290052	99777	0	40480

Table A.50.: Output Scenario 1

Test 5.23

	load	biomass PP max.gen.	nuclear power PP max.gen.	brown coal PP max.gen.	hard coal PP max.gen.	gas and steam PP max.gen.	heating oil PP max.gen.	run-of-river PP max.gen.	run-of-river PP inflow	wind PP max.gen.	wind PP wind strength	storage PP max.gen.	storage PP volume
	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh
00:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	14000	3327	40480
00:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	14075	3327	
01:00	18646	1210	4730	5900	7000	5680	1510	1300	1165	14500	14150	3327	
01:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	14200	3327	
02:00	17798	1210	4730	5900	7000	5680	1510	1300	1165	14500	14175	3327	
02:30	17516	1210	4730	5900	7000	5680	1510	1300	1165	14500	14175	3327	
03:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	14125	3327	
03:30	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	14075	3327	
04:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	14025	3327	
04:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	13975	3327	
05:00	18928	1210	4730	5900	7000	5680	1510	1300	1165	14500	14000	3327	
05:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	13825	3327	
06:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	13800	3327	
06:30	20906	1210	4730	5900	7000	5680	1510	1300	1165	14500	13775	3327	
07:00	22036	1210	4730	5900	7000	5680	1510	1300	1165	14500	13425	3327	
07:30	23166	1210	4730	5900	7000	5680	1510	1300	1165	14500	13175	3327	
08:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	13250	3327	
08:30	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	13075	3327	
09:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	12975	3327	
09:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	13175	3327	
10:00	27969	1210	4730	5900	7000	5680	1510	1300	1165	14500	13525	3327	
10:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	13700	3327	
11:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	13800	3327	
11:30	28534	1210	4730	5900	7000	5680	1510	1300	1165	14500	14000	3327	
12:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	14025	3327	
12:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	14100	3327	
13:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	14175	3327	
13:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	14225	3327	
14:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	14375	3327	
14:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	14450	3327	
15:00	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	14625	3327	
15:30	25991	1210	4730	5900	7000	5680	1510	1300	1165	14500	14500	3327	
16:00	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	14550	3327	
16:30	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	14450	3327	
17:00	24296	1210	4730	5900	7000	5680	1510	1300	1165	14500	14400	3327	
17:30	24578	1210	4730	5900	7000	5680	1510	1300	1165	14500	14300	3327	
18:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	14050	3327	
18:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	13800	3327	
19:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	13450	3327	
19:30	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	13125	3327	
20:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	12975	3327	
20:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	12850	3327	
21:00	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	12850	3327	
21:30	23731	1210	4730	5900	7000	5680	1510	1300	1165	14500	12975	3327	
22:00	23448	1210	4730	5900	7000	5680	1510	1300	1165	14500	13175	3327	
22:30	22318	1210	4730	5900	7000	5680	1510	1300	1165	14500	13250	3327	
23:00	21188	1210	4730	5900	7000	5680	1510	1300	1165	14500	13350	3327	
23:30	20623	1210	4730	5900	7000	5680	1510	1300	1165	14500	13500	3327	
sum	1130324	58080	227040	283200	336000	272640	72480	62400	55920	696000	662000	159696	40480

