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Quantum-Quench in N-Chain Luttinger-Liquids

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I am grateful to my parents Christine and Herbert for giving me human and financial backing. I also want to thank my prospectively parents in law, Elfriede and Peter for always making me feel welcome.
I am grateful for lots of interesting discussions about science, with my brother Patrick. He often remember me, that we should keep in mind the possibility to see beyond ones own nose.
Finally I would like to thank my great love Sabine for her patience of sharing me with physics and for making me happy since so many years.

Abstract

Motivated by the open question about the time evolution of many-body-quantum systems, the time evolution of correlations in Luttinger liquids, after suddenly switching on the interactions is studied. In the so-called quantum quench, the Hamiltonian is changed over a time scale much shorter than any other time scales of the system. The system before and after the quench is diagonalized with the help of a Bogoliubov transformation. This allows the exact computation of time dependent correlation functions, by using the analytical method of bosonization.
In this thesis such a quench is generalized to a model of N parallel Luttinger liquids, where density-density interactions between the chains are switched on. Assuming translational invariance in the direction perpendicular to the chains, uncoupled chains in transverse momentum space are obtained. Due to this, it is again feasible to diagonalize the Hamiltonian after the quench. In this way, direct and reverse Bogoliubov transformation allow to write the time evolution of the boson operators after the interaction quench, in terms of operators in which the system was diagonal before the quench. Now, knowing the time dependence of the boson creation and destruction operators, the one particle equal time correlation is computed. Thermodynamic and large time limit of the correlation function are considered. The critical exponent of the correlation function is compared to the coupled N-chain system in the ground state, and shows a different value. Further, the behavior dependent on the range of the interactions, is examined. In particular, the spatial slow decaying and at wave vector $q = 0$ diverging Coulomb interaction is investigated.

Kurzfassung

Motiviert von der offenen Frage über die Zeitentwicklung in Vielkörpersystemen der Quantenmechanik, wird die Zeitentwicklung von Korrelationen in Luttinger Flüssigkeiten, nach dem plötzlichen Einschalten der Wechselwirkungen untersucht. In dem sogenannten ‘Quanten-Quench’ wird der Hamilton-Operator, in einer Zeitskala, die viel kürzer ist als alle anderen Zeitskalen des Systems, verändert. Das System vor und nach dem Quench wird mit Hilfe der Bogoliubov Transformation diagonalisiert. Somit lassen sich die zeitabhängigen Korrelationsfunktionen mittels Verwendung der analytischen Methode der Bosonisierung exakt bestimmen.
In dieser Arbeit wird der Quench auf ein Modell mit N parallelen Ketten verallgemeinert, wobei Dichte-Dichte Wechselwirkungen zwischen den Ketten eingeschaltet werden.

Unter der Voraussetzung von Translationsinvarianz in der Richtung quer zu den Ketten, erhält man im Impulsraum ungekoppelte Ketten. Aufgrund dessen wird es wiederum möglich den Hamilton Operator nach dem Quench in eine diagonale Form zu bringen. Unter Verwendung einer direkten und inversen Bogoliubov Transformation wird es möglich die Operatoren nach dem Quench, durch Operatoren in denen das System vor dem Quench diagonal war, auszudrücken. Mit den nun bekannten Zeitabhängikeiten, der Bosonen Erzeugungs und Vernichtungsoperatoren, ist es dann möglich die Einteilchen Korrelationsfunktion zu bestimmen. Der Übergang zum thermodynamischen Limit und Langzeitlimit wird durchgeführt. Der kritische Exponent wird verglichen mit dem Fall vom bereits Anfangs voll gekoppelten System von N parallelen Ketten im Gleichgewicht und zeigt einen abweichenden Wert. Weiters wird das Verhalten in Abhängigkeit der Reichweite der Wechselwirkungen analysiert. Insbesondere wird auch die örtlich langsam abfallende, beim Wellenvektor $q = 0$ divergierende Coulomb Wechselwirkung untersucht.

Contents

1. Motivation

Sometimes, when I talk to people about my work the question arises, whether 1D systems can have any meaning in our 3D world beyond being a model system for calculations of merely academical interest. And the answer is, yes, they can. Clearly, it is true that there can exist no material entities which have zero extension in one or more directions. But this is also not necessary for showing 1D behavior. The quantum mechanical behavior of our physical world makes it possible. Consider the simple quantum problem of particles in a box. The possible energies are quantized and the distances between the energy levels in a particular direction are inverse proportional to the extension in this direction squared. So when the extension of the system in two directions is very small, the energy levels in this direction are far separated from each other. So for the system having finite bandwidth and not too much energy, the only accessible energy excitations are localized in the one remaining direction, where the energy distances are small (Appendix:G.1). So in the sense of quantum mechanics, the system behaves really as a 1-dimensional one. As a short exercise, one can calculate the required diameter of such a wire for realistic values of bandwidth and mass, and will get approximately $1nm$. The creation of such systems was a great challenge for a long time. Recently, great experimental progress led to many approaches to produce 1D quantum systems. Examples are: cleaved edge overgrowth [27, 1], atomic chains deposited on step edges [23], makromolecules like carbon nanotubes [4, 10], and cold atomic gases in optical lattices, which become very interesting, because there are techniques available which allow to tune the interactions between the particles [6].

Next, to get a little foretaste of the fascination of 1D systems, one has to realize that "one dimension is different". Although, Fermi liquid theory [14] is a very powerful theory to describe higher dimensions (2D,3D,...), it completely breaks down in 1D. In higher dimensions, the excited states of the interacting system are obtained by adiabatically switching on the interactions from the noninteracting system. Then there emerge quasiparticles, which can be described by the same quantum numbers as free fermions. The quasiparticles keep the same properties with renormalized mass, finite lifetime and with some residual interactions between quasiparticles. Although this is somewhat sloppy, one can already imagine, why this picture in 1D cannot work. Because of Fermi principle and because of the dimensional constraint, a moving fermion always affects the other fermions along its way it. So one can argue, that a interacting fermion in 1D can never be described as a quasiparticle, but rather, this highly correlated system must be described by a sort of collective excitation. That actually this is the case and can be done, we will see in the chapter about Luttinger liquids.

But because maybe we do not want to wait any longer, we briefly list the exotic properties of 1D already here.

First of all, a added fermion state consisting of charge and spin is not stable in a 1D system. It breaks up in collective excitations carrying charge and spin, with different velocities. This is the famous "spin charge separation". Further, the correlation functions show non-universal (interaction dependent) power-law behavior. Which means that there are correlations on all length scales and no long range order can emerge. Also the local density of states vanishes as a power law when reaching Fermi energy(see Appendix: G.2) and so in a first guess one would expect strongly suppressed one particle tunneling between two 1D chains. I will not say more about this issue yet, but this and the related, really important, not yet clarified question about the maintenance of non Fermi liquid behavior in higher dimensions is one of the most exciting questions of physics today.

And more, new questions arise with progress in experimental methods, which give new possibilities, for the investigation of time dependent correlations out of equilibrium. One can produce time dependent interaction strengths, between ultra cold atoms trapped in optical lattices, by using Feshbach resonance[6, 19]. One of the most fundamental questions, which becomes now accessible, is if am isolated many-body quantum system reaches a stationary state, when it evolves in time after it was prepared in a initial state, which is not an eigenstate of its Hamiltonian. And if it reaches a stationary state can it be described by a standard statistical ensemble? This question provide a new exciting playground for further theoretical investigation. From a technical point of view the the power of a Luttinger liquid approach is its rigorous formulation based on operator identities, which makes it exactly solvable for forward scattering and linearized dispersion relation. Moreover its validity is in principle not dependent on the strength of the interactions and it constitute the universality class for the low energy properties of all gapless 1D quantum systems. But now lets begin, I hope you will enjoy your journey to the 1D world!

2. Introduction to Luttinger liquid theory

2.1. Brief introduction to Fermi liquid theory and its failure in 1D

Let us start with a brief introduction to the main concepts of Fermi liquid theory, and by picturing the reasons for its failure in 1D. By the way, a comprehensive standard reference for an introduction into the 1D Fermi-liquids is given by J. Voit in [25] and I will follow in main parts its spirit. Here I also want to mention the Book of Giamarchi [5], from which I get lots of inspiration. In this chapter and in most parts of my work physical constants like $\hbar$ are set equal to one.

2.1.1. The Landau Fermi liquid theory

For free fermions we get the exact result by solving the single-particle Schrödinger equation. This results in a dispersion relation $\epsilon(k) = k^2/2m$. The non interacting ground state is constructed by filling up the energy-levels up to the Fermi level ϵ_F (see fig.: 2.1). Now what happens if there are interactions between the particles? It is a striking and also surprising fact that many 3D metals, although the interactions are not weak, are well described by the simple model of non interacting quasiparticles, renormalized from the interaction. Landau solved this riddle by his work "The theory of a Fermi liquid "[14].

The main result is, that quasiparticle states evolve continuously from the free electron states and carry the same charge, spin and momentum. Quasiparticles essentially keep the same properties except that the mass m is replaced by renormalized mass m^* and they have a finite lifetime. Fortunately, the lifetime becomes very large for quasiparticles in the vicinity of the Fermi energy. The physical reason is, that due to the limited available phase space, the decayprobability of a quasiparticle becomes small upon approaching the Fermi surface. Only a small fraction of electrons can be scattered out of it. In other words the Fermi exclusion principle, and not too high energy(compared to Fermi energy) are the ingredients, sufficient for producing only low density of excitations and the applicability of Fermi liquid theory. So we have only small derivations of the momentum distribution from the ground state, $\delta n(\mathbf{k}) = n(\mathbf{k}) - n_0(\mathbf{k})$ and the change of energy can be written as an expansion in $\delta n(\mathbf{k})$:

$$\delta E = \sum_{\mathbf{k}} \left[\varepsilon_0(\mathbf{k}) - \mu\right] \delta n(\mathbf{k}) + \frac{1}{2} \sum_{\mathbf{k},\mathbf{k'}} \delta n(\mathbf{k}) f(\mathbf{k},\mathbf{k'}) \delta n(\mathbf{k'}) + \dots \quad , \tag{2.1}$$

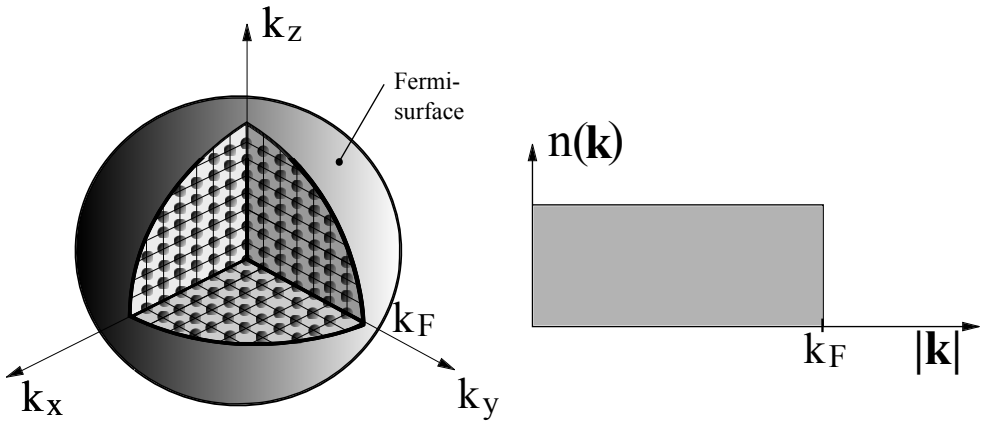

Figure 2.1.: Ground state of noninteracting Fermions where, due to Pauli exclusion principle, each energy level is filled with one fermion up to Fermi energy. This results in a Fermi sphere in momentum space(left picture). The right part of the figure is the momentum distribution, which shows constant particle density up to Fermi momentum and zero above.

Where the quasiparticle interaction is defined as:

$$f(\mathbf{k},\mathbf{k}') = 2\pi i z_{\mathbf{k}} z_{\mathbf{k}'} \lim_{\omega\to 0}\lim_{\mathbf{q}\to 0} \Gamma(\mathbf{k}, E_F, \mathbf{k}', E_F; \mathbf{q}, \omega) \quad . \tag{2.2}$$

with the particle-hole interaction vertex $\Gamma(\mathbf{k}, E, \mathbf{k'}, E'; \mathbf{q}, \omega)$, which describes the scattering of two particles from the initial state $(\mathbf{k}, \mathbf{k}')$ to the final state $(\mathbf{k}-\mathbf{q}, \mathbf{k}'+\mathbf{q})$. The electron Greens function is given by

$$G(\mathbf{k},\omega) = \frac{1}{\omega - \varepsilon_0(\mathbf{k}) - \Sigma(\mathbf{k},\omega)} \quad . \tag{2.3}$$

In general this equation has poles in the complex ω plane. Assuming that the singularities lie close to the real axis we can expand around $\omega = 0$ and $k = k_F$ and obtain a form similar to free electrons:

$$G(\mathbf{k},\omega) \approx \frac{Z}{\omega - \tilde{\epsilon}(\mathbf{k})} \quad ; \qquad Z^{-1} = 1 - \left.\frac{\partial \Sigma}{\partial \omega}\right|_{\omega=0} \quad . \tag{2.4}$$

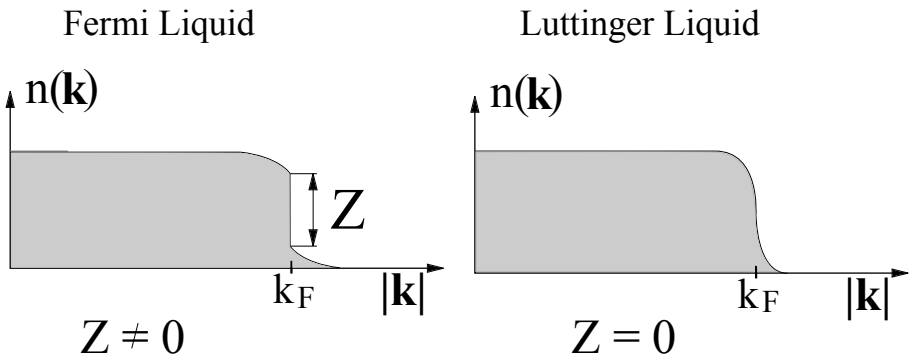

Figure 2.2.: Ground state momentum distribution $n(k)$ of a Fermi liquid compared to the one of a Luttinger liquid. Regardless to the strength of the interactions the Fermi liquid always has a jump on Fermi surface due to the quasiparticle pole. The Luttinger liquid has a continuous fading of n(k) on Fermi surface with a characteristic interaction dependent power law:
$n(k) - n(k_F) = \pm|k - k_F|^{\alpha}$.

The spectral function $A(\mathbf{k}, \omega)$, which is a delta function for free electrons, acquires the form of a Lorenzian for the quasiparticles, centered at $\omega = \epsilon(\mathbf{k})$. Its width, proportional to the inverse quasiparticle lifetime, goes to zero for $\mathbf{k} \to \mathbf{k}_F$. The total weight Z of the peaks can be interpreted as the probability, that an additional electron ends up in an eigenstate of the many particle Hamiltonian, which then coherently evolves with a given momentum. $(1 - Z)$ is the remaining part of the wave function, which decays into a continuum of eigenstates and represents the incoherent part of the spectrum. The spectral weight Z is also a renormalization of the jump on the Fermi surface (see fig.: 2.2). The infinite lifetime of the free particle excitations become finite in the interacting case, but it diverges when one goes closer to Fermi energy. This is mainly because there is lesser available phase space for scattering, the nearer to ϵ_F the quasiparticles are. Due to this fact, in most cases the finite lifetime can be neglected. What a great pity (or maybe not) that we will see soon, as successful Fermi liquid theory is in 2D and 3D as useless it is in 1D. On this place it is also worth noting, that in 3D Fermi liquid theory also fails in the case of attractive interactions, because of pairing Bardeen-Cooper-Schrieffer (BCS)

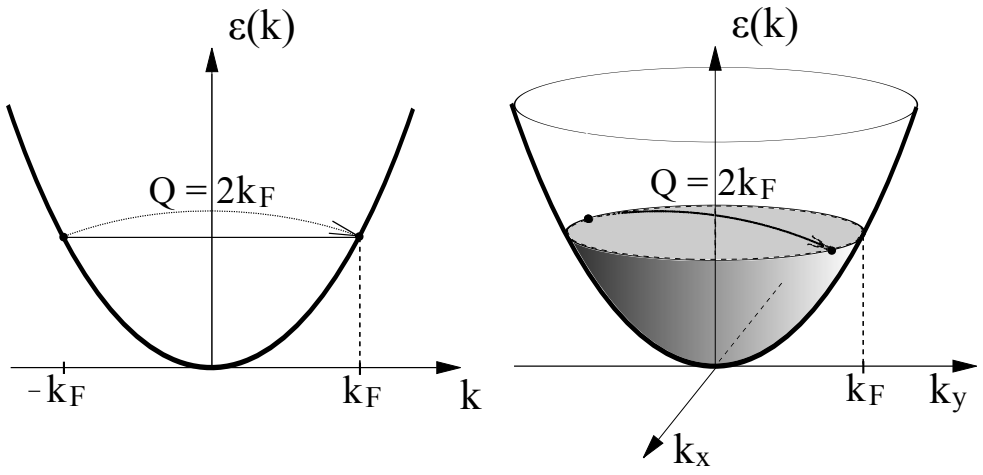

Figure 2.3.: Nesting properties of the Fermi surface. In one dimension(left picture) a translation of $Q = 2k_F$ brings the left and the right Fermi point(which is the whole Fermi surface) into each other. In higher dimensions(right picture) points are of measure zero and so the arising divergences are less severe.

instability, which opens a gap at Fermi energy and makes conventional superconductivity possible.