Table A.51.: Input - scenario 2

	run-on-river MWh	wind MWh	biomass MWh	nuclear power MWh	brown coal MWh	hard coal MWh	gas and steam MWh	heating oil MWh	storage power plant MWh
00:00	1165	14000	1210	3401	0	0	0	0	0
00:30	1165	14075	1210	2761	0	0	0	0	0
01:00	1165	14150	1210	2121	0	0	0	0	0
01:30	1165	14200	1210	1506	0	0	0	0	0
02:00	1165	14175	1210	1248	0	0	0	0	0
02:30	1165	14175	1210	966	0	0	0	0	0
03:00	1165	14125	1210	733	0	0	0	0	0
03:30	1165	14075	1210	783	0	0	0	0	0
04:00	1165	14025	1210	833	0	0	0	0	0
04:30	1165	13975	1210	1731	0	0	0	0	0
05:00	1165	14000	1210	2553	0	0	0	0	0
05:30	1165	13825	1210	3011	0	0	0	0	0
06:00	1165	13800	1210	3601	0	0	0	0	0
06:30	1165	13775	1210	4730	26	0	0	0	0
07:00	1165	13425	1210	4730	1506	0	0	0	0
07:30	1165	13175	1210	4730	2886	0	0	0	0
08:00	1165	13250	1210	4730	5900	0	0	0	18
08:30	1165	13075	1210	4730	3049	0	0	0	3327
09:00	1165	12975	1210	4730	3431	0	0	0	3327
09:30	1165	13175	1210	4730	5146	0	0	0	0
10:00	1165	13525	1210	4730	5756	0	0	0	1583
10:30	1165	13700	1210	4730	5900	0	0	0	981
11:00	1165	13800	1210	4730	5900	0	0	0	599
11:30	1165	14000	1210	4730	5900	0	0	0	1529
12:00	1165	14025	1210	4730	5900	0	0	0	91
12:30	1165	14100	1210	4730	5900	0	0	0	581
13:00	1165	14175	1210	4730	5558	0	0	0	0
13:30	1165	14225	1210	4730	5791	0	0	0	0
14:00	1165	14375	1210	4730	5900	0	0	0	24
14:30	1165	14450	1210	4730	5566	0	0	0	0
15:00	1165	14625	1210	4730	4826	0	0	0	0
15:30	1165	14500	1210	4730	4386	0	0	0	0
16:00	1165	14550	1210	4730	4053	0	0	0	0
16:30	1165	14450	1210	4730	2458	0	0	0	0
17:00	1165	14400	1210	4730	2791	0	0	0	0
17:30	1165	14300	1210	4730	3173	0	0	0	0
18:00	1165	14050	1210	4730	2922	0	0	0	3327
18:30	1165	13800	1210	4730	2889	0	0	0	3327
19:00	1165	13450	1210	4730	2391	0	0	0	3327
19:30	1165	13125	1210	4730	5478	0	0	0	0
20:00	1165	12975	1210	4730	3714	0	0	0	3327
20:30	1165	12850	1210	4730	2144	0	0	0	3327
21:00	1165	12850	1210	4730	731	0	0	0	3327
21:30	1165	12975	1210	4730	324	0	0	0	3327
22:00	1165	13175	1210	4730	0	0	0	0	3168
22:30	1165	13250	1210	4730	0	0	0	0	1963
23:00	1165	13350	1210	4730	733	0	0	0	0
23:30	1165	13500	1210	4730	18	0	0	0	0
sum	55920	662000	58080	190798	123046	0	0	0	40480

Table A.52.: Output - scenario 2

Test 5.24

	load	biomass PP max.gen.	nuclear power PP max.gen.	brown coal PP max.gen.	hard coal PP max.gen.	gas and steam PP max.gen.	heating oil PP max.gen.	run-of-river PP max.gen.	run-of-river PP inflow	wind PP max.gen.	wind PP wind strength	storage PP max.gen.	storag PP volume
	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh
00:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	14000	3327	40480
00:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	14075	3327	
01:00	18646	1210	4730	5900	7000	5680	1510	1300	1165	14500	14150	3327	
01:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	14200	3327	
02:00	17798	1210	4730	5900	7000	5680	1510	1300	1165	14500	14175	3327	
02:30	17516	1210	4730	5900	7000	5680	1510	1300	1165	14500	14175	3327	
03:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	14125	3327	
03:30	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	14075	3327	
04:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	14025	3327	
04:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	13975	3327	
05:00	18928	1210	4730	5900	7000	5680	1510	1300	1165	14500	14000	3327	
05:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	13825	3327	
06:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	13800	3327	
06:30	20906	1210	4730	5900	7000	5680	1510	1300	1165	14500	13775	3327	
07:00	22036	1210	4730	5900	7000	5680	1510	1300	1165	14500	13425	3327	
07:30	23166	1210	4730	5900	7000	5680	1510	1300	1165	14500	13175	3327	
08:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	13250	3327	
08:30	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	13075	3327	
09:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	12975	3327	
09:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	13175	3327	
10:00	27969	1210	4730	5900	7000	5680	1510	1300	1165	14500	13525	3327	
10:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	13700	3327	
11:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	13800	3327	
11:30	28534	1210	4730	5900	7000	5680	1510	1300	1165	14500	14000	3327	
12:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	14025	3327	
12:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	14100	3327	
13:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	14175	3327	
13:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	14225	3327	
14:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	14375	3327	
14:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	14450	3327	
15:00	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	14625	3327	
15:30	25991	1210	4730	5900	7000	5680	1510	1300	1165	14500	14500	3327	
16:00	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	14550	3327	
16:30	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	14450	3327	
17:00	24296	1210	4730	5900	7000	5680	1510	1300	1165	14500	14400	3327	
17:30	24578	1210	4730	5900	7000	5680	1510	1300	1165	14500	14300	3327	
18:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	14050	3327	
18:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	13800	3327	
19:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	13450	3327	
19:30	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	13125	3327	
20:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	12975	3327	
20:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	12850	3327	
21:00	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	12850	3327	
21:30	23731	1210	4730	5900	7000	5680	1510	1300	1165	14500	12975	3327	
22:00	23448	1210	4730	5900	7000	5680	1510	1300	1165	14500	13175	3327	
22:30	22318	1210	4730	5900	7000	5680	1510	1300	1165	14500	13250	3327	
23:00	21188	1210	4730	5900	7000	5680	1510	1300	1165	14500	13350	3327	
23:30	20623	1210	4730	5900	7000	5680	1510	1300	1165	14500	13500	3327	
sum	1130324	58080	227040	283200	336000	272640	72480	62400	55920	696000	662000	159696	40480

Table A.53.: Input - scenario 3

	run-on-river MWh	wind MWh	biomass MWh	nuclear power MWh	brown coal MWh	hard coal MWh	gas and steam MWh	heating oil MWh	storage power plant MWh
00:00	1165	14000	1210	0	3401	0	0	0	0
00:30	1165	14075	1210	0	2761	0	0	0	0
01:00	1165	14150	1210	0	2121	0	0	0	0
01:30	1165	14200	1210	0	1506	0	0	0	0
02:00	1165	14175	1210	0	1248	0	0	0	0
02:30	1165	14175	1210	0	966	0	0	0	0
03:00	1165	14125	1210	0	733	0	0	0	0
03:30	1165	14075	1210	0	783	0	0	0	0
04:00	1165	14025	1210	0	833	0	0	0	0
04:30	1165	13975	1210	0	1731	0	0	0	0
05:00	1165	14000	1210	0	2553	0	0	0	0
05:30	1165	13825	1210	0	3011	0	0	0	0
06:00	1165	13800	1210	0	3601	0	0	0	0
06:30	1165	13775	1210	0	4756	0	0	0	0
07:00	1165	13425	1210	0	5900	0	0	0	336
07:30	1165	13175	1210	0	5900	0	0	0	1716
08:00	1165	13250	1210	0	5900	4748	0	0	0
08:30	1165	13075	1210	0	5900	5206	0	0	0
09:00	1165	12975	1210	0	5900	5588	0	0	0
09:30	1165	13175	1210	0	5900	3651	0	0	325
10:00	1165	13525	1210	0	5900	6169	0	0	0
10:30	1165	13700	1210	0	5900	5711	0	0	0
11:00	1165	13800	1210	0	5900	5329	0	0	0
11:30	1165	14000	1210	0	5900	2932	0	0	3327
12:00	1165	14025	1210	0	5900	4821	0	0	0
12:30	1165	14100	1210	0	5900	5311	0	0	0
13:00	1165	14175	1210	0	5900	4388	0	0	0
13:30	1165	14225	1210	0	5900	1294	0	0	3327
14:00	1165	14375	1210	0	5900	1427	0	0	3327
14:30	1165	14450	1210	0	5900	1069	0	0	3327
15:00	1165	14625	1210	0	5900	3656	0	0	0
15:30	1165	14500	1210	0	5900	3216	0	0	0
16:00	1165	14550	1210	0	5900	2883	0	0	0
16:30	1165	14450	1210	0	5900	1288	0	0	0
17:00	1165	14400	1210	0	5900	1621	0	0	0
17:30	1165	14300	1210	0	5900	2003	0	0	0
18:00	1165	14050	1210	0	5900	1752	0	0	3327
18:30	1165	13800	1210	0	5900	1719	0	0	3327
19:00	1165	13450	1210	0	5900	1221	0	0	3327
19:30	1165	13125	1210	0	5900	4308	0	0	0
20:00	1165	12975	1210	0	5900	2544	0	0	3327
20:30	1165	12850	1210	0	5900	974	0	0	3327
21:00	1165	12850	1210	0	5900	0	0	0	2888
21:30	1165	12975	1210	0	5900	0	0	0	2481
22:00	1165	13175	1210	0	5900	0	0	0	1998
22:30	1165	13250	1210	0	5900	0	0	0	793
23:00	1165	13350	1210	0	5463	0	0	0	0
23:30	1165	13500	1210	0	4748	0	0	0	0
sum	55920	662000	58080	0	229015	84829	0	0	40480