2.1.2. Breakdown Fermi liquid theory in 1D

There are many ways to show that Fermi liquid theory is not appropriate in 1D. First of all, handwaving but useful to imagine is, that a moving fermion in 1D always has to push away its neighbor particles. This is a consequence of the interaction and the dimensional constraints. Intuitively follows, and turns out to be right, that momentum transfer can never be neglected in 1D. But this is a prerequisite in the formulation of Fermi liquid theory (see equation 2.2). Also special in 1D is that the whole Fermi surface consist only of two points, which can be matched identically into each other by a translation of $Q = \pm k_F$. This is also called perfect nesting (see fig.: 2.3). In consequence of this, the particle-hole susceptibility (the so called Peierls channel) becomes divergent. Mean field treatment like in BCS theory predicts for repulsive interactions a charge density wave instability with a finite transition temperature. However, of course at finite temperatures no long range order can exist in 1D. A closer investigation shows, that Cooper and Peierls

channel interfere and destroy the feasibility of mean field treatment. If one still wants to do perturbation theory, one probably has a hard time by doing parquet approximation [2]. Due to these competing, partly canceling, instabilities, the systems behavior at $T = 0$ is related to the behavior of systems in higher dimensions at critical temperature. The role of the temperature is played here by quantum fluctuations and so such systems are called quantum critical.

Another way to see that Fermi liquid theory does not work, is to calculate the self energy in second order perturbation theory. This becomes feasible when the system is limited to small momentum transfer (g_4 processes). It means that only interactions between particles on the same side of the Fermi surface(=two points in 1D) are treated. For particles with opposite spin only the Feynman diagram called polarization insertion gives a contribution to the self energy. So the self energy can be calculated and inserted into equation 2.3. What follows is that the equation which should tell us the energy for the quasiparticle excitations thus has now two solutions. Unfortunately (or maybe not) going to higher order perturbation theory creates more and more poles ending in a branch cut.
For forward scattering between particles on opposite sides of the Fermi surface (g_2 processes), there is a pole in the Greens function (2.3), but it vanish when $k \to k_F$ like $z_k \propto -1/\ln|rk - k_F|$. Higher orders in perturbation theory lead to higher powers of the logarithm. Summing up ends in a power law and does not restore a quasiparticle pole. (for more details see: [25]; pages: 10 - 13 and references therein)

2.2. The Tomonaga-Luttinger model

Here again let us follow in main points: [25].
Let me first introduce the basic spirit of Luttinger liquid approach in a few sentences. Using rigorous formulation based on operator identities, the Tomonaga-Luttinger model is exactly solvable for forward scattering and linearized dispersion relation. Both, kinetic and interaction terms of the Hamiltonian can be written in a form quadratic in boson operators, which can then be exactly diagonalized by Bogoliubov transformation. Also, all correlation functions can be calculated exactly by representing the fermion fields with boson operators using bosonization identity[7]. Its validity is in principle not dependent on the strength of the interactions. Moreover, it constitutes the universality class for the low energy properties of all gap-less 1D quantum systems.

2.2.1. Collective bosonic excitations

As we have seen in the previous chapter, fermionic quasiparticles are not an appropriate description in 1D. Soon we will see that collective bosonic excitations will be a much more practicable choice. This is because of the remarkable properties of particle-hole excitations $c^\dagger_{k+q}c_k\,|FS\rangle$ in 1D compared to its higher dimensional counterparts(see fig.: 2.4, 2.5).

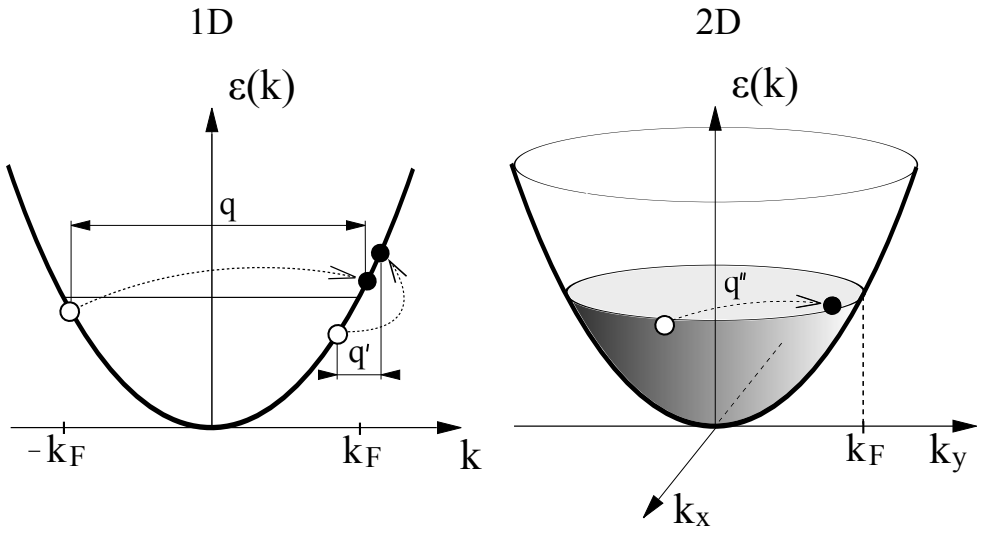

Figure 2.4.: Particle hole excitations for 1D and 2D. In 2D and 3D an excitation with specified amount energy can be generated having an arbitrary momentum. In 1D due to the geometrical constraints energy and momentum can not be chosen independently from each other.

The energy of a particle-hole excitation is:

$$E_k(q) = \epsilon(k+q) - \epsilon(k) \quad , \tag{2.5}$$

where $\epsilon(k)$ must be occupied and $\epsilon(k+q)$ must be empty. By insertion of a quadratic dispersion in k one gets exactly a linear dispersion for the average of excitation energy $E_k(q)$:

$$E(q) = \frac{k_F q}{m} \quad , \qquad \delta E(q) = \frac{q^2}{m} \quad ; \qquad \text{for k } \in [k_F - q, k_F] \quad . \tag{2.6}$$

Obviously the bosonic particle-hole excitations in 1D provide an analogical concept to the fermionic quasiparticles in higher dimensions. With $q \to 0$ the uncertainty of their energy $\delta E(q)$ approaches zero much faster than the mean value $E(q)$. So the quasiparticles have well defined momentum q and energy $E(q)$. Their lifetime is $\gamma \propto 1/\delta E(q)$. This, and what we have to realize next, are the main ingredients for a successful theory of 1D fermions. What is already intuitive clear is, that there would be a way to,

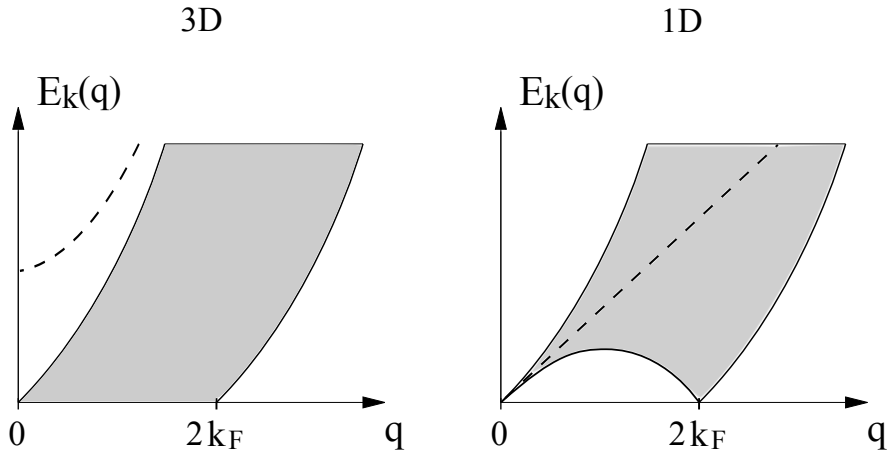

Figure 2.5.: Particle hole spectrum for 3D and 1D. In 2D and 3D particle hole excitations can decay in its lower energy counterparts. In 1D this does not happen, because the white area in the phase space(picture on the right) is not available for particle hole excitations. Also the collective plasmon excitations in RPA approximation are drawn in the figure as dotted lines. One can see while in 3D it has a quadratic dispersion, in 1D it is a linear one, which proceeds inside the area where the particle hole excitations lie.

at least approximately, treat fermionic two particle interactions as interaction between bosonic densities. But perhaps more surprising and also essential is the fact, that in 1D also the kinetic energy can be written in the form of densities. This relation we will also see below is known as Kronig identity [13].

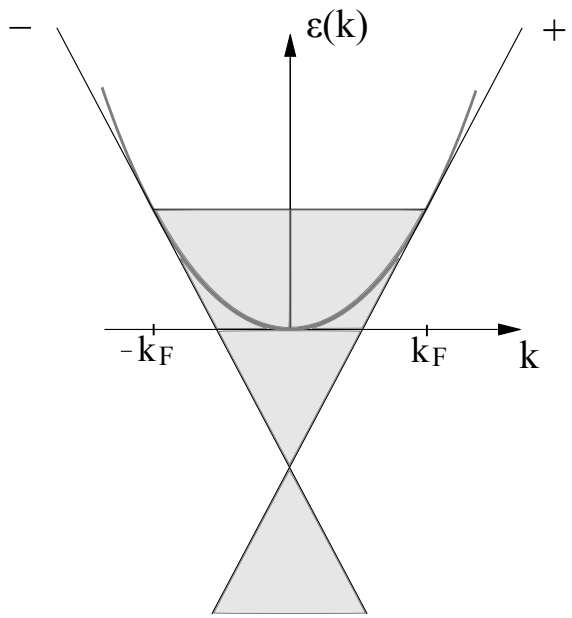

Figure 2.6.: Linearizing the dispersion relation and including unphysical but inert energy states with k from $-\infty$ to ∞ lead to the Tomonaga-Luttinger model.

2.2.2. General Hamiltionian of a 1D fermionic system

The general Hamiltonian for a 1D fermionic system(suppressing the spin indices for simplicity) consists of a kinetic energy and a two particle interaction term:

$$H = H_0 + H_{int} \quad , \tag{2.7}$$

$$H_0 = \sum_k \xi_k c_k^\dagger c_k \quad , \tag{2.8}$$

$$H_{int} = \frac{1}{L} \sum_{k,k',q\neq 0} V(q) c_k^\dagger c_{k'}^\dagger c_{k'-q} c_{k+q} \quad . \tag{2.9}$$

Now it was the idea of Tomonaga[24] to use a linearized dispersion relation(see fig.: 2.6), what is perfectly justified in the low energy sector(small $|k - k_F|$) in which we can approximate:

$$\xi_k \approx (|k| - k_F)\, v_F \tag{2.10}$$

Only electrons near the Fermi points contribute to the properties of the system, the others, lie deeper in the Fermi sea, can not be excited and consequently are inert. It follows that the electrons can be separated into left moving ones '−' (around the left

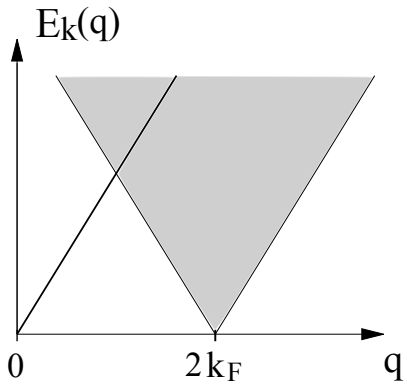

Figure 2.7.: Particle hole spectrum for linearized dispersion relation. Where excitations have exactly defined energy $E(q)$ for every momentum q, when momentum transfer q is small.

Fermi point) and right moving ones '+'(around the right Fermi point) respectively. Note that low energies are meant in comparison to Fermi energy. Typical for excitations in solid state are for example a few meV and Fermi energies are typical of the order of eV. So we write the destruction operator for an electron with momentum k as destruction operators near the right or left Fermi points dependent on the sign of k.

$$c_k = c_{k,+}\Theta(k) + c_{k,-}\Theta(-k) \tag{2.11}$$

And the same for the creation operators respectively. After insertion of this operators into equation 2.9, the interaction splits up into intra-branch interactions, which are the one between particles moving in the same direction(called g_4), and inter-branch interactions, which are the one between particles moving in the opposite direction(called g_2). A calculation for spin-less fermions can be found in Appendix [F]. For fermions with spin it makes also a difference whether the particles change their branch (backward scattering), or if they stay on the same side of the Fermi surface after scattering(forward scattering). In general additional interaction processes (g_1, g_3) appear, beside the ones, written in the Tomonaga-Luttinger Hamiltionian(see equation: 2.12 and fig.:2.8). In particular g_3 processes, the so called umklapp scattering, spoils charge current conservation. However renormalization group analysis (e.g. [22]) can be shown that umklapp scattering becomes irrelevant, except for commensurate band filling. Physically, this emerges because

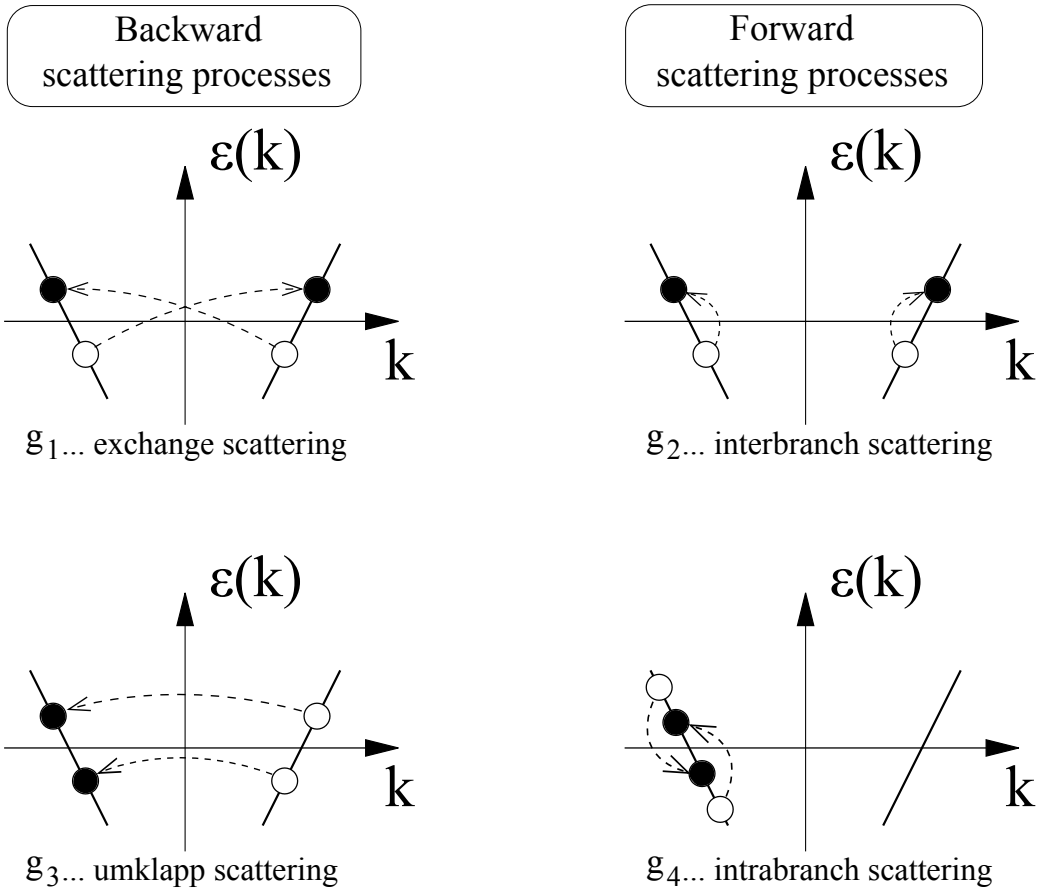

Figure 2.8.: Scattering processes in Tomonaga-Luttinger model.

it violates momentum conservation. The g_1 processes, called exchange or backward scattering spoil spin current conservation and become irrelevant for repulsive interactions. For attractive interactions however it opens a gap and can only be treated by the Sine-Gordon model and so far exactly solved only on the Luther-Emery point[15].

2.2.3. The Tomonaga-Luttinger Hamiltionian

[24][17][8]
We write the Hamiltonian for the model in the form, first used by Luttinger, with linearized infinite dispersion $k \in [-\infty, \infty]$ for both the left and the right mover branch (see fig.: 2.6). All states with large negative energy are occupied and do not influence the low energy properties of the model. Since there can't be reference to an infinite particle number, normal ordering ': ... :' (which is defined by right side of the equation

2.16) is used.
The Tomonaga-Luttinger Hamiltonian reads:

$$H = H_0 + H_2 + H_4 \quad , \tag{2.12}$$

$$H_0 = \sum_{r,k,s} v_F(rk - k_F) : c^\dagger_{rks} c_{rks} : \quad , \tag{2.13}$$

$$H_2 = \frac{1}{L} \sum_{q\neq 0,s,s'} \left[g_{2\parallel}(q)\delta_{s,s'} + g_{2\perp}(q)\delta_{s,-s'} \right] \rho_{+,s}(q)\rho_{-,s'}(-q) \quad , \tag{2.14}$$

$$H_4 = \frac{1}{2L} \sum_{r,q\neq 0,s,s'} \left[g_{4\parallel}(q)\delta_{s,s'} + g_{4\perp}(q)\delta_{s,-s'} \right] : \rho_{r,s}(q)\rho_{r,s'}(-q) : \quad , \tag{2.15}$$

with the density fluctuation or also called momentum shifting operators:

$$\rho_{r,s}(q) = \sum_k : c^\dagger_{r,k+q,s} c_{r,k,s} := \sum_k \left(c^\dagger_{r,k+q,s} c_{r,k,s} - \delta_{q,0} \langle c^\dagger_{r,k,s} c_{r,k,s} \rangle_0 \right) \quad . \tag{2.16}$$

To keep track of the indices, here a short collection:

$s = \uparrow, \downarrow$	$r = +, -$
$\uparrow \ldots$ spin up	$+ \ldots$ right mover
$\downarrow \ldots$ spin down	$- \ldots$ left mover
$g \ldots$ *interaction between* :	
$_2 \ldots$ opposite branch	$_\perp \ldots$ opposite spin
$_4 \ldots$ same branch	$_\parallel \ldots$ same spin

As already mentioned before, the crucial point is that the kinetic energy part of the Hamiltonian can also be represented with bosonic density fluctuation operators. This is due to the fact that these operators have following commutation with the kinetic term(equation: 2.13).

$$[H_0, \rho_{r,s}(q)] = v_F r q \rho_{r,s}(q) \tag{2.17}$$

With the help of this, one gets the so called Kronig identity [13]:

$$\begin{aligned} H_0 &= \sum_{r,k,s} v_F(rk - k_F) : c^\dagger_{rks} c_{rks} : \\ &= \frac{\pi v_F}{L} \sum_{r,q\neq 0,s} : \rho_{r,s}(q)\rho_{r,s}(-q) : \quad + C(N_{+,s}, N_{-,s}) \end{aligned} \tag{2.18}$$

The $N_{r,s} = \rho_{r,s}(q = 0)$ are the deviation of the number of particles relative to the Fermi sea , which has an infinite number of particles that fill the energy levels up to the Fermi

energy. The total current is given by number of right movers minus number of left movers:

$$J_s = N_{+,s} - N_{-,s} \quad , \tag{2.19}$$

and the total charge by:

$$N_s = N_{+,s} + N_{-,s} \quad . \tag{2.20}$$

For completeness one should mention that the density fluctuation operators obey commutation relations, known as the Kac-Moody algebra in field theory:

$$[\rho_{r,s}(q), \rho_{r',s'}(-q')] = -\delta_{r,r'}\delta_{s,s'}\delta_{q,q'}\frac{rqL}{2\pi} \quad . \tag{2.21}$$

To further develop the Hamiltonian, we transform the density fluctuation operators in a way that they obey canonical boson operator relations:

$$\begin{aligned} b_s(q) &= -i\sqrt{\frac{2\pi}{|q|\,L}}[\Theta(q)\rho_{+,s}(-q) - \Theta(-q)\rho_{-,s}(q)] \quad , \\ b_s^\dagger(q) &= -i\sqrt{\frac{2\pi}{|q|\,L}}[\Theta(q)\rho_{+,s}(q) - \Theta(-q)\rho_{-,s}(-q)] \quad , \end{aligned} \tag{2.22}$$

$$\left[b_s(q), b_{s'}^\dagger(q')\right] = \delta_{s,s'}\delta_{q,q'} \quad . \tag{2.23}$$

In this way, the Hamiltonian acquires the form:

$$\begin{aligned} H = \frac{1}{2}\sum_{s,s',q\neq 0}\Big\{&\Big[\big(v_F + g_{4\parallel}(q)\big)\delta_{s,s'} + g_{4\perp}(q)\delta_{s,-s'}\Big]|q| \times \big(b_s^\dagger(q)b_{s'}(q) + h.c.\big) \\ &+\Big[-g_{2\parallel}(q)\delta_{s,s'} + g_{2\perp}(q)\delta_{s,-s'}\Big]|q| \times \big(b_s(q)b_{s'}(-q) + h.c.\big)\Big\} \\ &+ C(J_s, N_s) \quad . \end{aligned} \tag{2.24}$$

In the next step we introduce spin and charge variables. This is done by taking the symmetric and antisymmetric combinations of the operators with different spin. Symmetric

combination gives the charge operators. By taking the antisymmetric combination the spin operators are obtained:

$$b_\rho(q) = \frac{1}{\sqrt{2}}(b_\uparrow(q) + b_\downarrow(q)) \ ,$$
$$b_\sigma(q) = \frac{1}{\sqrt{2}}(b_\uparrow(q) - b_\downarrow(q)) \ . \tag{2.25}$$

In this way our H gets a simpler form, as it fully decouples into independent charge and spin part:

$$H = \frac{1}{2}\sum_{\nu,q\neq 0}\Big\{\big(v_F + g_{4\nu}(q)\big)|q| \times \Big(b_\nu^\dagger(q)b_\nu(q) + h.c.\Big) - g_{2\nu}(q)|q| \times \Big(b_\nu(q)b_\nu(-q) + h.c.\Big)\Big\} + C(J_\nu, N_\nu) \ . \tag{2.26}$$

Where the interaction parameters transform as:

$$(\nu = \rho, \sigma); \quad g_{i,\rho} = \frac{1}{2}(g_{i,\parallel} + g_{i,\perp}); \quad g_{i,\sigma} = \frac{1}{2}(g_{i,\parallel} - g_{i,\perp}) \ . \tag{2.27}$$

2.3. Bogoliubov-Transformation

In the last chapter we obtained a Hamiltonian quadratic in boson operators. Fortunately we can get rid of the off diagonal terms and are able to get a Hamiltonian diagonal in other boson operators. This can be done by Bogoliubov transformation, which is a unitary transformation in a way that the commutation relations are conserved.
Starting from a Hamiltonian in following matrix form:

$$H = \begin{pmatrix} b^\dagger & b \end{pmatrix} \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} b \\ b^\dagger \end{pmatrix} \tag{2.28}$$

we carry on a transformation to new bosons:

$$\begin{pmatrix} \tilde{b} \\ \tilde{b}^\dagger \end{pmatrix} = \begin{pmatrix} \cosh\varphi & \sinh\varphi \\ \sinh\varphi & \cosh\varphi \end{pmatrix} \begin{pmatrix} b \\ b^\dagger \end{pmatrix} \quad \text{with} \qquad \gamma = -\tanh 2\varphi \quad , \tag{2.29}$$

We require that H is diagonal in the new $\tilde{b}$ and that the commutation rules are fulfilled. By taking $\gamma = -\tanh 2\varphi$ we obtain:

$$\begin{pmatrix} \tilde{b}^\dagger & \tilde{b} \end{pmatrix} \begin{pmatrix} \sqrt{1-\gamma^2} & 0 \\ 0 & \sqrt{1-\gamma^2} \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{b}^\dagger \end{pmatrix} \quad , \tag{2.30}$$

it is easy to see that:

$$[\tilde{b}, \tilde{b}^\dagger] = ... = \left(|\cosh(\tilde{\varphi})|^2 - |sinh(\tilde{\varphi})|^2\right) \cdot [b, b^\dagger] = 1 \quad . \tag{2.31}$$

Using this to diagonalize the Hamiltonian we identify,

$$\tanh 2\tilde{\varphi}_\nu(q) = \frac{g_{2\nu}(q)}{v_F + g_{4\nu}(q)} \quad . \tag{2.32}$$

In this way, the new diagonal elements of the Hamiltonian become:

$$\tilde{v}_\nu(q) \cdot |q| = (v_F + g_{4\nu}(q)) \sqrt{1 - \left(\frac{g_{2\nu}(q)}{v_F + g_{4\nu}(q)}\right)^2} \cdot |q| \tag{2.33}$$

$$= \sqrt{(v_F + g_{4\nu}(q))^2 - g_{2\nu}(q)^2} \cdot |q| \quad . \tag{2.34}$$

allowing to identify new velocities.
Finally we arrive at a Hamiltonian diagonal in the new boson operators:

$$\tilde{H} = \frac{1}{2} \sum_{\nu, q \neq 0} \tilde{v}_\nu(q)|q| \left(\tilde{b}^\dagger_\nu(q)\tilde{b}_\nu(q) + h.c.\right) \quad + \; C(J_\nu, N_\nu) \quad . \tag{2.35}$$

The constant is called particle mode and is of the form(for details see e.g. [25]):

$$C(J_\nu, N_\nu) = \frac{1}{L}\left[v_{N\nu}(q){N_\nu}^2 + v_{J\nu}(q){J_\nu}^2\right] \quad , \tag{2.36}$$

with:

$$v_{N\nu}(q) = v_\nu(q)/K_\nu \; ; \qquad v_{J\nu}(q) = v_\nu(q)K_\nu \quad . \tag{2.37}$$

K_ν is called renormalized coupling constant:

$$K_\nu(q) = \sqrt{\frac{v_F + g_{4\nu}(q) - g_{2\nu}(q)}{v_F + g_{4\nu}(q) + g_{2\nu}(q)}} \quad . \tag{2.38}$$

The Hamiltonian contains, bosonic and fermionic excitations. The velocities of the bosonic ones are given by the v_ν, which are normally different for spin and charge in the presence of interactions ($K_\nu \neq 1$) and lead to spin-charge separation. The velocities of the fermionic excitations are given by $v_{N,\nu}$, which correspond to charge excitations and $v_{J,\nu}$, which describe current excitations.

- $v_{N,\rho}$ represents the shift of chemical potential under variation of Fermi wave vector $\delta\mu = v_{N,\rho}\delta k_F$.
- $v_{N,\sigma}$ is the relation of magnetic field to magnetization $M = v_{N,\sigma}(k_{F\uparrow} - k_{F\downarrow})$.
- Finally, $v_{J,\nu}$ measures the required energy to create persistent charge($\nu = \rho$) and spin($\nu = \sigma$) currents respectively.

It is worth noting, that the parameters v_ν and K_ν together are sufficient to describe the low energy physics of 1D quantum systems without gap.

2.4. More about Bosonization

Bosonization also allows to express the fermion field operators $\psi_{r,s}(x)$, $\psi^\dagger_{r,s}(x)$ in terms of the canonical boson operators $b_s(q)$ and $b^\dagger_s(q)$. This is very useful, because we are interested in correlation functions and they can be calculated straightforward in this formulation, as can be seen further below. The bosonic representation of the fermion fields was discovered by Mattis[18], and by Luther and Peschel[16]. Later it was brought to its major form of a operator identity by Heidenreich et al. [9] and finally by Haldane[8]. A rigorous, and in my opinion also illuminating, Introduction to constructive bosonization approach is given in [26].

2.4.1. Bosonization requirements

For possible other applications one should note that the linear dispersion is not necessary for doing bosonization. This is because the bosonization identity has the status of an operator identity, valid for every state in the whole Fock space and can be derived without defining any form for the dispersion relation. But then kinetic energy becomes complicated. Actually necessary that the formalism above work, in its form presented above, the dispersion must be linear and the interaction must be of g_2 and g_4 type only.

2.4.2. Why it works

The fundamental reason why bosonization works is that the Fock space $\mathcal{F}$ can be reorganized as a sum over Hilbert spaces $\mathcal{H}_\mathcal{N}$ with fixed particle numbers $\mathcal{N}$.

$$\mathcal{F} = \sum_{\oplus\mathcal{N}} \mathcal{H}_\mathcal{N} \tag{2.39}$$

Within each Hilbert space $\mathcal{H}_\mathcal{N}$ every configuration can be reached by acting of bosonic operators on its ground state(filled Fermi sea). For moving from one Hilbert space, with a given number of particles $\mathcal{H}_\mathcal{N}$, to another Hilbert space with an different number of particles $\mathcal{H}_{\mathcal{N}'}$, operators which can raise and lower the number of fermions are required. In addition, these operators, called Klein factors η_r, $\eta^\dagger_r$, ensure anti-commutation of the fermion fields.

2.4.3. The bosonization identity

Here let us follow [26]. A detailed investigation therein leads to an expression for the original fermion operators which contains Klein factors and an exponential in bosons. This works, because the action of a fermion field operator on an arbitrary state in a Hilbert space, with a defined number of particles, is an eigenstate of the boson operator and so can be represented as a coherent state of bosons. This can be shown by using the definition of the bosonic operators:

$$\begin{aligned} b_s(q) &= -i\sqrt{\frac{2\pi}{|q|\,L}}[\Theta(q)\rho_{+,s}(-q) - \Theta(-q)\rho_{-,s}(q)] \quad , \\ b_s^\dagger(q) &= -i\sqrt{\frac{2\pi}{|q|\,L}}[\Theta(q)\rho_{+,s}(q) - \Theta(-q)\rho_{-,s}(-q)] \quad , \end{aligned} \tag{2.40}$$

$$\rho_{r,s}(q) = \sum_k : c^\dagger_{r,k+q,s} c_{r,k,s} : \tag{2.41}$$

and the Fourier transform of the fermion operators:

$$\psi_{r,s}(x) = \sqrt{\frac{2\pi}{L}} \sum_k e^{-ikx} c_{r,k,s} \quad , \tag{2.42}$$

where one obtain:

$$\begin{aligned} [b_{s'}(q), \psi_{r,s}(x)] &= \delta_{r,r'}\delta_{s,s'}\alpha(q,x)\psi_{r,s}(x) \\ [b^\dagger_{s'}(q), \psi_{r,s}(x)] &= \delta_{r,r'}\delta_{s,s'}\alpha^*(q,x)\psi_{r,s}(x) \quad . \end{aligned} \tag{2.43}$$

Because $b_s(q)|\vec{N}\rangle_0 = 0$ it can easily be seen that $\psi_{r,s}(x)|\vec{N}\rangle_0$ is an eigenstate of $b_s(q)$ with eigenvalue $\alpha(q,x)$.
This lead to the conclusion that this state must have an coherent-state representation in boson operators. After some calculations one get write down the bosonization identity. Here let us only look at the result, in a for our purposes well suited form. For details about the derivation see e.g. [26].

$$\Psi_r(x,t) = \frac{\eta_r}{\sqrt{\alpha}} \exp\left(i\Phi_r(x)\right) \tag{2.44}$$

with the boson field:

$$\begin{aligned} \Phi_r(x,t) = \sum_{q>0} \left(\frac{2\pi}{qL}\right)^{1/2} e^{-\frac{\alpha q}{2}} \left[b^\dagger(q,t)\cdot e^{-irqx} + b(q,t)\cdot e^{irqx}\right] \\ + \; r\varphi_{0,r} \; + \; \frac{\pi}{L} N_r x \quad . \end{aligned} \tag{2.45}$$

α ... short distance cutoff	N_r ... number right movers
η_r ... Klein factor	$[\varphi_{0,r}, N_r] = i$

At this point I want to mention that there are many different notations with different definitions of the fields. Hopefully in order to help the reader to make easier connections with other literature, I summarize at least one notation, which is used quite often in the literature. It was to my knowledge first used by Kane and Fisher [12] and is related to the notation used here as follows:

$$\begin{aligned}
\phi(x) &= \frac{1}{2\sqrt{\pi}}\left[\bar{\Phi}_+(x) + \bar{\Phi}_-(x)\right] \ldots \text{ displacement field} \\
\rho(x) &= \partial_x\, \phi(x) \ldots\ldots\ldots\ldots \text{ density field} \\
\theta(x) &= \frac{1}{2\sqrt{\pi}}\left[\bar{\Phi}_+(x) - \bar{\Phi}_-(x)\right] \ldots \text{ phase field} \\
\Pi(x) &= \partial_x\, \theta(x) \ldots\ldots\ldots\ldots \text{ momentum field}
\end{aligned}$$

$$\begin{aligned}
&\text{with :} \quad [\phi(x), \Pi(x')] = i\delta(x - x') \\
&\bar{\Phi}(x) = \Phi(x) - \varphi_0 - \frac{\pi}{L} N x \quad .
\end{aligned} \tag{2.46}$$

This lead to the form of the Hamiltonian:

$$H \propto \int dx \left(v_J \Pi(x)^2 + v_N \rho(x)^2 \right) \quad , \tag{2.47}$$

in which one can easy see similarities to its classical counterparts, e.g. an elastic string.

2.5. Physical properties

We have already exactly solved the Tomonaga-Luttinger Hamiltonian in section (2.2.3). There we obtained the full energy spectrum. Not satisfied with that? No, because additional really interesting entities in many body quantum theory are correlation functions, which (and this is maybe the most beautiful property of the model) can be computed exactly by straightforward calculation using bosonization identity (equation: 2.44). Naturally, with the correlation function one can build the Greens function and from that one get the spectral function. With that, one has the key to many experimental accessible quantities.
Next, I should mention that thermodynamics of Luttinger liquid is essentially the same, as is predicted by Fermi liquid theory in 1D. Compared to noninteracting fermion systems

only proportionality constants are replaced by renormalized ones. As we will see, a qualitative difference emerges, with observables which can only be expressed with correlation functions. This quantities all show anomalous (=interaction dependent) power law behavior.

2.5.1. Thermodynamical properties

The interacting system is described by bosons which are noninteracting, because we have a linear dispersion relation. 1D bosons have a specific heat which is linear in temperature. This is also true for fermions (linear dispersion in arbitrary dimension).

$$C(T) = \frac{dE}{dT} = \ldots = \gamma T \quad , \qquad \gamma = \frac{\gamma_0}{2}\left(\frac{v_F}{v_\rho} + \frac{v_F}{v_\sigma}\right) \quad , \tag{2.48}$$

with the coefficient from free electron case:

$$\gamma_0 = \frac{2\pi k_B^2}{3 v_F} \quad . \tag{2.49}$$

The compressibility κ defined as the relative change of volume per change of pressure, is written in 1D with the definition of pressure at zero temperature $p = -(\partial E_0/\partial L)_N$ and a quadratic N dependence of E_0 (see e.g. equation: 2.36) as:

$$\kappa = -\frac{1}{L}\left(\frac{\partial L}{\partial p}\right)_N = \frac{1}{L}\left(\frac{\partial^2 E_0}{\partial N_\rho^2}\right)_L^{-1} = \frac{2}{v_{N\rho}} \tag{2.50}$$

and half of this value for spinless case.