Table A.54.: Output - scenario 3

Test 5.25

	load	biomass PP max.gen.	nuclear power PP max.gen.	brown coal PP max.gen.	hard coal PP max.gen.	gas and steam PP max.gen.	heating oil PP max.gen.	run-of-river PP max.gen.	run-of-river PP inflow	wind PP max.gen.	wind PP wind strength	storage PP max.gen.	storage PP volume
	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh
00:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	40480
00:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	11753	3327	
01:00	18646	1210	4730	5900	7000	5680	1510	1300	1165	14500	12236	3327	
01:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	12558	3327	
02:00	17798	1210	4730	5900	7000	5680	1510	1300	1165	14500	12397	3327	
02:30	17516	1210	4730	5900	7000	5680	1510	1300	1165	14500	12397	3327	
03:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	12075	3327	
03:30	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	11753	3327	
04:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	11431	3327	
04:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	11109	3327	
05:00	18928	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	
05:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	10143	3327	
06:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	9982	3327	
06:30	20906	1210	4730	5900	7000	5680	1510	1300	1165	14500	9821	3327	
07:00	22036	1210	4730	5900	7000	5680	1510	1300	1165	14500	9499	3327	
07:30	23166	1210	4730	5900	7000	5680	1510	1300	1165	14500	8855	3327	
08:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	7728	3327	
08:30	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	6601	3327	
09:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	7084	3327	
09:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	7567	3327	
10:00	27969	1210	4730	5900	7000	5680	1510	1300	1165	14500	8211	3327	
10:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	9338	3327	
11:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	9982	3327	
11:30	28534	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	
12:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	11431	3327	
12:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	11914	3327	
13:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	9397	3327	
13:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	8719	3327	
14:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	7685	3327	
14:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	6168	3327	
15:00	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	5295	3327	
15:30	25991	1210	4730	5900	7000	5680	1510	1300	1165	14500	4490	3327	
16:00	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	8812	3327	
16:30	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	10168	3327	
17:00	24296	1210	4730	5900	7000	5680	1510	1300	1165	14500	11846	3327	
17:30	24578	1210	4730	5900	7000	5680	1510	1300	1165	14500	12202	3327	
18:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	13592	3327	
18:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	12982	3327	
19:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	11728	3327	
19:30	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	12335	3327	
20:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	10669	3327	
20:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	9864	3327	
21:00	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	4164	3327	
21:30	23731	1210	4730	5900	7000	5680	1510	1300	1165	14500	4669	3327	
22:00	23448	1210	4730	5900	7000	5680	1510	1300	1165	14500	6957	3327	
22:30	22318	1210	4730	5900	7000	5680	1510	1300	1165	14500	6440	3327	
23:00	21188	1210	4730	5900	7000	5680	1510	1300	1165	14500	7084	3327	
23:30	20623	1210	4730	5900	7000	5680	1510	1300	1165	14500	8050	3327	
sum	1130324	58080	227040	283200	336000	272640	72480	62400	55920	696000	462991	159696	40480