Also for the spin susceptibility one simply obtains a constant:

$$\chi = \frac{1}{L}\left(\frac{\partial^2 E_0}{\partial N_\sigma^2}\right)_L^{-1} = \frac{2}{v_{N\sigma}} \quad . \tag{2.51}$$

Indeed, the results are similar to the ones for the non interacting Fermi gas, but with renormalized velocities entering the equations, which is also obtained in Fermi liquid theory. Therefore, up to this point no exciting things happened. The Luttinger liquid shows no difference in the thermodynamics, compared to Fermi liquid theory. Fortunately, thermodynamics is not all one can know about physical systems. As we will see soon, essentially new physics emerge in 1D and this is described in an elegant way by using the bosonization identity.

2.5.2. Correlation functions

Under the physical quantities accessible by correlation functions are spin and charge density waves, singlet and triplet superconducting correlations, the single-particle spectral function $A(\omega, k)$ and from that the momentum distribution function $n(k)$ and the single-particle density of states $n(E)$. Experimental realizations, which probe the single-particle spectral function, are angle resolved photoemission spectroscopy and tunneling into the 1D quantum system. In the former, photons with definite direction and energy hit the probe and extract electrons from the metal. Energy and momentum resolved intensity of the electrons is measured and so one gets information about spectral function, from which the intensity can be calculated. In the later, the system is contacted by leads, or by a tip of a scanning tunneling microscope and the tunneling current is measured. The current depends on the local density of states of the lead/tip and the system. On the other hand, local density of states can be derived again from the local spectral function.
Lets first analyze the one-particle equal time correlation function also called one-particle density matrix:

$$\left\langle \psi(x)\psi^\dagger(0)\right\rangle_{H_0} = e^{-ik_F x}\left\langle \psi_+(x)\psi_+^\dagger(0)\right\rangle_{H_0} + e^{ik_F x}\left\langle \psi_-(x)\psi_-^\dagger(0)\right\rangle_{H_0} \quad . \tag{2.52}$$

Now we insert the bosonized fermion field(equation: 2.44) and get:

$$\left\langle \psi_+(x)\psi_+^\dagger(0)\right\rangle_{H_0} \propto \left\langle \exp\left(i\phi_+(x)\right)\exp\left(-i\phi_+(0)\right)\right\rangle_{H_0} \quad . \tag{2.53}$$

Next step is to use the operator identity:

$$e^A e^B = e^{\frac{1}{2}[A,B]}\, e^{A+B} \qquad \text{for: } [A,B] \in \mathbb{C} \quad , \tag{2.54}$$

and obtain:

$$\begin{aligned}\left\langle \psi_+(x)\psi_+^\dagger(0)\right\rangle_{H_0} &\propto \exp\Big(-\frac{1}{2}\big[\phi_+(x),\phi_+(0)\big]\Big)\cdot\left\langle \exp\Big(i\big(\phi_+(x)-\phi_+(0)\big)\Big)\right\rangle_{H_0} \\ &\propto \exp\Big(-\frac{1}{2}\big[\phi_+(x),\phi_+(0)\big]\Big)\cdot\exp\Big(i\frac{1}{2}\left\langle\big(\phi_+(x)-\phi_+(0)\big)^2\right\rangle_{H_0}\Big)\end{aligned} \tag{2.55}$$

where in the last line we used:

$$\left\langle e^{\hat{O}}\right\rangle = e^{\frac{1}{2}\langle\hat{O}^2\rangle} \quad , \tag{2.56}$$

valid for expectation values of operators linear in bosons.

The computation of the commutator is simple and one get:

$$\begin{aligned}\left[\phi_+(x),\phi_+(0)\right] &= \sum_{q>0}\frac{2\pi}{qL}e^{-\alpha q}\Big\{e^{-iqx}\underbrace{\left[b^\dagger(q),b(q)\right]}_{1} + e^{iqx}\underbrace{\left[b(q),b^\dagger(q)\right]}_{-1}\Big\}\\ &= \sum_{q>0}\frac{2\pi}{qL}e^{-\alpha q}\sin(qx)\\ &\overset{L\to\infty}{\propto} \arctan\frac{x}{\alpha}\\ &\overset{\alpha\to 0}{\propto} \operatorname{sgn}(x)\quad , \end{aligned} \tag{2.57}$$

and for the expectation value we insert the Bogoliubov transformed form of the operators $(\tilde{b},\tilde{b}^\dagger)$, in which the Hamiltonian is diagonal and in ground state:

$$\left(\phi_+(x)-\phi_+(0)\right) = \sum_{q>0}\sqrt{\frac{2\pi}{qL}}e^{-\frac{\alpha q}{2}}\left[b^\dagger(q)\left(e^{-iqx}-1\right)+b(q)\left(e^{iqx}-1\right)\right]$$

$$\begin{aligned} b^\dagger(q) &= \sinh\phi(q)\cdot\tilde{b}(-q)+\cosh\phi(q)\cdot\tilde{b}^\dagger(q)\\ b(q) &= \cosh\phi(q)\cdot\tilde{b}(q)\quad +\sinh\phi(q)\cdot\tilde{b}^\dagger(-q)\quad . \end{aligned}$$

Only the diagonal terms have a expectation value different form zero. The diagonal terms are particle number operators. Their expectation value is given by Bose-Einstein distribution, which for temperature zero simply means, that all bosons n_0 are in the $q=0$ ground state. $n(q=0)=n_0, n(q\neq 0)=0$

$$\begin{aligned}\left\langle\left(\phi_+(x)-\phi_+(0)\right)^2\right\rangle_{H_0} &= \sum_{q>0}\frac{2\pi}{qL}e^{-\alpha q}\;2\left(1-\cos(qx)\right)\\ &\cdot\Big\{\cosh^2\phi(q)\underbrace{\left\langle\tilde{b}^\dagger(q)\tilde{b}(q)+\tilde{b}(q)\tilde{b}^\dagger(q)\right\rangle_{H_0}}_{2n_0(q)-1}+\sinh^2\phi(q)\underbrace{\left\langle\tilde{b}(-q)\tilde{b}^\dagger(-q)+\tilde{b}^\dagger((-q)\tilde{b}-q)\right\rangle_{H_0}}_{2n_0(q)-1}\Big\}\\ &= \sum_{q>0}\frac{2\pi}{qL}e^{-\alpha q}\cos(qx)\;\cosh 2\phi(q)\end{aligned} \tag{2.58}$$

Where the sum can be evaluated by introducing momentum independent ϕ, so that

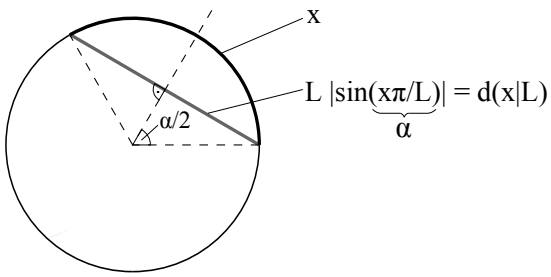

Figure 2.9.: Illustration of the cord function d(x|L) with L being the circumference and x the arc-length distance between two points on the circle.

$\sinh\phi(q) \approx e^{-|q|R_0/2}\sinh\phi(0)$, ($R_0$ order of interaction range) and using summation formula:

$$\sum_{q>0}\frac{2\pi}{qL}e^{-\alpha q}\cos(qx) = -\ln\left(\frac{2\pi}{L}d(x+i\alpha|L)\right)+\frac{\pi}{L}\alpha \quad , \tag{2.59}$$

with $d(z|L) = L|\sin(z\pi/L)|$ being the cord function.
Now we get the following result for the single-particle equal time correlation function:

$$\left\langle\psi_+(x)\psi_+^\dagger(0)\right\rangle_{H_0} = \frac{c\,\mathrm{sgn}(x)}{d(x|L)^{\cosh 2\varphi}} \quad . \tag{2.60}$$

With c being a finite constant.
In the limit of large L one can simply replace the cord function $d(x|L)$ by $|x|$ and get the famous anomalous interaction dependent power law behavior of the correlation function.

$$\left\langle\psi_+(x)\psi_+^\dagger(0)\right\rangle_{H_0} = \frac{c}{|x|^{\cosh 2\varphi}} \quad . \tag{2.61}$$

To get the time dependence of the correlation functions, one uses the time dependence of the operators in which the Hamiltonian is diagonal:

$$\tilde{b}(q,t) = e^{-i\tilde{v}(q)|q|t}\,\tilde{b}(q) \tag{2.62}$$

This gives the same result as simple replacement of the coordinates x by the light cone coordinates for right movers $(+)$ and left movers$(-)$: $x \pm \tilde{v}t$,

$$\left\langle \psi_+(x,t)\psi_+^\dagger(0,0)\right\rangle_{H_0} = \frac{c'\ \mathrm{sgn}(x+\tilde{v}t)}{d(x+\tilde{v}t|L)^{\cosh 2\varphi}} \quad . \tag{2.63}$$

One can also calculate the Greens function and look at its decay in time for e.g. right movers and get another well known result (α called the anomalous exponent):

$$\begin{aligned} G_+(x,t) \ &= -i\Theta(t)\left\langle\left\{\psi(x,t),\psi(x,0)\right\}\right\rangle_{H_0} \\ &\overset{L\to\infty}{\propto} \ |t|^{-\cosh 2\varphi} = |t|^{-\alpha+1} \quad . \end{aligned} \tag{2.64}$$

By Fourier transformation in time one gets the density of states(see fig. in Appendix: G.2):

$$n(\omega) \propto \int dt e^{i\omega t} t^{-\cosh 2\varphi} \ \propto \ \omega^{\cosh 2\varphi - 1} = \omega^\alpha \quad . \tag{2.65}$$

The spatial decay of Greensfunction at $t = 0$:

$$G_+(x,t) \propto |x|^{-\alpha-1} \quad . \tag{2.66}$$

Its Fourier transformation in space gives the result for the momentum distribution function (see also fig.:2.2):

$$n(k) - n(k_F) \propto \ -\mathrm{sgn}(k-k_F)|k-k_F|^\alpha \quad . \tag{2.67}$$

So all above quantities (and some others not listed here) have the form of a power law with a common interaction dependent exponent. Once again for clarity, I write down the connection of the different interaction dependent quantities one can find here and in literature:

$$\alpha = \ \cosh 2\varphi - 1 = \ \frac{1}{2}(K + 1/K) - 1 \tag{2.68}$$

$$K = \sqrt{\frac{v_F + g_4 - g_2}{v_F + g_4 + g_2}} \tag{2.69}$$

The same calculations can be done taking fermions with spin. As already seen before, the spin- and charge-part of the Hamiltonian separate and so to the correlation function. One gets a product of the correlation function for charge and spin, and taking half of the above value for α and taking K_ρ and K_σ.

Density Correlations.
Here I follow [5]. Next thing to do is to define various operators, which allow to compute spin/charge density and superconducting correlations. Finding out which correlation decay slowest, or rather which susceptibility diverges, makes it possible to draw a phase diagram. Here I will only introduce the parts of the operators appearing in phase diagram and write down their exponents of the asymptotic power law decay.

Charge density-density correlation function consists of the parts depending on the wave vektor: $q = 0,\ \pm 2k_F,\ \pm 4k_F, \ldots$ called $\rho\ , CDW\ , 4k_F$ respectively:

$$\begin{aligned} O_\rho(x) &= \sum_{r,s} \rho_{r,s} \\ O_{CDW}(x) &= \sum_{s} \psi^\dagger_{+,s}(x)\psi_{-,s}(x) \\ O_{4k_F}(x) &= \sum_{s} \psi^\dagger_{+,s}(x)\psi^\dagger_{+,-s}(x)\psi_{-,-s}(x)\psi_{-,s}(x) \quad . \end{aligned} \tag{2.70}$$

In principle the same for spin density-density correlation function, but only relevant in the phase diagram and so here necessary to mention is the $2k_F$, SDW with a standing for its x, y, and z component:

$$O_{SDW,a}(x) = \sum_{s,s'} \psi^\dagger_{+,s}(x)\sigma^a_{s,s'}\psi_{-,-s'}(x) \quad . \tag{2.71}$$

Moreover, the singlet superconducting (SS) and triplet superconducting (TS) operators turn out to be slowest decaying for attractive interactions:

$$\begin{aligned} O_{SS}(x) &= \sum_{s,s'} s\psi^\dagger_{+,s}(x)\delta_{s,s'}\psi^\dagger_{-,-s'}(x) \\ O_{TS,a}(x) &= \sum_{s,s'} s\psi^\dagger_{+,s}(x)\sigma^a_{s,s'}\psi^\dagger_{-,-s'}(x) \quad . \end{aligned} \tag{2.72}$$

All that operators can be expressed in terms of boson operators by insertion of the fermion field from equation: 2.44. With that, one can then calculate correlation functions of the form:

$$R_0(x,t) = -i\Theta(t)\left\langle \left[O(x,t), O^\dagger(0,0)\right]\right\rangle \quad . \tag{2.73}$$

It turns out that all have a part asymptotically decaying as a power law(2_{k_F}charge and spin density waves in addition oscillate with $\cos(2k_F x)$, SS and TS not oscillating):

$$R_o(x,t) \approx x^{-\alpha} \quad . \tag{2.74}$$

The exponents are respectively(note that for spin rotation invariance $K_\sigma = 1$):

$$\begin{aligned}
\alpha_\rho &= 2 \\
\alpha_{CDW} &= K_\rho + K_\sigma \\
\alpha_{4_{k_F}} &= 4K_\rho \\
\alpha_{SDW,x/y} &= K_\rho + K_\sigma^{-1} \\
\alpha_{SDW,z} &= K_\rho + K_\sigma \\
\alpha_{SS} &= K_\rho^{-1} + K_\sigma \\
\alpha_{TS_0} &= K_\rho^{-1} + K_\sigma \\
\alpha_{TS_{\pm 1}} &= K_\rho^{-1} + K_\sigma^{-1}
\end{aligned} \tag{2.75}$$

Now we recall that the response to an external perturbation in linear response theory is given by the Kubo formula:

$$\delta\left\langle A(x,t)\right\rangle = \left\langle A(x,t)\right\rangle_H - \left\langle A(x,t)\right\rangle_{H_0} = \int dx'dt'\chi(x-x',t-t')V(x',t') \quad , \tag{2.76}$$

with $V(x,t)$ being an external perturbation and the full Hamiltonian:

$$H = H_0 + \int dx V(x,t)B(x,t) \quad . \tag{2.77}$$

And the susceptibility computed by the retarded correlation function:

$$\chi(x-x',t-t') = -\Theta(t-t')\left\langle \left[A(x,t), B(x',t')\right]_\pm \right\rangle_{H_0} \quad . \tag{2.78}$$

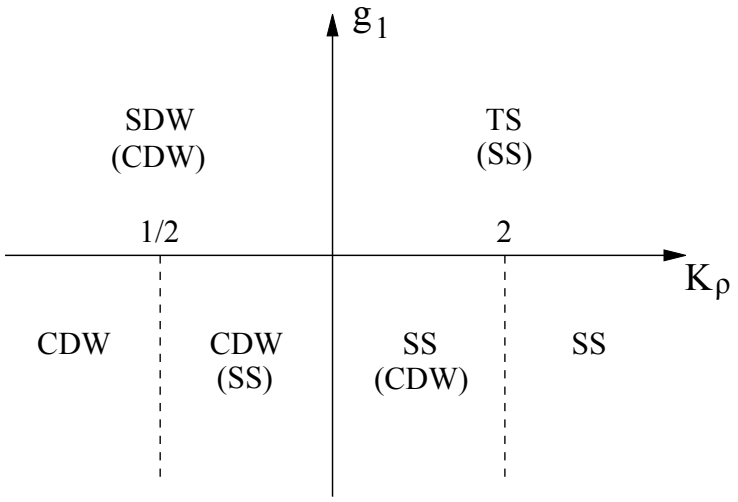

Figure 2.10.: Taken from [25]. Phase diagram for interacting spin-1/2 fermions in 1D for spin isotropic interactions. The phases correspond to the dominating fluctuations. For the backscattering term $g_1 \geq 0$ the spectrum is gap-less, for $g_1 < 0$ a gap opens. SDW and CDW is written for spin and charge density wave. SS and TS stands for singlet and triplet superconducting fluctuations respectively.

By doing such a calculation one get for the susceptibilities:

$$\chi \approx \omega^{\alpha-2} \quad . \tag{2.79}$$

For diverging susceptibilities, the system has a finite response even for infinitesimal perturbation. This allows to draw a phase diagram with the dominating fluctuations. The case when backscattering becomes relevant ($g_1 < 0$) leads to the opening of a spin-gap and has to be discussed separately, but I will include it in the figure here.

For repulsive long range and/or strong interactions the $4k_F$ fluctuations are most divergent and lead to the formation of a Wigner crystal. For very strong attractive interactions K_ρ becomes infinite. This instability leads to a phase separation because of electrons 'clumping to droplets'.

3. Time Evolution pictures

Not to bore advanced readers with this basic quantum mechanical stuff, but more as a preface to what follows, I write down what quantum mechanics tells us about time evolution. Especially in next chapter all operators will be written in the Heisenberg picture. The fact that the time evolution of operators has a simple form, when the Hamiltonian is diagonal in this operators, will be used to get more complicated time evolution in operators, in which the Hamiltonian is not diagonal. In principle, all information we can get from quantum mechanical systems are the expectation values of its operators, which correspond to physical observables. So lets start with the Schrödinger equation and its formal solution:

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle \tag{3.1}$$

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle \quad . \tag{3.2}$$

So we can write expectation value of an operator as:

$$\underbrace{\overbrace{\langle\psi(t)|}^{\langle\psi_0|e^{iHt}}}_{A(t)} A \overbrace{e^{-iHt}|\psi_0\rangle}^{|\psi(t)\rangle} = \langle A\rangle_{(t)} \tag{3.3}$$

Grouping as done by overbracing, puts the time evolution into the state and is called Schrödinger picture. By putting the time evolution into the operators we obtain the Heisenberg picture. Often used is also Dirac or interaction picture, where most of the time evolution is put into the operators, but a rest, coming from a perturbation, is put into the states. Here we will use the Heisenberg picture and so I will only describe that briefly.

Easily one derives the equation of motion which describes the time evolution of an operator. This is done by taking the total time derivative of the operator in Heisenberg picture $A(t)$:

$$\dot{A}(t) = \frac{d}{dt}\left(e^{iHt}Ae^{-iHt}\right)$$

$$= e^{iHt}\left(iHA - iAH + \partial_t A\right)e^{iHt}$$

$$= i\big[H, A(t)\big] + \left(\partial_t A\right)(t) \quad . \tag{3.4}$$

In the next chapter we will use creation and destruction operators of bosons and fermions. So we will apply the equation of motion with a time independent Hamiltonian diagonal in some operators to get their time evolution:

$$H = \sum_n \epsilon_n a_n^\dagger a_n \tag{3.5}$$

$$\dot{a}(t) = i\big[H, a_n(t)\big] = ie^{iHt}\big[H, a_n\big]e^{-iHt}$$

$$= ie^{iHt}\sum_{n'}\epsilon_{n'}\underbrace{\left[a_{n'}^\dagger a_{n'}, a_n\right]}_{-\delta_{n,n'}a_{n'}}e^{-iHt} = -\epsilon_n e^{iHt}a_n e^{-iHt}$$

$$= -\epsilon_n a_n(t) \quad . \tag{3.6}$$

So we get the time dependence of the destruction operator by the solution of this differential equation:

$$a_n(t) = e^{-i\epsilon_n t}a_n \tag{3.7}$$

The same calculation, or simply Hermitian conjugation, gives us the result for the creation operator:

$$a_n^\dagger(t) = e^{i\epsilon_n t}a_n^\dagger \quad . \tag{3.8}$$

4. Quench in N parallel spin-full LL-chains

4.1. Diagonalizing uncoupled LL-chains at t<0

At $t < 0$, we have N non interacting chains. We use bosonic density fluctuation(or interpreted in an other way: momentum shifting) operators in canonical form like in (2.2.3), now for each chain λ:

$$b_{\lambda,s}(q) = -i\sqrt{\frac{2\pi}{|q|\,L}}[\Theta(q)\rho_{\lambda,+,s}(-q) - \Theta(-q)\rho_{\lambda,-,s}(q)]$$
$$b^\dagger_{\lambda,s}(q) = -i\sqrt{\frac{2\pi}{|q|\,L}}[\Theta(q)\rho_{\lambda,+,s}(q) - \Theta(-q)\rho_{\lambda,-,s}(-q)] \quad ,$$

$$\rho_{\lambda,r,s}(q) = \sum_k :c^\dagger_{\lambda,r,k+q,s}c_{\lambda,r,k,s}: \quad \text{and} \quad \left\{c_{\lambda,r,k,s}, c^\dagger_{\lambda',r',k',s'}\right\} = \delta_{\lambda,\lambda'}\delta_{r,r'}\delta_{k,k'}\delta_{s,s'} \quad . \tag{4.1}$$

$s = \uparrow, \downarrow$	$r = +, -$
$\uparrow \ldots$ spin up	$+ \ldots$ right mover
$\downarrow \ldots$ spin down	$- \ldots$ left mover

The operators $b_{\lambda,s}(q)$, $b^\dagger_{\lambda',s'}$ obey the canonical boson commutation relations:

$$\left[b_{\lambda,s}(q), b^\dagger_{\lambda',s'}(q')\right] = \delta_{\lambda,\lambda'}\delta_{s,s'}\delta_{q,q'} \quad . \tag{4.2}$$

The so called zero modes, or also called particle modes (see equations 2.26, 2.36), are conserved by our Hamiltonians. Therefor, they do not give interesting contribution to our quench dynamics, so we work in the sector of the Hilbert space, where $(N = J = 0)$. Then our initial Hamiltonian, consisting of N identical replica of a Luttinger liquid without interactions between them, can be written as:

$$H_{init} = \frac{1}{2}\sum_{\lambda,s,s',q\neq 0}\Big\{\Big[\big(v_F + g_{4\parallel}(q)\big)\delta_{s,s'} + g_{4\perp}(q)\delta_{s,-s'}\Big]|q| \times \big(b^\dagger_{\lambda,s}(q)b_{\lambda,s'}(q) + h.c.\big)$$
$$+\Big[-g_{2\parallel}(q)\delta_{s,s'} + g_{2\perp}(q)\delta_{s,-s'}\Big]|q| \times \big(b_{\lambda,s}(q)b_{\lambda,s'}(-q) + h.c.\big)\Big\} \quad . \tag{4.3}$$

Next it is convenient to introduce spin and charge variables:

$$b_{\lambda,\rho}(q) = \frac{1}{\sqrt{2}}(b_{\lambda,\uparrow}(q) + b_{\lambda,\downarrow}(q) \quad ,$$
$$b_{\lambda,\sigma}(q) = \frac{1}{\sqrt{2}}(b_{\lambda,\uparrow}(q) - b_{\lambda,\downarrow}(q) \quad . \tag{4.4}$$

And now it happens the same as in (2.2.3), our H_{init} decouples in charge and spin part. Now separately for each chain:

$$H_{init} = \frac{1}{2}\sum_{\lambda,\nu,q\neq 0}\Big\{\big(v_F + g_{4\nu}(q)\big)|q| \times \Big(b^\dagger_{\lambda,\nu}(q)b_{\lambda,\nu}(q) + h.c.\Big) - g_{2\nu}(q)|q| \times \Big(b_{\lambda,\nu}(q)b_{\lambda,\nu}(-q) + h.c.\Big)\Big\} \tag{4.5}$$

$$(\nu = \rho,\sigma); \quad g_{i,\rho} = \frac{1}{2}(g_{i,\parallel} + g_{i,\perp}); \quad g_{i,\sigma} = \frac{1}{2}(g_{i,\parallel} - g_{i,\perp}) \quad . \tag{4.6}$$

Therefor each chain for its own can be diagonalized by Bogoliubov-transformation for charge and spin part:

$$\tilde{b}_{\lambda,\nu}(q) = \cosh(\tilde{\varphi}_\nu)\cdot b_{\lambda,\nu}(q) + \sinh(\tilde{\varphi}_\nu)\cdot b^\dagger_{\lambda,\nu}(-q) \quad . \tag{4.7}$$

With $\tilde{\varphi}$ called the coherence factor:

$$\tanh 2\tilde{\varphi}_\nu(q) = \frac{g_{2\nu}(q)}{v_F + g_{4\nu}(q)} \quad , \tag{4.8}$$

and renormalized velocities for charge and spin sector:

$$\tilde{v}_\nu(q) = \sqrt{(v_F + g_{4\nu}(q))^2 - g_{2\nu}(q)^2} \quad , \tag{4.9}$$

we obtain the diagonal Hamiltonian:

$$\tilde{H}_{init} = \frac{1}{2}\sum_{\lambda,\nu,q\neq 0}\tilde{v}_\nu(q)|q|\left(\tilde{b}^\dagger_{\lambda,\nu}(q)\tilde{b}_{\lambda,\nu}(q) + h.c.\right) \quad , \tag{4.10}$$

whose ground state we define as the state in which the system is prepared before the quench.

4.2. Interaction Quench at t=0

At $t = 0$ we switch on momentum transfer scattering between the chains. The Hamiltonian in terms of fermion operators is given by:

$$H_{final} = \sum_{\lambda,k,s} (|k| - k_F)\, v_F c^\dagger_{\lambda,k,s} c_{\lambda,k,s} + \frac{1}{L} \sum_{\lambda,\lambda',k,k',q\neq 0,s,s'} V(q) c^\dagger_{\lambda,k,s} c^\dagger_{\lambda',k',s'} c_{\lambda',k'-q,s'} c_{\lambda,k+q,s} \quad , \tag{4.11}$$

which can be brought into its bosonic form with identical transformations as done above:

$$H_{final} = \frac{1}{2} \sum_{\lambda,\lambda',\nu,q\neq 0} \Big\{ \big(v_F \delta_{\lambda,\lambda'} + g_{4\nu,\lambda\lambda'}(q)\big)|q| \times \Big(b^\dagger_{\lambda,\nu}(q) b_{\lambda',\nu}(q) + h.c.\Big) - g_{2\nu,\lambda\lambda'}(q)|q| \times \Big(b_{\lambda,\nu}(q) b_{\lambda',\nu}(-q) + h.c.\Big)\Big\} \tag{4.12}$$

From now on we work with a translation invariant system with periodic boundary conditions in transverse direction. The interactions between two chains are not dependent on the indices of each chain, but only on the distance between them. Obviously the natural thing to do, is to introduce a Fourier transform in the direction transverse to the chains. A detailed calculation is done in appendix [A]:

$$\begin{aligned} g_{i,\nu}(\mathbf{q}) &= \sum_\lambda \exp[-iq_\perp (R_\lambda - R_{\lambda'})] g_{i\nu,\lambda\lambda'}(q) \\ &= \sum_\lambda \exp[-iq_\perp R_\lambda] g_{i\nu,\lambda 0}(q) \quad ; \qquad \text{with the choice: } R_{\lambda'} = 0 \\ b_\nu(\mathbf{q}) &= \frac{1}{\sqrt{N}} \sum_\lambda \exp(iq_\perp R_\lambda) b_{\nu,\lambda}(q) \quad ; \qquad \text{and } \ \mathbf{q} = (q_\perp, q) \end{aligned}$$

$$\begin{aligned} R_\lambda &= a \cdot \lambda \quad && \text{with } a \text{ being the distance between the chains} \\ & && \text{and } \lambda = 0, \ldots, N-1 \\ q_\perp &= \frac{2\pi n}{L} && \text{with } L = Na \text{ the extension in transverse direction} \\ & && \text{and } n = 0, \ldots, N-1 \end{aligned} \tag{4.13}$$

So the Hamiltonian can be written in the Fourier transformed form as:

$$H_{final} = \frac{1}{2} \sum_{\nu,q_\perp,q\neq 0} \Big\{ \big(v_F + g_{4\nu}(\mathbf{q})\big)|q| \times \Big(b^\dagger_\nu(\mathbf{q}) b_\nu(\mathbf{q}) + h.c.\Big) - g_{2\nu}(\mathbf{q})|q| \times \Big(b_\nu(\mathbf{q}) b_\nu(-\mathbf{q}) + h.c.\Big)\Big\} \quad . \tag{4.14}$$

Note: For the uncoupled chains we obtain one LL in transverse momentum space. In the final coupled case we get N different LL, but fortunately without interactions between the LL.

So it is once again possible to diagonalize by Bogoliubov transformation. Now for each transverse momentum mode, and again spin and charge separately:

$$\bar{b}_\nu(\mathbf{q}) = \cosh\left(\bar{\varphi}_\nu(\mathbf{q})\right) \cdot b_\nu(\mathbf{q}) + \sinh\left(\bar{\varphi}_\nu(\mathbf{q})\right) \cdot b_\nu^\dagger(-\mathbf{q}) \quad . \tag{4.15}$$

With the coherence factor:

$$\tanh 2\bar{\varphi}_\nu(\mathbf{q}) = \frac{g_{2\nu}(\mathbf{q})}{v_F + g_{4\nu}(\mathbf{q})} \quad , \tag{4.16}$$

and renormalized velocities

$$\bar{v}_\nu(\mathbf{q}) = \sqrt{\left(v_F + g_{4\nu}(\mathbf{q})\right)^2 - \left(g_{2\nu}(\mathbf{q})^2\right)} \quad , \tag{4.17}$$

we get the diagonal Hamiltonian after the quench:

$$\bar{H}_{final} = \frac{1}{2} \sum_{\nu,\mathbf{q}\neq 0} \bar{v}_\nu(\mathbf{q})|q| \left(\bar{b}_\nu^\dagger(\mathbf{q})\bar{b}_\nu(\mathbf{q}) + h.c.\right) \quad . \tag{4.18}$$

4.3. Computation of the one particle correlation function

To calculate the correlation functions we calculate the time dependence of the creation and destruction operators. Without loss of generality, the first step is to compute the time evolution of spin ↑ particle destruction operators on chain 1 after the quench $b_{1,\uparrow}(\mathrm{q})$, in terms of operators in which the system was diagonal (and prepared to the ground state) before the quench $\tilde{b}_{\lambda,\nu}(q)$. This can be done by some calculations including Bogoliubov transform(2.3) and inverse Bogoliubov transform. Further, the simple time evolution of operators, in which the Hamiltonian is diagonal, can be used to evaluate the complicated time evolution of the system after the quench(see also chapter 3 about Heisenberg picture). A detailed computation is done in appendix [B] and gives the result:

$$b_{1,\uparrow}(q,t) = \frac{1}{\sqrt{2}N} \sum_{\nu,q_\perp,\lambda} \exp(iq_\perp R_\lambda) \Big\{ \mathbf{A}_\nu(\mathbf{q},t) \cdot \tilde{b}_{\nu,\lambda}(q) + \mathbf{B}_\nu(\mathbf{q},t) \cdot \tilde{b}^\dagger_{\nu,\lambda}(-q) \Big\} \tag{4.19}$$

with:

$$\begin{aligned} \mathbf{A}_\nu(\mathbf{q},t) &= \cosh\tilde{\varphi}_\nu(q) \cdot \alpha_\nu(\mathbf{q},t) - \sinh\tilde{\varphi}_\nu(q) \cdot \beta_\nu(\mathbf{q},t) \\ \mathbf{B}_\nu(\mathbf{q},t) &= -\sinh\tilde{\varphi}_\nu(q) \cdot \alpha_\nu(\mathbf{q},t) + \cosh\tilde{\varphi}_\nu(q) \cdot \beta_\nu(\mathbf{q},t) \end{aligned} \tag{4.20}$$

and:

$$\begin{aligned} \alpha_\nu(\mathbf{q},t) &= \cos\bar{v}_\nu(\mathbf{q})|q|t - i\sin\bar{v}_\nu(\mathbf{q})|q|t \cdot \cosh 2\bar{\varphi}_\nu(\mathbf{q}) \\ \beta_\nu(\mathbf{q},t) &= -i\sin\bar{v}_\nu(\mathbf{q})|q|t \cdot \sinh 2\bar{\varphi}_\nu(\mathbf{q}) \end{aligned} \tag{4.21}$$

It is easy to see that for $N = 2$ one recovers the result of the 2 chain model calculated by E. Perfetto[20].