Table A.55.: Input - scenario 4

	run-on-river MWh	wind MWh	biomass MWh	nuclear power MWh	brown coal MWh	hard coal MWh	gas and steam MWh	heating oil MWh	storage power plant MWh
00:00	1165	11270	1210	0	5900	231	0	0	0
00:30	1165	11753	1210	0	5083	0	0	0	0
01:00	1165	12236	1210	0	4035	0	0	0	0
01:30	1165	12558	1210	0	3148	0	0	0	0
02:00	1165	12397	1210	0	3026	0	0	0	0
02:30	1165	12397	1210	0	2744	0	0	0	0
03:00	1165	12075	1210	0	2783	0	0	0	0
03:30	1165	11753	1210	0	3105	0	0	0	0
04:00	1165	11431	1210	0	3427	0	0	0	0
04:30	1165	11109	1210	0	4597	0	0	0	0
05:00	1165	11270	1210	0	5283	0	0	0	0
05:30	1165	10143	1210	0	5900	793	0	0	0
06:00	1165	9982	1210	0	5900	1519	0	0	0
06:30	1165	9821	1210	0	5900	2810	0	0	0
07:00	1165	9499	1210	0	5900	4262	0	0	0
07:30	1165	8855	1210	0	5900	6036	0	0	0
08:00	1165	7728	1210	0	5900	7000	3270	0	0
08:30	1165	6601	1210	0	5900	7000	1419	0	3261
09:00	1165	7084	1210	0	5900	7000	1152	0	3327
09:30	1165	7567	1210	0	5900	7000	0	0	2584
10:00	1165	8211	1210	0	5900	7000	1156	0	3327
10:30	1165	9338	1210	0	5900	7000	0	0	3073
11:00	1165	9982	1210	0	5900	7000	2147	0	0
11:30	1165	11270	1210	0	5900	7000	1989	0	0
12:00	1165	11431	1210	0	5900	7000	415	0	0
12:30	1165	11914	1210	0	5900	7000	497	0	0
13:00	1165	9397	1210	0	5900	7000	2166	0	0
13:30	1165	8719	1210	0	5900	7000	0	0	3127
14:00	1165	7685	1210	0	5900	7000	1117	0	3327
14:30	1165	6168	1210	0	5900	7000	2351	0	3327
15:00	1165	5295	1210	0	5900	7000	2659	0	3327
15:30	1165	4490	1210	0	5900	7000	2899	0	3327
16:00	1165	8812	1210	0	5900	7000	1621	0	0
16:30	1165	10168	1210	0	5900	5570	0	0	0
17:00	1165	11846	1210	0	5900	4175	0	0	0
17:30	1165	12202	1210	0	5900	4101	0	0	0
18:00	1165	13592	1210	0	5900	5537	0	0	0
18:30	1165	12982	1210	0	5900	5864	0	0	0
19:00	1165	11728	1210	0	5900	6270	0	0	0
19:30	1165	12335	1210	0	5900	5098	0	0	0
20:00	1165	10669	1210	0	5900	7000	1177	0	0
20:30	1165	9864	1210	0	5900	7000	287	0	0
21:00	1165	4164	1210	0	5900	7000	1247	0	3327
21:30	1165	4669	1210	0	5900	7000	460	0	3327
22:00	1165	6957	1210	0	5900	7000	0	0	1216
22:30	1165	6440	1210	0	5900	7000	0	0	603
23:00	1165	7084	1210	0	5900	5829	0	0	0
23:30	1165	8050	1210	0	5900	4298	0	0	0
sum	55920	462991	58080	0	261431	223393	28029	0	40480

Table A.56.: Output - scenario 4

Test 5.26

	load MWh	biomass PP max.gen. MWh	nuclear power PP max.gen. MWh	brown coal PP max.gen. MWh	hard coal PP max.gen. MWh	gas and steam PP max.gen. MWh	heating oil PP max.gen. MWh	run-of-river PP max.gen. MWh	run-of-river PP inflow MWh	wind PP max.gen. MWh	wind PP wind strength MWh	storage PP max.gen. MWh	storage PP volume MWh
00:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	40480
00:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	11753	3327	
01:00	18646	1210	4730	5900	7000	5680	1510	1300	1165	14500	12236	3327	
01:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	12558	3327	
02:00	17798	1210	4730	5900	7000	5680	1510	1300	1165	14500	12397	3327	
02:30	17516	1210	4730	5900	7000	5680	1510	1300	1165	14500	12397	3327	
03:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	12075	3327	
03:30	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	11753	3327	
04:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	11431	3327	
04:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	11109	3327	
05:00	18928	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	
05:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	10143	3327	
06:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	9982	3327	
06:30	20906	1210	4730	5900	7000	5680	1510	1300	1165	14500	9821	3327	
07:00	22036	1210	4730	5900	7000	5680	1510	1300	1165	14500	9499	3327	
07:30	23166	1210	4730	5900	7000	5680	1510	1300	1165	14500	8855	3327	
08:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	7728	3327	
08:30	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	6601	3327	
09:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	7084	3327	
09:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	7567	3327	
10:00	27969	1210	4730	5900	7000	5680	1510	1300	1165	14500	8211	3327	
10:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	9338	3327	
11:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	9982	3327	
11:30	28534	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	
12:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	11431	3327	
12:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	11914	3327	
13:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	9397	3327	
13:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	8719	3327	
14:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	7685	3327	
14:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	6168	3327	
15:00	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	5295	3327	
15:30	25991	1210	4730	5900	7000	5680	1510	1300	1165	14500	4490	3327	
16:00	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	8812	3327	
16:30	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	10168	3327	
17:00	24296	1210	4730	5900	7000	5680	1510	1300	1165	14500	11846	3327	
17:30	24578	1210	4730	5900	7000	5680	1510	1300	1165	14500	12202	3327	
18:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	13592	3327	
18:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	12982	3327	
19:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	11728	3327	
19:30	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	12335	3327	
20:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	10669	3327	
20:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	9864	3327	
21:00	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	4164	3327	
21:30	23731	1210	4730	5900	7000	5680	1510	1300	1165	14500	4669	3327	
22:00	23448	1210	4730	5900	7000	5680	1510	1300	1165	14500	6957	3327	
22:30	22318	1210	4730	5900	7000	5680	1510	1300	1165	14500	6440	3327	
23:00	21188	1210	4730	5900	7000	5680	1510	1300	1165	14500	7084	3327	
23:30	20623	1210	4730	5900	7000	5680	1510	1300	1165	14500	8050	3327	
sum	1130324	58080	227040	283200	336000	272640	72480	62400	55920	696000	462991	159696	40480