Knowing the time dependence of our operators after the quench, we can now compute, without loss of generality, the equal time one particle spin $\uparrow$, right mover correlation function on chain 1 for $t \geq 0$:

$$C^{\uparrow}_{+,1}(x,t) = \frac{1}{\alpha}\left\langle \exp(iH_{final}t)\Psi_{1,\uparrow,+}(x)\Psi^{\dagger}_{1,\uparrow,+}(0)\exp(-iH_{final}t)\right\rangle_{\tilde{H}_{init}} \quad . \tag{4.22}$$

To express the fermion fields with our computed canonical boson operators we use the Bosonization Identity [7]:

$$\Psi_{1,\uparrow,+}(x,t) = \frac{\eta_+}{\sqrt{\alpha}}\exp\left(i\Phi_{1,\uparrow,+}(x)\right) \tag{4.23}$$

with the boson field:

$$\Phi_{1,\uparrow,+}(x,t) = \sum_{q>0}\left(\frac{2\pi}{qL}\right)^{1/2} e^{-\frac{\alpha q}{2}}\left[b^{\dagger}_{1,\uparrow}(q,t)\cdot e^{-iqx} + b_{1,\uparrow}(q,t)\cdot e^{iqx}\right] + \varphi_{0,+} + \frac{\pi}{L}N_+ x \quad . \tag{4.24}$$

$\alpha \ldots$ short distance cutoff	$N_+ \ldots$ number right movers
$\eta_+ \ldots$ Klein factor	$[\varphi_{0,+}, N_+] = i$

Using the operator identity(Baker-Hausdorf formula):

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]} \quad , \quad \text{for } [A,B] \in \mathbb{C} \tag{4.25}$$

and the expectation value of operators linear in bosons:

$$\left\langle e^{\hat{O}}\right\rangle = e^{\frac{1}{2}\langle \hat{O}^2\rangle} \quad , \tag{4.26}$$

follows for the correlation function:

$$\begin{aligned} C^{\uparrow}_{+,1}(x,t) &= \frac{1}{\alpha}\left\langle \exp\left(i\Phi_{1,\uparrow,+}(x,t)\right)\exp\left(i\Phi_{1,\uparrow,+}(0,t)\right)\right\rangle_{\tilde{H}_{init}} \\ &= \frac{1}{\alpha}\left\langle \exp\left\{i(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0,t)) - \frac{1}{2}[\Phi_{1,\uparrow,+}(x,t), \Phi_{1,\uparrow,+}(0,t)]\right\}\right\rangle_{\tilde{H}_{init}} \\ &= \frac{1}{\alpha}\exp\left\{-\frac{1}{2}\left\langle \left(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0,t)\right)^2\right\rangle_{\tilde{H}_{init}} - \frac{1}{2}\left[\Phi_{1,\uparrow,+}(x,t), \Phi_{1,\uparrow,+}(0,t)\right]\right\} \quad . \end{aligned} \tag{4.27}$$

In that way, we only have to calculate an expectation value and the commutator of boson fields consisting of our boson operators, from which we now know the exact time dependence(equation: 4.19):

$$\begin{aligned}
&\left[\Phi_{1,\uparrow,+}(x,t),\Phi_{1,\uparrow,+}(0,t)\right] = \\
&= \sum_{q>0}\left(\frac{\pi}{qNa}\right) e^{-\alpha q}\left[b^\dagger_{1,\uparrow}(q,t)\cdot e^{-iqx} + b_{1,\uparrow}(q,t)\cdot e^{iqx}, b^\dagger_{1,\uparrow}(q,t) + b_{1,\uparrow}(q,t)\right] \\
&= \sum_{q>0}\left(\frac{\pi}{qNa}\right) e^{-\alpha q}\Big(e^{-iqx}\underbrace{\left[b^\dagger_{1,\uparrow}(q,t), b_{1,\uparrow}(q,t)\right]}_{-1} + e^{iqx}\underbrace{\left[b^\dagger_{1,\uparrow}(q,t), b_{1,\uparrow}(q,t)\right]}_{1}\Big) \\
&= \sum_{q>0}\left(\frac{\pi}{qNa}\right) e^{-\alpha q}\big(2i\sin qx\big) \quad . \qquad (4.28)
\end{aligned}$$

The evaluation of the expectation value is more complicated. The lengthy algebra can be found in appendix [C]. In the main text is only given the result:

$$\begin{aligned}
&\left\langle\left(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0,t)\right)^2\right\rangle_{\tilde{H}_{init}} = \\
&= \sum_{q>0}\left(\frac{\pi}{qNa}\right) e^{-\alpha q}\left\langle\left(b^\dagger_{1,\uparrow}(q,t)\cdot\left(e^{-iqx}-1\right) + b_{1,\uparrow}(q,t)\cdot\left(e^{iqx}-1\right)\right)^2\right\rangle_{\tilde{H}_{init}} \\
&= \sum_{q>0}\left(\frac{\pi}{qN^2a}\right) e^{-\alpha q} 2(1-\cos qx)\sum_{\nu,q_\perp}\left(\left|\mathbf{A}_\nu(\mathbf{q},t)\right|^2 + \left|\mathbf{B}_\nu(\mathbf{q},t)\right|^2\right) \quad . \qquad (4.29)
\end{aligned}$$

When we bring the parts together, we get an expression for the correlation function:

$$C^{\uparrow}_{+,1}(x,t) = \frac{1}{\alpha}\exp\left\{-\sum_{q>0}\frac{\pi}{qN^2a}e^{-\alpha q}\left(Ni\sin qx + (1-\cos qx)\sum_{\nu,q_\perp}\left(\left|\mathbf{A}_\nu(\mathbf{q},t)\right|^2 + \left|\mathbf{B}_\nu(\mathbf{q},t)\right|^2\right)\right)\right\} \qquad (4.30)$$

$$\begin{aligned}
\left|\mathbf{A}_\nu(\mathbf{q},t)\right|^2 + \left|\mathbf{B}_\nu(\mathbf{q},t)\right|^2 &= \frac{1}{2}\cosh\left(2\tilde{\varphi}_\nu(q) - 4\bar{\varphi}_\nu(\mathbf{q})\right)\cdot(1-\cos 2\bar{v}_\nu(\mathbf{q})|q|t) \\
&\quad + \frac{1}{2}\cosh 2\tilde{\varphi}_\nu(q)\cdot(1+\cos 2\bar{v}_\nu(\mathbf{q})|q|t) \quad . \qquad (4.31)
\end{aligned}$$

The absolute values squared are calculated in appendix [D].

The dependence of the equations above on the interaction parameters is of the form: $\cosh(\operatorname{arctanh}(X)) = 1/\sqrt{1-X^2}$, with $X = g_2(q)/(v_F + g_4(q))$ (see also equation 4.16). This function varies slowly for parameters $|X| << 1$, which is true because $v_F >> g_i(q)$. Moreover, for realistic inter-chain interactions, $g_2(q)$ is much smaller than $g_4(q)$ (see e.g. the discussion of two parallel nanotubes on the end of Perfetto's paper [20]). So for our further computation we take φ's not dependent on q.

Now it is possible to compute the sum by using:

$$\sum_{q>0} \frac{2\pi}{qL} e^{-\alpha q} \cos qz = -\ln\left[\frac{2\pi}{L} d(z + i\alpha|L)\right] + \frac{\pi}{L}\alpha \qquad (L = Na)$$

$$d(z|L) = \frac{L}{\pi} \sin\left(\frac{\pi}{L} z\right) \quad \ldots \text{ cord function}$$

(for its geometrical interpretation see fig.: 2.9) (4.32)

This gives the final result for the one particle spin ↑, right mover equal time correlation function: (for detailed computation see appendix[A5])

$$C^{\uparrow}_{+,1}(x,t) \propto \prod_{\nu,q_\perp} \left\{ \frac{1}{d(x|L)^{\frac{1}{4}\left[\cosh\left(2\tilde{\varphi}_\nu - 4\bar{\varphi}_\nu(q_\perp)\right) + \cosh 2\tilde{\varphi}_\nu\right]}} \right.$$
$$\left. \times \left(\frac{d\big(x + 2\bar{v}_\nu(q_\perp)t\big) \cdot d\big(x - 2\bar{v}_\nu(q_\perp)t\big)}{d\big(2\bar{v}_\nu(q_\perp)t\big)^2} \right)^{\frac{1}{8}\left[\cosh\left(2\tilde{\varphi}_\nu - 4\bar{\varphi}(q_\perp)\right) - \cosh 2\tilde{\varphi}_\nu\right]} \right\} . \qquad (4.33)$$

Here again this equation gives for the number of chains $N = 2$ the same result as calculated by Perfetto in [20].

4.4. Thermodynamic limit and asymptotic long time behavior

Next, let us take the thermodynamic limit ($L \to \infty$) and the asymptotic limit of long times ($t \to \infty$). First we do the thermodynamic limit, where we can simply replace the cord function $d(x|L)$ in equation (4.33) by $|x|$:

$$C^{\uparrow}_{+,1}(x,t) \overset{L\to\infty}{\propto} \prod_{\nu,q_\perp} \left\{ \frac{1}{|x|^{\frac{1}{4}\left[\cosh\left(2\tilde{\varphi}_\nu - 4\bar{\varphi}_\nu(q_\perp)\right)+\cosh 2\tilde{\varphi}_\nu\right]}} \times \left(\frac{\left|x^2 - \left(2\bar{v}_\nu(q_\perp)t\right)^2\right|}{\left(2\bar{v}_\nu(q_\perp)t\right)^2} \right)^{\frac{1}{8}\left[\cosh\left(2\tilde{\varphi}_\nu - 4\bar{\varphi}(q_\perp)\right)-\cosh 2\tilde{\varphi}_\nu\right]} \right\} \tag{4.34}$$

Taking that result, we can write the asymptotic limit ($t \to \infty$):

$$C^{\uparrow}_{+,1}(x,t) \overset{\substack{t\to\infty \\ L\to\infty}}{\propto} \prod_{\nu,q_\perp} \frac{1}{|x|^{\frac{1}{4}\left[\cosh\left(2\tilde{\varphi}_\nu - 4\bar{\varphi}_\nu(q_\perp)\right)+\cosh 2\tilde{\varphi}_\nu\right]}} \quad . \tag{4.35}$$

We can compare this with the case of chains coupled from the beginning, being in equilibrium ground state (calculation see appendix [E]).

$$C^{\uparrow}_{+,1}(x,t) \overset{\substack{equilibrium \\ L\to\infty}}{\propto} \prod_{\nu,q_\perp} \frac{1}{|x|^{\frac{1}{2}[\cosh 2\bar{\varphi}_\nu(q_\perp)]}} \quad . \tag{4.36}$$

Obviously we observe that the stationary state, which is reached in thermodynamic limit after the quench, is not the ground state of coupled chains. This emerges from the model, because the non-equilibrium distribution of bosonic excitations introduced by the quench is a constant of motion. Moreover energy conservation disable relaxation to the ground state (see also discussion in [3]). That for realistic cases (intra chain interactions stronger than intra chain interactions) the exponent $\alpha_{asympt} > \alpha_{equil}$, is also clear because in equilibrium case, the system has minimized the repulsive interactions [20].
Furthermore one can take the limit ($t \to 0$) in equation (4.33) and get:

$$C^{\uparrow}_{+,1}(x,t) \overset{t\to 0}{\propto} \prod_{\nu,q_\perp} \frac{1}{d(x)^{\frac{1}{2}[\cosh 2\tilde{\varphi}_\nu]}} \quad , \tag{4.37}$$

which is of course identical with the result for uncoupled chains, as it should.

4.5. Investigation for different forms of interactions

Until now we have not specified the concrete form of the interactions. With that I mean especially the spatial decay. It shows that we more or less have great freedom in doing this. The point in the above calculation where the form of the interactions becomes relevant is, where we do the evaluation of the sum in equation 4.30. But this was anyhow only done to simplify the computation. There we use q independent φ's (see discussion below equation 4.30). Extreme short range (delta function) interactions in space of course make no problems, because their Fourier transform is a constant. But the approximation is also good, for in q equally fast fading g_2 and g_4 interactions. Maybe one can expect problems with long range unsreened Coulomb interactions which diverge at $q = 0$. In general, when the potential has a finite Fourier transform, the asymptotic behavior is given by the $q = 0$ part of the potential. Otherwise, when there is a singularity, the q dependence must be kept during calculation.

4.5.1. Long range Coulomb interactions

In general the unscreened Coulomb interaction, for distances larger than some cutoff length is: $V(r) = e^2/r$. The cutoff is of the order of some minimal particle distance. For example it is given by the lattice constant, or in a continuum model by the extension in transverse direction. The form of the coulomb interaction, which take such a cutoff into account is (see chapter 'Refinements' in [5]):

$$\frac{e^2}{\sqrt{r^2 + d^2}} \quad , \tag{4.38}$$

which avoid the singularity at $r = 0$ and allow Fourier transform, which gives:

$$V(q) = 2e^2 K_0(|q|d) \ , \qquad K_0 \ldots \text{ modified Bessel function of zeroth order} \ . \tag{4.39}$$

As we can see is the small distance r, or equivalent large wave-vektor q, behavior now dependent on the cutoff d. Of course this should not be true for the here interesting long distance, small q behavior and one can find:

$$V(q \to 0) \approx \int dx \frac{1}{x} \approx \log(1/q) \tag{4.40}$$

From that, the charge part of the spectrum obtains the form:

$$\omega(q) = |q| v_\rho(q) \propto |q| \log^{1/2}(1/q) \quad , \tag{4.41}$$

which is identically to the the plasmon dispersion relation in 1D. Note, the long range Coulomb interaction only influence the charge part, the spin part of the spectrum stays unchanged.

Next let's see what happens for a system with long range Coulomb interactions, consisting of many parallel chains. This was to my knowledge, in the context of Tomonaga-Luttinger liquids and using bosonization, first done by Schulz [21]. Then we will look if there are any consequences for our interaction quench. For a system consisting of many parallel chains we could treat the coulomb interaction without introducing a cutoff.

$$V(r) = \frac{e^2}{r_{\lambda,\lambda'}(x-x')} \quad ; \qquad r_{\lambda,\lambda'}(x-x') = \sqrt{(R_\lambda - R_{\lambda'})^2 - (x-x')^2} \quad . \tag{4.42}$$

Here one can do Fourier transformation and identify the small momentum transfer interaction coefficients as:

$$g_{j\rho}(\mathbf{q}) = \frac{4\pi e^2}{a^2} \sum_G \frac{1}{\epsilon_\parallel q^2 + \epsilon_\perp (q_\perp + G)^2} \quad , \qquad (j = 2, 4) \tag{4.43}$$

with $G = (2\pi/a)$ being the transversal reciprocal lattice vector and $\epsilon_\parallel, \epsilon_\perp$ some background dielectric constants. For the interesting long-range, small q behavior we can restrict ourselves to the $G = 0$ part of the term above. As done by Schulz [21] we also allow in real space delta function like and consequently in q-space constant intra-chain contributions:

$$g_{j\rho}(\mathbf{q}) = \frac{4\pi e^2}{a^2} \frac{1}{\epsilon_\parallel q^2 + \epsilon_\perp q_\perp^2} + g_{j0} \quad . \tag{4.44}$$

In this way we can insert this into the renormalized charge velocity:

$$\begin{aligned} \bar{v}_\rho(\mathbf{q}) &= \sqrt{\left(v_F + g_{4\rho}(\mathbf{q})\right)^2 - \left(g_{2\rho}(\mathbf{q})^2\right)} \\ &= \sqrt{\left[\left(v_F + g_{40}\right)^2 - g_{20}^2\right] + \frac{8\pi e^2}{a^2} \cdot \frac{v_F + g_{40} - g_{20}}{\epsilon_\parallel q^2 + \epsilon_\perp q_\perp^2}} \end{aligned} \tag{4.45}$$

In this way we get for the excitation energy:

$$\bar{v}_\rho(\mathbf{q})|q| = \sqrt{\left[\left(v_F + g_{40}\right)^2 - g_{20}^2\right] q^2 + \frac{8\pi e^2}{a^2} \cdot \frac{v_F + g_{40} - g_{20}}{\epsilon_\parallel q^2 + \epsilon_\perp q_\perp^2} q^2} \tag{4.46}$$

Setting $\epsilon_\parallel = \epsilon_\perp = \epsilon$ and taking the limit $|\mathbf{q}| \to 0$ gives:

$$\bar{v}_\rho(\mathbf{q})|q| \;=\; \frac{8\pi e^2}{a^2 \epsilon_\parallel} \cdot (v_F + g_{40} - g_{20}) \quad . \tag{4.47}$$

This allows interpretation as a plasma frequency. Moreover one can see that the charge excitations for $\mathbf{q} = 0$ now have finite energy. But there are excitations with $q_\perp \neq 0$

and $q = 0$, which have zero energy. Next we will look for the consequences to the one particle correlation functions. Until equation (4.30) the calculation is the same as for short range interactions. But due the divergence at $q = 0$ the explanation, why one can take q independent parameters, for the further evaluation of the sum might be wrong. Again, we consider that the parameter dependence in the exponents is of the form:

$$\cosh 2\tilde{\varphi}(q) \quad \text{and} \quad \cosh\left(2\tilde{\varphi}(q) - 4\bar{\varphi}(\mathbf{q})\right) \quad , \tag{4.48}$$

with:

$$2\tilde{\varphi}(q) = \frac{g_2(q)}{g_4(q) + v_F} \tag{4.49}$$

Next we insert the coulomb form for the interactions and assume that the g_4 coulomb interaction is larger than the g_2 interaction, which is in general true for realistic cases. Further we introduce a ratio between them: $C_R = g_2/g_4$. With that, we obtain the result for the coefficients in the $q \to 0$ limit:

$$2\tilde{\varphi}(q) = \operatorname{arctanh} \frac{C_R \frac{4\pi e^2}{a^2}\frac{1}{\epsilon_\parallel q^2} + g_{20}}{\frac{4\pi e^2}{a^2}\frac{1}{\epsilon_\parallel q^2} + g_{40} + v_F} \overset{q\to 0}{=} \operatorname{arctanh} C_R \tag{4.50}$$

$$\begin{aligned} 2\tilde{\varphi}(q) - 4\bar{\varphi}(\mathbf{q}) &= \operatorname{arctanh} \frac{C_R \frac{4\pi e^2}{a^2}\frac{1}{\epsilon_\parallel q^2} + g_{20}}{\frac{4\pi e^2}{a^2}\frac{1}{\epsilon_\parallel q^2} + g_{40} + v_F} - \operatorname{arctanh} \frac{2C_R \frac{4\pi e^2}{a^2}\frac{1}{\epsilon_\parallel q^2} + 2g_{20}}{\frac{4\pi e^2}{a^2}\frac{1}{\epsilon_\parallel q^2} + g_{40} + v_F} \\ &= \operatorname{arctanh} C_R - \operatorname{arctanh} 2C_R \end{aligned} \tag{4.51}$$

As one can see if we include long range Coulomb interactions in both, in g_2 and in g_4 no divergences arise at $q = 0$. Moreover for C_R small compared to 1, the exponents again vary very slowly with q. So we can come to the conclusion that the Coulomb interactions do not change the results derived above, as long as $g_4 > g_2$.

5. Summary and Conclusions

Arrived at the end of our journey, lets look back and summarize.
We started with a general Hamiltonian, expressed in fermionic creation and destruction operators, describing the problem of uncoupled chains of spin-full fermions. Using a linearized dispersion relation (the Tomonaga-Luttinger model) allowed us to write the Hamiltonian in terms of canonical boson operators. By means of Bogoliubov transformation it was possible to obtain an Hamiltonian, diagonal in the new bosons. In this way, the creation and destruction operators for the new bosons obtained a simple time evolution in Heisenberg picture.

At T=0, we switched on the interaction between the chains. Assuming translational invariance in the direction perpendicular to the chains, we obtained uncoupled chains in transverse momentum space. In this way, it was again feasible to diagonalize the Hamiltonian. After doing some direct and reverse Bogoliubov transformation it was possible to write the time evolution of the operators after the interaction quench, in terms of operators in which the system was diagonal before the quench (4.19). This, for the further computation important result, was compared to the work of E. Perfetto [20] who treated the special case of 2 chains. As easily can be seen, our result is identical when we set $N = 2$.