Table A.57.: Input - scenario 5

	run-on-river MWh	wind MWh	biomass MWh	nuclear power MWh	brown coal MWh	hard coal MWh	gas and steam MWh	heating oil MWh	storage power plant MWh	unused wind energy MWh
00:00	1165	10000	1210	0	5900	1501	0	0	0	1270
00:30	1165	10000	1210	0	5900	936	0	0	0	1753
01:00	1165	10000	1210	0	5900	371	0	0	0	2236
01:30	1165	10000	1210	0	5706	0	0	0	0	2558
02:00	1165	10000	1210	0	5423	0	0	0	0	2397
02:30	1165	10000	1210	0	5141	0	0	0	0	2397
03:00	1165	10000	1210	0	4858	0	0	0	0	2075
03:30	1165	10000	1210	0	4858	0	0	0	0	1753
04:00	1165	10000	1210	0	4858	0	0	0	0	1431
04:30	1165	10000	1210	0	5706	0	0	0	0	1109
05:00	1165	10000	1210	0	5900	653	0	0	0	1270
05:30	1165	10000	1210	0	5900	936	0	0	0	143
06:00	1165	9982	1210	0	5900	1519	0	0	0	0
06:30	1165	9821	1210	0	5900	2810	0	0	0	0
07:00	1165	9499	1210	0	5900	4262	0	0	0	0
07:30	1165	8855	1210	0	5900	6036	0	0	0	0
08:00	1165	7728	1210	0	5900	7000	3270	0	0	0
08:30	1165	6601	1210	0	5900	7000	1419	0	3261	0
09:00	1165	7084	1210	0	5900	7000	1152	0	3327	0
09:30	1165	7567	1210	0	5900	7000	0	0	2584	0
10:00	1165	8211	1210	0	5900	7000	1156	0	3327	0
10:30	1165	9338	1210	0	5900	7000	0	0	3073	0
11:00	1165	9982	1210	0	5900	7000	2147	0	0	0
11:30	1165	10000	1210	0	5900	7000	3259	0	0	1270
12:00	1165	10000	1210	0	5900	7000	1846	0	0	1431
12:30	1165	10000	1210	0	5900	7000	2411	0	0	1914
13:00	1165	9397	1210	0	5900	7000	2166	0	0	0
13:30	1165	8719	1210	0	5900	7000	0	0	3127	0
14:00	1165	7685	1210	0	5900	7000	1117	0	3327	0
14:30	1165	6168	1210	0	5900	7000	2351	0	3327	0
15:00	1165	5295	1210	0	5900	7000	2659	0	3327	0
15:30	1165	4490	1210	0	5900	7000	2899	0	3327	0
16:00	1165	8812	1210	0	5900	7000	1621	0	0	0
16:30	1165	10000	1210	0	5900	5738	0	0	0	168
17:00	1165	10000	1210	0	5900	6021	0	0	0	1846
17:30	1165	10000	1210	0	5900	6303	0	0	0	2202
18:00	1165	10000	1210	0	5900	7000	2129	0	0	3592
18:30	1165	10000	1210	0	5900	7000	1846	0	0	2982
19:00	1165	10000	1210	0	5900	7000	998	0	0	1728
19:30	1165	10000	1210	0	5900	7000	433	0	0	2335
20:00	1165	10000	1210	0	5900	7000	1846	0	0	669
20:30	1165	9864	1210	0	5900	7000	287	0	0	0
21:00	1165	4164	1210	0	5900	7000	1247	0	3327	0
21:30	1165	4669	1210	0	5900	7000	460	0	3327	0
22:00	1165	6957	1210	0	5900	7000	0	0	1216	0
22:30	1165	6440	1210	0	5900	7000	0	0	603	0
23:00	1165	7084	1210	0	5900	5829	0	0	0	0
23:30	1165	8050	1210	0	5900	4298	0	0	0	0
sum	55920	422462	58080	0	278450	236213	38719	0	40480	40529