Afterwards, we computed the one particle equal time correlation function, by using the bosonization identity. A detailed computation finally leaded to the result (4.33), likewise including the 2-chain case [20].

Remarkable, the thermodynamic limit of the correlation function, after the quench (4.35), shows a different result compared to the correlation function calculated in equilibrium ground state for initially coupled chains (4.36). This can be understood as a consequence of energy conservation, which disables relaxation to the ground state.

The investigation of the long range Coulomb interaction (4.5.1) leaded to the conclusion, that it do not change the results, for the quench dynamics computed above, at least for the realistic case where $g_4 > g_2$.

For the ones who have missed something up to now and to remember the others that we treated a model and models are seldom complete, of course we must mention that tunneling between the chains play some role. We should note, that experimental realizations of coupled chains are supposed to have a small, but always finite inter-chain hopping amplitude $t_\perp$.

Under many possible ways to take hopping between Luttinger-liquids into account are following three strategies.
One approach is to solve the problem including hopping, but without interactions be-

tween the chains. Already this is a difficult problem, because interchain hopping can be a relevant perturbation to the Luttinger liquid state within the individual chains. Afterwards one can include interchain interactions, which in turn influence whether the hopping is a relevant perturbation. However, renormalization group analysis in this context is also criticized because it for example does not take into account the spin-charge separation.
Another strategy used is to solve the coupled chains without hopping using bosonization, like it was done here. This needs to introduce the hopping afterwards, what become feasible using perturbation theory.
An account which allow equal treatment of interaction and hopping will be e.g. bosonization for higher dimensions. This approach presuppose a linearization of higher dimensional dispersion relation at the Fermi surface.
Finally, I want to mention, that I have also dealt with this topics, however it seems all these strategies have its difficulties and a more detailed investigation goes beyond the feasible for my diploma work. Obviously, when the electrons are allowed to move perpendicular to the chains, one can expect a dimensional crossover to a Fermi liquid in 2 or 3D. However, also interesting might be the question how much of the Luttinger liquid physics can survive in higher dimensions and whether it might act as a model for other higher dimensional non Fermi liquids.

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A. Appendix - Transverse Fourier transform

In section 4.2 we introduce the Fourier transform in the direction which is transverse to the chains. Here I do the explicit calculation. Lets start with the definition of the Fourier transformed creation and destruction operators:

$$\begin{aligned} b^\dagger_{\nu,\lambda}(q) &= \frac{1}{\sqrt{N}} \sum_{q_\perp} \exp(iq_\perp R_\lambda) b^\dagger_\nu(\mathbf{q}) \\ b_{\nu,\lambda}(q) &= \frac{1}{\sqrt{N}} \sum_{q_\perp} \exp(-iq_\perp R_\lambda) b_\nu(\mathbf{q}) \quad , \end{aligned} \tag{A.1}$$

$$\text{with: } \mathbf{q} = (q_\perp, q) \quad , \tag{A.2}$$

and insert it in part of the Hamiltonian (4.12)

$$\begin{aligned} &\sum_{\lambda,\lambda'} g_{4\nu,\lambda\lambda'}(q) \cdot b^\dagger_{\lambda,\nu}(q) b_{\lambda',\nu}(q) = \\ &= \sum_{\lambda,\lambda'} g_{4\nu,\lambda\lambda'}(q) \left(\frac{1}{\sqrt{N}} \sum_{q_\perp} \exp(iq_\perp R_\lambda) b^\dagger_\nu(\mathbf{q}) \right) \left(\frac{1}{\sqrt{N}} \sum_{q_{\perp'}} \exp(-iq'_\perp R_{\lambda'}) b_\nu(\mathbf{q}') \right) \\ &= \sum_{\lambda,\lambda'} g_{4\nu,\lambda\lambda'}(q) \frac{1}{N} \sum_{q_\perp,q_\perp}{}' \exp\left(i(q_\perp - q'_\perp) \underbrace{R_\lambda}_{\uparrow}\right) \exp\left(iq'_\perp (R_\lambda - R_{\lambda'})\right) b^\dagger_\nu(\mathbf{q}) b_\nu(\mathbf{q}') \end{aligned}$$

Because the system is translation invariant it can not depend on an arbitrary choice of the origin R_λ. So $q'_\perp$ must be equal $q_\perp$. $\longrightarrow \exp\left(i(q_\perp - q'_\perp)R_\lambda\right) \stackrel{!}{=} N\delta'_{q_\perp,q_\perp}$

$$= \sum_{q_\perp} \underbrace{\sum_{\lambda} \exp\left(iq'_\perp (R_\lambda - R_{\lambda'})\right) g_{4\nu,\lambda\lambda'}(q)}_{= g_{4\nu}(\mathbf{q})} b^\dagger_\nu(\mathbf{q}) b_\nu(\mathbf{q}) \tag{A.3}$$

Where we found the definition of the Fourier transformed interaction (4.13). So we

obtain one term of the Hamiltonian (A.4). We can do the same for the term:

$$\sum_{\lambda,\lambda'} g_{2\nu,\lambda\lambda'}(q) \cdot b^{\dagger}_{\lambda,\nu}(q) b_{\lambda',\nu}(q) =$$
$$= \ldots = \sum_{q_\perp} \underbrace{\sum_{\lambda} \exp\left(iq'_\perp (R_\lambda - R_{\lambda'})\right) g_{2\nu,\lambda\lambda'}(q)}_{= g_{2\nu}(\mathbf{q})} b_\nu(\mathbf{q}) b_\nu(-\mathbf{q})$$

The term with the Fermi velocity multiplies with $\delta_{\lambda,\lambda'}$ which gives following result:

$$\sum_{\lambda,\lambda'} v_F \delta_{\lambda,\lambda'} \cdot b^{\dagger}_{\lambda,\nu}(q) b_{\lambda',\nu}(q) = \sum_{\lambda} v_F \cdot b^{\dagger}_{\lambda,\nu}(q) b_{\lambda,\nu}(q) =$$
$$= v_F \left(\frac{1}{\sqrt{N}} \sum_{q_\perp} \exp(iq_\perp R_\lambda) b_\nu(\mathbf{q}') \right) \left(\frac{1}{\sqrt{N}} \sum_{q_{\perp'}} \exp(-iq'_\perp R_\lambda) b_\nu(\mathbf{q}') \right)$$
$$= \frac{1}{N} \sum_{q_\perp, q_\perp}{}' v_F \underbrace{\exp\left(i(q_\perp - q'_\perp) R_\lambda\right)}_{N\delta_{q_\perp, q'_\perp}} b^{\dagger}_{\nu}(\mathbf{q}) b_{,\nu}(\mathbf{q}')$$
$$= \sum_{q_\perp} v_F b^{\dagger}_{\nu}(\mathbf{q}) b_\nu(\mathbf{q})$$

Now collecting the terms gives us the Fourier transformed form of the whole Hamiltonian:

$$H_{final} = \frac{1}{2} \sum_{\nu, q_\perp, q \neq 0} \Big\{ \big(v_F + g_{4\nu}(\mathbf{q})\big)|q| \times \Big(b^{\dagger}_{\nu}(\mathbf{q}) b_\nu(\mathbf{q}) + h.c. \Big) - g_{2\nu}(\mathbf{q})|q| \times \Big(b_\nu(\mathbf{q}) b_\nu(-\mathbf{q}) + h.c. \Big) \Big\} \tag{A.4}$$

B. Appendix - Time evolution of the operators

In this appendix we will derive the time evolution of the creation $b_{\lambda,s}(q,t)$ and destruction $b_{\lambda,s}(q,t)$ operators by using Bogoliubov transformation (2.3) and the simple form of time evolution of operators in which the Hamiltonian is diagonal (3.7, 3.8). Without loss of generality we compute the operators for chain 1 (the chains are all equal) and spin $\uparrow$ (we have spin rotation invariance). $\mathscr{F}\mathscr{T}$ is written for doing Fourier transformation. The tilde (e.g.: $\tilde{b}$) is written for quantities and operators in diagonalized system with no interaction between the chains(see equations: 4.7-4.10). The bar (e.g.: $\bar{b}$) for the ones in diagonalized full interacting system(see equations: 4.15-4.18). Operators and quantities without tilde or bar are the ones from original not diagonalized model(see equations: 4.5, A.4).

$$b_{1,\uparrow}(q,t) = \frac{1}{\sqrt{2}} \sum_{\nu} \underbrace{b_{1,\nu}(q,t)}_{\mathscr{F}\mathscr{T}\searrow} \qquad \nu = \rho,\ \sigma$$

$$= \frac{1}{\sqrt{2}} \sum_{\nu} \overbrace{\frac{1}{\sqrt{N}} \sum_{q_\perp} \exp\big(-iq_\perp R_1\big) \cdot b_\nu(\mathbf{q},t)}$$

$$\Big\downarrow \text{ choosing coordinate origin at first chain. } \rightarrow \text{ so set: } R_1 = 0$$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} b_\nu(\mathbf{q},t)$$

$$\Big\downarrow \text{ BOGOLIUBOV TRANSFORMATION (diagonalization of the full interacting system)}$$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} \Big\{ \cosh\bar{\varphi}_\nu(\mathbf{q}) \cdot \bar{b}_\nu(\mathbf{q},t) - \sinh\bar{\varphi}_\nu(\mathbf{q}) \cdot \bar{b}^\dagger_\nu(-\mathbf{q},t) \Big\}$$

$$\Big\downarrow \text{ insertion of time evolution of operators, in which Hamiltonian is diagonal}$$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} \Big\{ \exp\big(-i\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t\big) \cosh\bar{\varphi}_\nu(\mathbf{q}) \cdot \bar{b}_\nu(\mathbf{q}) - \exp\big(i\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t\big) \sinh\bar{\varphi}_\nu(\mathbf{q}) \cdot \bar{b}^\dagger_\nu(-\mathbf{q}) \Big\}$$

$\Big\downarrow$ BOGOLIUBOV TRANSFORMATION^{-1} (full interacting system)

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} \Big\{ \exp\big(-i\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t\big) \cosh\bar{\varphi}_\nu(\mathbf{q}) \Big(\cosh\big(\bar{\varphi}_\nu(\mathbf{q})\big) \cdot b_\nu(\mathbf{q}) + \sinh\big(\bar{\varphi}_\nu(\mathbf{q})\big) \cdot b^\dagger_\nu(-\mathbf{q}) \Big) - \exp\big(i\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t\big) \sinh\bar{\varphi}_\nu(\mathbf{q}) \Big(\sinh\big(\bar{\varphi}_\nu(\mathbf{q})\big) \cdot b_\nu(\mathbf{q}) + \cosh\big(\bar{\varphi}_\nu(\mathbf{q})\big) \cdot b^\dagger_\nu(-\mathbf{q}) \Big) \Big\}$$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} \Big\{ \exp\big(-i\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t\big) \Big(\cosh^2\bar{\varphi}_\nu(\mathbf{q}) \cdot b_\nu(\mathbf{q}) + \sinh\bar{\varphi}_\nu(\mathbf{q}) \cosh\bar{\varphi}_\nu(\mathbf{q}) \cdot b^\dagger_\nu(-\mathbf{q}) \Big) - \exp\big(i\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t\big) \Big(\sinh^2\bar{\varphi}_\nu(\mathbf{q}) \cdot b_\nu(\mathbf{q}) + \sinh\bar{\varphi}_\nu(\mathbf{q}) \cosh\bar{\varphi}_\nu(\mathbf{q}) \cdot b^\dagger_\nu(-\mathbf{q}) \Big) \Big\}$$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} \Big\{ \overbrace{\Big(\cos\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t - i\sin\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t \cosh 2\bar{\varphi}_\nu(\mathbf{q}) \Big)}^{=:\,\alpha_\nu(\mathbf{q},t)} \cdot b_\nu(\mathbf{q}) - \underbrace{i\sin\bar{v}_\nu(\mathbf{q})|\mathbf{q}|t \sinh 2\bar{\varphi}_\nu(\mathbf{q})}_{=:\,\beta_\nu(\mathbf{q},t)} \cdot b^\dagger_\nu(-\mathbf{q}) \Big\}$$

$\Big\downarrow \mathscr{F}\mathscr{T}^{-1}$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp} \Big\{ \alpha_\nu(\mathbf{q},t) \frac{1}{\sqrt{N}} \sum_\lambda \exp\big(iq_\perp R_\lambda\big) b_{\nu,\lambda}(q) + \beta_\nu(\mathbf{q},t) \frac{1}{\sqrt{N}} \sum_\lambda \exp\big(iq_\perp R_\lambda\big) b^\dagger_{\nu,\lambda}(-q) \Big\}$$

$$= \frac{1}{\sqrt{2N}} \sum_{\nu,q_\perp,\lambda} \exp\big(iq_\perp R_\lambda\big) \Big\{ \alpha_\nu(\mathbf{q},t) b_{\nu,\lambda}(q) + \beta_\nu(\mathbf{q},t) b^\dagger_{\nu,\lambda}(-q) \Big\}$$

$$\Big\downarrow \text{BOGOLIUBOV TRANSFORMATION (diagonalization on the chains, for isolated chains)}$$

$$= \frac{1}{\sqrt{2}N} \sum_{\nu,q_\perp,\lambda} \exp\left(iq_\perp R_\lambda\right) \Big\{ \alpha_\nu(\mathbf{q},t) \Big(\cosh\tilde{\varphi}_\nu \cdot \tilde{b}_{\lambda,\nu}(q) - \sinh\tilde{\varphi}_\nu \cdot \tilde{b}^\dagger_{\lambda,\nu}(-q) \Big) + \beta_\nu(\mathbf{q},t) \Big(-\sinh\tilde{\varphi}_\nu \cdot \tilde{b}_{\lambda,\nu}(q) + \cosh\tilde{\varphi}_\nu \cdot \tilde{b}^\dagger_{\lambda,\nu}(-q) \Big) \Big\}$$

$$= \frac{1}{\sqrt{2}N} \sum_{\nu,q_\perp,\lambda} \exp\left(iq_\perp R_\lambda\right) \Big\{ \overbrace{\Big(\cosh\tilde{\varphi}_\nu\ \alpha_\nu(\mathbf{q},t) - \sinh\tilde{\varphi}_\nu\ \beta_\nu(\mathbf{q},t) \Big)}^{=:\,\mathbf{A}_\nu(\mathbf{q},t)} \cdot \tilde{b}_{\lambda,\nu}(q) \underbrace{\Big(-\sinh\tilde{\varphi}_\nu\ \alpha_\nu(\mathbf{q},t) + \cosh\tilde{\varphi}_\nu\ \beta_\nu(\mathbf{q},t) \Big)}_{=:\,\mathbf{B}_\nu(\mathbf{q},t)} \cdot \tilde{b}^\dagger_{\lambda,\nu}(-q) \Big\}$$

So finally we have derived equation (4.19) of the main text.