Table A.58.: Output - scenario 5

Test 5.27

	load	biomass max.gen.	nuclear power max.gen.	brown coal max.gen.	hard coal max.gen.	gas and steam max.gen.	heating oil max.gen.	run-of-river max.gen.	run-of-river inflow	wind max.gen.	wind wind strength	storage max.gen.	storage volume	solar max.gen.	solar useable solar radiation
	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh	MWh
00:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327	40480	12500	0
00:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	11753	3327		12500	0
01:00	18646	1210	4730	5900	7000	5680	1510	1300	1165	14500	12236	3327		12500	0
01:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	12558	3327		12500	0
02:00	17798	1210	4730	5900	7000	5680	1510	1300	1165	14500	12397	3327		12500	0
02:30	17516	1210	4730	5900	7000	5680	1510	1300	1165	14500	12397	3327		12500	0
03:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	12075	3327		12500	0
03:30	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	11753	3327		12500	0
04:00	17233	1210	4730	5900	7000	5680	1510	1300	1165	14500	11431	3327		12500	0
04:30	18081	1210	4730	5900	7000	5680	1510	1300	1165	14500	11109	3327		12500	0
05:00	18928	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327		12500	0
05:30	19211	1210	4730	5900	7000	5680	1510	1300	1165	14500	10143	3327		12500	0
06:00	19776	1210	4730	5900	7000	5680	1510	1300	1165	14500	9982	3327		12500	0
06:30	20906	1210	4730	5900	7000	5680	1510	1300	1165	14500	9821	3327		12500	0
07:00	22036	1210	4730	5900	7000	5680	1510	1300	1165	14500	9499	3327		12500	500
07:30	23166	1210	4730	5900	7000	5680	1510	1300	1165	14500	8855	3327		12500	500
08:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	7728	3327		12500	1000
08:30	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	6601	3327		12500	1000
09:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	7084	3327		12500	1500
09:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	7567	3327		12500	1500
10:00	27969	1210	4730	5900	7000	5680	1510	1300	1165	14500	8211	3327		12500	3000
10:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	9338	3327		12500	3000
11:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	9982	3327		12500	4000
11:30	28534	1210	4730	5900	7000	5680	1510	1300	1165	14500	11270	3327		12500	4300
12:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	11431	3327		12500	4500
12:30	27686	1210	4730	5900	7000	5680	1510	1300	1165	14500	11914	3327		12500	4500
13:00	26838	1210	4730	5900	7000	5680	1510	1300	1165	14500	9397	3327		12500	4500
13:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	8719	3327		12500	4500
14:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	7685	3327		12500	4500
14:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	6168	3327		12500	4500
15:00	26556	1210	4730	5900	7000	5680	1510	1300	1165	14500	5295	3327		12500	4000
15:30	25991	1210	4730	5900	7000	5680	1510	1300	1165	14500	4490	3327		12500	3000
16:00	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	8812	3327		12500	2000
16:30	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	10168	3327		12500	2000
17:00	24296	1210	4730	5900	7000	5680	1510	1300	1165	14500	11846	3327		12500	500
17:30	24578	1210	4730	5900	7000	5680	1510	1300	1165	14500	12202	3327		12500	500
18:00	27404	1210	4730	5900	7000	5680	1510	1300	1165	14500	13592	3327		12500	0
18:30	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	12982	3327		12500	0
19:00	26273	1210	4730	5900	7000	5680	1510	1300	1165	14500	11728	3327		12500	0
19:30	25708	1210	4730	5900	7000	5680	1510	1300	1165	14500	12335	3327		12500	0
20:00	27121	1210	4730	5900	7000	5680	1510	1300	1165	14500	10669	3327		12500	0
20:30	25426	1210	4730	5900	7000	5680	1510	1300	1165	14500	9864	3327		12500	0
21:00	24013	1210	4730	5900	7000	5680	1510	1300	1165	14500	4164	3327		12500	0
21:30	23731	1210	4730	5900	7000	5680	1510	1300	1165	14500	4669	3327		12500	0
22:00	23448	1210	4730	5900	7000	5680	1510	1300	1165	14500	6957	3327		12500	0
22:30	22318	1210	4730	5900	7000	5680	1510	1300	1165	14500	6440	3327		12500	0
23:00	21188	1210	4730	5900	7000	5680	1510	1300	1165	14500	7084	3327		12500	0
23:30	20623	1210	4730	5900	7000	5680	1510	1300	1165	14500	8050	3327		12500	0
sum	1130324	58080	227040	283200	336000	272640	72480	62400	55920	696000	462991	159696	40480	600000	59300

Table A.59.: Input - scenario 6

	run-on-river MWh	wind MWh	biomass MWh	nuclear power MWh	brown coal MWh	hard coal MWh	gas and steam MWh	heating oil MWh	storage power plant MWh	solar power plant MWh
00:00	1165	11270	1210	0	5900	231	0	0	0	0
00:30	1165	11753	1210	0	5083	0	0	0	0	0
01:00	1165	12236	1210	0	4035	0	0	0	0	0
01:30	1165	12558	1210	0	3148	0	0	0	0	0
02:00	1165	12397	1210	0	3026	0	0	0	0	0
02:30	1165	12397	1210	0	2744	0	0	0	0	0
03:00	1165	12075	1210	0	2783	0	0	0	0	0
03:30	1165	11753	1210	0	3105	0	0	0	0	0
04:00	1165	11431	1210	0	3427	0	0	0	0	0
04:30	1165	11109	1210	0	4597	0	0	0	0	0
05:00	1165	11270	1210	0	5283	0	0	0	0	0
05:30	1165	10143	1210	0	5900	793	0	0	0	0
06:00	1165	9982	1210	0	5900	1519	0	0	0	0
06:30	1165	9821	1210	0	5900	2810	0	0	0	0
07:00	1165	9499	1210	0	5900	3762	0	0	0	500
07:30	1165	8855	1210	0	5900	5536	0	0	0	500
08:00	1165	7728	1210	0	5900	5943	0	0	3327	1000
08:30	1165	6601	1210	0	5900	7000	353	0	3327	1000
09:00	1165	7084	1210	0	5900	6652	0	0	3327	1500
09:30	1165	7567	1210	0	5900	4846	0	0	3238	1500
10:00	1165	8211	1210	0	5900	7000	0	0	1483	3000
10:30	1165	9338	1210	0	5900	7000	0	0	73	3000
11:00	1165	9982	1210	0	5900	5147	0	0	0	4000
11:30	1165	11270	1210	0	5900	4689	0	0	0	4300
12:00	1165	11431	1210	0	5900	2915	0	0	0	4500
12:30	1165	11914	1210	0	5900	2997	0	0	0	4500
13:00	1165	9397	1210	0	5900	4666	0	0	0	4500
13:30	1165	8719	1210	0	5900	5627	0	0	0	4500
14:00	1165	7685	1210	0	5900	6944	0	0	0	4500
14:30	1165	6168	1210	0	5900	7000	0	0	1178	4500
15:00	1165	5295	1210	0	5900	7000	0	0	1986	4000
15:30	1165	4490	1210	0	5900	7000	0	0	3226	3000
16:00	1165	8812	1210	0	5900	6621	0	0	0	2000
16:30	1165	10168	1210	0	5900	3570	0	0	0	2000
17:00	1165	11846	1210	0	5900	3675	0	0	0	500
17:30	1165	12202	1210	0	5900	3601	0	0	0	500
18:00	1165	13592	1210	0	5900	5537	0	0	0	0
18:30	1165	12982	1210	0	5900	5864	0	0	0	0
19:00	1165	11728	1210	0	5900	6270	0	0	0	0
19:30	1165	12335	1210	0	5900	5098	0	0	0	0
20:00	1165	10669	1210	0	5900	7000	0	0	1177	0
20:30	1165	9864	1210	0	5900	7000	0	0	287	0
21:00	1165	4164	1210	0	5900	7000	1247	0	3327	0
21:30	1165	4669	1210	0	5900	7000	460	0	3327	0
22:00	1165	6957	1210	0	5900	7000	0	0	1216	0
22:30	1165	6440	1210	0	5900	4276	0	0	3327	0
23:00	1165	7084	1210	0	5900	2502	0	0	3327	0
23:30	1165	8050	1210	0	5900	971	0	0	3327	0
sum	55920	462991	58080	0	261431	190062	2060	0	40480	59300

Table A.60.: Output - scenario 6