C. Appendix - Expectation values of time dependent fields

Here we calculate the expectation value of a boson field which arises in the computation of the fermion one particle correlation function (4.27).

$$\Big\langle \big(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0,t)\big)^2 \Big\rangle_{\tilde{H}_{init}} =$$

$$= \sum_{q>0} \left(\frac{2\pi}{qN}\right) e^{-\alpha q} \Big\langle \Big(\big(e^{-iqx} - 1\big) \cdot b^\dagger_{1,\uparrow}(q,t) + \big(e^{-iqx} - 1\big) \cdot b_{1,\uparrow}(q,t) \Big)^2 \Big\rangle_{\tilde{H}_{init}}$$

$$\Big\downarrow \text{ insertion of the time evolution for } b_{1,\uparrow}(q,t) \text{ and } b^\dagger_{1,\uparrow}(q,t) \text{ (4.19)}$$

$$= \sum_{q>0} \left(\frac{\pi}{2qN^3}\right) e^{-\alpha q} \Big\langle \Big\{ (e^{-iqx} - 1) \sum_{\nu,q_\perp,\lambda} \exp(-iq_\perp R_\lambda) \Big(\mathbf{A}^*_\nu(\mathbf{q},t) \cdot \tilde{b}^\dagger_{\nu,\lambda}(q) + \mathbf{B}^*_\nu(\mathbf{q},t) \cdot \tilde{b}_{\nu,\lambda}(-q) \Big)$$

$$+ (e^{iqx} - 1) \sum_{\nu',q'_\perp,\lambda'} \exp(iq_{\perp'} R_{\lambda'}) \Big(\mathbf{A}_\nu(\mathbf{q}',t) \cdot \tilde{b}_{\nu',\lambda'}(q) + \mathbf{B}_\nu(\mathbf{q}',t) \cdot \tilde{b}^\dagger_{\nu',\lambda'}(-q) \Big) \Big\}^2 \Big\rangle_{H_{init}}$$

The system consists initially of independent chains. Each chain (Luttinger liquid) is prepared to its interacting ground state. Remember $\tilde{b}_{1,\uparrow}(q,t)$ and $\tilde{b}^\dagger_{1,\uparrow}(q,t)$ are the operators in which the initial system is diagonal. So expectation values of following form, and its hermitian conjugates, are simply zero:

$$\Big\langle \tilde{b}^2(q) \Big\rangle_{H_{init}} = \Big\langle \tilde{b}^\dagger(-q)\tilde{b}^2(q) \Big\rangle_{H_{init}} = \Big\langle \tilde{b}^\dagger_\lambda(q)\tilde{b}_{\lambda'}(q) \Big\rangle_{H_{init}} = \Big\langle \tilde{b}^\dagger_\nu(q)\tilde{b}_{\nu'}(q) \Big\rangle_{H_{init}} = 0$$

So in our equation above we can take $\lambda = \lambda',\ \nu = \nu'$.

$$= \sum_{q>0} \left(\frac{\pi}{2qN^3}\right) e^{-\alpha q} \Big\langle \Big\{ \big(1 - \cos qx\big) \sum_{\nu,q_\perp,q'_\perp} \overbrace{\sum_\lambda \exp\big(i(q_\perp - q_{\perp'})R_\lambda\big)}^{N\delta_{q_\perp,q_{\perp'}}}$$

$$\times 2\Big(\mathbf{A}^*_\nu(\mathbf{q},t)\mathbf{A}_\nu(\mathbf{q}',t) \cdot \tilde{b}^\dagger_{\nu,\lambda}(q)\tilde{b}_{\nu,\lambda}(q) + \mathbf{B}^*_\nu(\mathbf{q},t)\mathbf{B}_\nu(\mathbf{q}',t) \cdot \tilde{b}_{\nu,\lambda}(-q)\tilde{b}^\dagger_{\nu,\lambda}(-q) \Big) \Big\} \Big\rangle_{H_{init}}$$

We can do the summation over λ and get the overbraced term equal to delta function $\delta_{q_\perp,q_{\perp'}}$ because the only λ dependent terms beside the $\exp(i(q_\perp - q_{\perp'})R_\lambda)$ term are the

ones with the $\tilde{b}$ and $\tilde{b}^\dagger$ operators. But their expectation value is not dependent on the chain index λ, because they are initially prepared all alike.

$$= \sum_{q>0} \left(\frac{\pi}{qN^2}\right) e^{-\alpha q} \Bigg\langle \Bigg\{ \big(1-\cos qx\big) \sum_{\nu,q_\perp} \Big(|\mathbf{A}_\nu(\mathbf{q},t)|^2 \cdot \big(\tilde{b}^\dagger_{\nu,\lambda}(q)\tilde{b}_{\nu,\lambda}(q) + \tilde{b}_{\nu,\lambda}(q)\tilde{b}^\dagger_{\nu,\lambda}(q)\big) \\ + |\mathbf{B}_\nu(\mathbf{q},t)|^2 \cdot \big(\tilde{b}^\dagger_{\nu,\lambda}(-q)\tilde{b}_{\nu,\lambda}(-q) + \tilde{b}_{\nu,\lambda}(-q)\tilde{b}^\dagger_{\nu,\lambda}(-q)\big)\Big) \Bigg\} \Bigg\rangle_{H_{init}}$$

Now we remember again, that for the system being in ground state in this operators applies:

$$\left[\left\langle \tilde{b}^\dagger_{\nu,\lambda}(q)\tilde{b}_{\nu,\lambda}(q) \right\rangle = n_0(q) \;\; ; \qquad \left\langle \tilde{b}_{\nu,\lambda}(-q)\tilde{b}^\dagger_{\nu,\lambda}(-q) \right\rangle = n_0(q) - 1 \right]$$

and:

$n_0(q) = 0 \quad \text{for} \quad q = 0 \quad .$

With that we get for our expectation value:

$$\left\langle \big(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0,t)\big)^2 \right\rangle_{\tilde{H}_{init}} = \\ = \sum_{q>0} \left(\frac{\pi}{qN^2}\right) e^{-\alpha q} \big(1-\cos qx\big) \sum_{\nu,q_\perp} \Big(|\mathbf{A}_\nu(\mathbf{q},t)|^2 + |\mathbf{B}_\nu(\mathbf{q},t)|^2 \Big) \tag{C.1}$$

With that let us proceed further in main text, at equation (4.29).

D. Appendix - Calculation of some coefficients

$$
\begin{aligned}
&|A|^2 = (\Re A)^2 + (\Im A)^2 \\
&= \big(\cosh\tilde{\varphi}_\nu(q)\cos\bar{v}_\nu(\mathbf{q})|q|t\big)^2 \;+\; \big(-\cosh\tilde{\varphi}_\nu(q)\cosh 2\bar{\varphi}_\nu(\mathbf{q})\sin\bar{v}_\nu(\mathbf{q})|q|t \\
&\qquad\qquad + \cosh\tilde{\varphi}_\nu(q)\cosh 2\bar{\varphi}_\nu(\mathbf{q})\sin\bar{v}_\nu(\mathbf{q})|q|t\big)^2 \\
&= \big(\cosh\tilde{\varphi}_\nu(q)\cos\bar{v}_\nu(\mathbf{q})|q|t\big)^2 - \frac{1}{4}\Big\{\big[\cosh\big(\tilde{\varphi}_\nu(q)+2\bar{\varphi}_\nu(\mathbf{q})\big)+\cosh\big(\tilde{\varphi}_\nu(q)-2\bar{\varphi}_\nu(\mathbf{q})\big)\big]\sin\bar{v}_\nu(\mathbf{q})|q|t \\
&\qquad\qquad + \big[\cosh\big(\tilde{\varphi}_\nu(q)+2\bar{\varphi}_\nu(\mathbf{q})\big)-\cosh\big(\tilde{\varphi}_\nu(q)-2\bar{\varphi}_\nu(\mathbf{q})\big)\big]\sin\bar{v}_\nu(\mathbf{q})|q|t\Big)\Big\}^2 \\
&= \cosh^2\tilde{\varphi}_\nu(q)\cos^2\bar{v}_\nu(\mathbf{q})|q|t \;+\; \cosh^2\big(\tilde{\varphi}_\nu(q)-2\bar{\varphi}_\nu(\mathbf{q})\big)\sin^2\bar{v}_\nu(\mathbf{q})|q|t
\end{aligned}
$$

$$
\begin{aligned}
&|B|^2 = (\Re B)^2 + (\Im B)^2 \\
&= \big(-\sinh\tilde{\varphi}_\nu(q)\cos\bar{v}_\nu(\mathbf{q})|q|t\big)^2 \;+\; \big(\sinh\tilde{\varphi}_\nu(q)\cosh 2\bar{\varphi}_\nu(\mathbf{q})\sin\bar{v}_\nu(\mathbf{q})|q|t \\
&\qquad\qquad - \cosh\tilde{\varphi}_\nu(q)\cosh 2\bar{\varphi}_\nu(\mathbf{q})\sin\bar{v}_\nu(\mathbf{q})|q|t\big)^2 \\
&= \big(\cosh\tilde{\varphi}_\nu(q)\cos\bar{v}_\nu(\mathbf{q})|q|t\big)^2 + \frac{1}{4}\Big\{\big[\sinh\big(\tilde{\varphi}_\nu(q)+2\bar{\varphi}_\nu(\mathbf{q})\big)-\sinh\big(\tilde{\varphi}_\nu(q)-2\bar{\varphi}_\nu(\mathbf{q})\big)\big]\sin\bar{v}_\nu(\mathbf{q})|q|t \\
&\qquad\qquad - \big[\sinh\big(\tilde{\varphi}_\nu(q)+2\bar{\varphi}_\nu(\mathbf{q})\big)+\sinh\big(\tilde{\varphi}_\nu(q)-2\bar{\varphi}_\nu(\mathbf{q})\big)\big]\sin\bar{v}_\nu(\mathbf{q})|q|t\Big)\Big\}^2 \\
&= \sinh^2\tilde{\varphi}_\nu(q)\cos^2\bar{v}_\nu(\mathbf{q})|q|t \;+\; \sinh^2\big(\tilde{\varphi}_\nu(q)-2\bar{\varphi}_\nu(\mathbf{q})\big)\sin^2\bar{v}_\nu(\mathbf{q})|q|t
\end{aligned}
$$

From that follows:

$$\left|\mathbf{A}_\nu(\mathbf{q},t)\right|^2 - \left|\mathbf{B}_\nu(\mathbf{q},t)\right|^2 = 1$$

$$\begin{aligned}\left|\mathbf{A}_\nu(\mathbf{q},t)\right|^2 + \left|\mathbf{B}_\nu(\mathbf{q},t)\right|^2 &= \frac{1}{2}\cosh\left(2\tilde{\varphi}_\nu(q) - 4\bar{\varphi}_\nu(\mathbf{q})\right)\cdot(1-\cos 2\bar{v}_\nu(\mathbf{q})|q|t) \\ &\quad + \frac{1}{2}\cosh 2\tilde{\varphi}_\nu(q)\cdot(1+\cos 2\bar{v}_\nu(\mathbf{q})|q|t)\end{aligned}$$

With this result we can perform the sum over q in the correlation function by insertion into equation: (4.30) and proceeding further in main text.

E. Appendix - Correlations of initially coupled chains

To enable reference and comparison to the result after the quench, we also compute the one particle correlation function, of chains which are already coupled from the beginning. As before, we use the general result for the correlation function in terms of boson fields (4.27):

$$C^{\uparrow}_{+,1}(x) = \frac{1}{\alpha}\exp\left\{ -\frac{1}{2}\Big\langle \big(\Phi_{1,\uparrow,+}(x) - \Phi_{1,\uparrow,+}(0)\big)^2 \Big\rangle_{\bar{H}_0} - \frac{1}{2}\big[\Phi_{1,\uparrow,+}(x), \Phi_{1,\uparrow,+}(0)\big] \right\} \quad .$$

Of course, as easily can be seen, the commutator gives exactly the same result as calculated in (4.28):

$$\big[\Phi_{1,\uparrow,+}(x), \Phi_{1,\uparrow,+}(0)\big] = \sum_{q>0}\left(\frac{2\pi}{qNa}\right) e^{-\alpha q}\big(i\sin qx\big) \quad .$$

Next we have to do the calculation of the expectation value, which is much simpler compared to the case with the quench:

$$\Big\langle \big(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0)\big)^2 \Big\rangle_{\bar{H}_0} = \sum_{q>0}\left(\frac{2\pi}{qNa}\right) e^{-\alpha q}\Big\langle \Big(b^{\dagger}_{1,\uparrow}(q)\cdot\big(e^{-iqx}-1\big) + b_{1,\uparrow}(q,t)\cdot\big(e^{iqx}-1\big)\Big)^2 \Big\rangle_{\bar{H}_0}$$

Here we have only to insert the Fourier and Bogoliubov transformed boson operators:

$$b_{1,\uparrow}(q) = \frac{1}{\sqrt{2}}\sum_{\nu} \underbrace{b_{1,\nu}(q)}_{\mathscr{F}\mathscr{T}\ \searrow} \qquad \nu = \rho,\ \sigma$$

$$= \frac{1}{\sqrt{2}}\sum_{\nu} \overbrace{\frac{1}{\sqrt{N}}\sum_{q_\perp}\exp\big(-iq_\perp R_1\big)\cdot b_\nu(\mathbf{q})}$$

$$\Big\downarrow \text{choosing coordinate origin at first chain. } \rightarrow \text{ so set: } R_1 = 0$$

$$= \frac{1}{\sqrt{2N}}\sum_{\nu,q_\perp} b_\nu(\mathbf{q},t)$$

$$\Big\downarrow \text{BOGOLIUBOV TRANSFORMATION (diagonalization of the full interacting system)}$$

$$= \frac{1}{\sqrt{2N}}\sum_{\nu,q_\perp}\Big\{\cosh\bar{\varphi}_\nu(\mathbf{q})\cdot\bar{b}_\nu(\mathbf{q}) - \sinh\bar{\varphi}_\nu(\mathbf{q})\cdot\bar{b}^{\dagger}_\nu(-\mathbf{q})\Big\}$$

This gives:

$$\left\langle \left(\Phi_{1,\uparrow,+}(x,t) - \Phi_{1,\uparrow,+}(0)\right)^2 \right\rangle_{\bar{H}_0} =$$

$$= \sum_{q>0} \left(\frac{2\pi}{qNa}\right) e^{-\alpha q}(1-\cos qx)$$

$$\left\{ \cosh^2 \bar{\varphi}_\nu(\mathbf{q}) \underbrace{\left\langle \bar{b}^\dagger_\nu(\mathbf{q})\bar{b}_\nu(\mathbf{q}) + \bar{b}_\nu(\mathbf{q})\bar{b}^\dagger_\nu(\mathbf{q}) \right\rangle_{\bar{H}_0}}_{2n_0(q)-1} + \sinh^2 \bar{\varphi}_\nu(\mathbf{q}) \underbrace{\left\langle \bar{b}_\nu(-\mathbf{q})\bar{b}^\dagger_\nu(-\mathbf{q})\bar{b}^\dagger_\nu(-\mathbf{q})\bar{b}_\nu(-\mathbf{q}) \right\rangle_{\bar{H}_0}}_{2n_0(q)-1} \right\}$$

the system is in ground state in these operators so $n_0(q) = 0$ for $q > 0$

$$= \sum_{q>0} \left(\frac{2\pi}{qNa}\right) e^{-\alpha q}(1-\cos qx)\cosh 2\bar{\varphi}_\nu(\mathbf{q})$$

In the last step we use summation formula (2.59) and get the result for the one particle correlation function for already initially coupled chains:

$$C^{\uparrow}_{+,1}(x,t) \propto \prod_{\nu,q_\perp} \frac{1}{d(x)^{\frac{1}{2}[\cosh 2\bar{\varphi}_\nu(q_\perp)]}} \quad . \tag{E.1}$$

In the main text we take the thermodynamic limit to get equation (4.36) and compare it with the asymptotic limit after the quench (equation: 4.35).

F. Appendix - Electron-electron interactions

In this appendix we decompose the interaction potential of the general Hamiltonian (2.7). We have introduced right moving '+' and left moving '-' fermions (2.11). Concerning this, we have to treat two different groups of interactions (see also fig.: 2.8). One with small momentum transfer $q \approx 0$, called forward scattering, where the fermions do not change the direction in which they are moving. The other one is regarding to fermions, which change their direction and so its momentum transfer is respectively $q \approx 2k_F$. The last one is called backward scattering.

Forward scattering ($q \approx 0$)

$$\begin{aligned} H_f &= \frac{1}{L}\sum_{k,k',q} \big(c^\dagger_{+,k}c_{+,k-q} + c^\dagger_{-,k}c_{-,k-q}\big)V(q)\big(c^\dagger_{+,k'}c_{+,k'+q} + c^\dagger_{-,k'}c_{-,k'+q}\big) \\ &= \frac{1}{L}\sum_{q}\Big(g_4\big(\rho_+(-q)\rho_+(q) + \rho_-(-q)\rho_-(q)\big) + g_2\big(\rho_+(-q)\rho_-(q) + \rho_-(-q)\rho_+(q)\big)\Big) \\ &= \frac{1}{2}\sum_{q\neq 0}\Big(g_4\big(b^\dagger(q)b(q) + h.c.\big) + g_2\big(b(q)b(-q) + h.c.\big)\Big) \;+\; C(N_+, N_-) \end{aligned} \tag{F.1}$$

Backward scattering ($q \approx 2k_F$)

$$\begin{aligned} H_b = \frac{1}{L}\sum_{k,k',\delta k}\; & c^\dagger_{+,k}c_{-,k-\delta k}V(2k_F+\delta k)c^\dagger_{-,k'}c_{+,k'+\delta k} \\ & + c^\dagger_{-,k}c_{+,k-\delta k}V(-2k_F-\delta k)c^\dagger_{+,k'}c_{-,k'+\delta k} \end{aligned}$$

Now we transform the parameters, $\delta k = q+k-k'$ and introduce the backward scattering parameter g_1, which can in good approximation be taken q-independent for small q.

$$\begin{aligned} H_b &= \frac{1}{L}\sum_{k,k',q} g_1\Big(c^\dagger_{+,k}c_{+,k+q}c_{-,k'-q}c^\dagger_{-,k'} + c^\dagger_{-,k}c_{-,k+q}c_{+,k'-q}c^\dagger_{+,k'}\Big) \\ &= \frac{1}{L}\sum_{q} g_1\big(\rho_+(q)\rho_-(-q) + \rho_-(q)\rho_+(-q)\big) \end{aligned} \tag{F.2}$$

This has the same form as the g_2 term above. So for spinless fermions the backscattering g_1 can be absorbed into the forward scattering term by a redefinition of g_2. For spinfull fermions this is not possible. In this case the backward scattering process is discussed away by renormalization group arguments.

G. Appendix - Figures

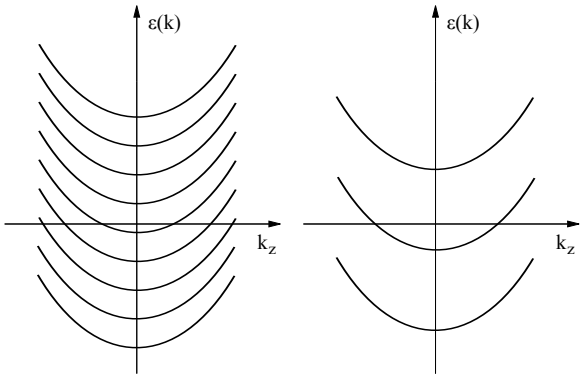

Figure G.1.: Schematic band structure for a quantum wire infinitely extended in z-direction and with more (left) and fewer (right) extension in the perpendicular direction. For the later case only band crosses Fermi energy.

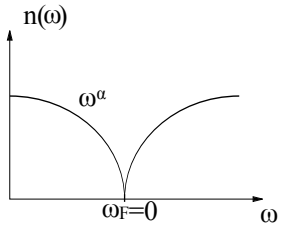

Figure G.2.: Single particle density of states $n(\omega)$ in 1D goes to zero as a power law when approaching Fermi energy. This is an other evidence that single particle excitation do not survive near Fermi level and due to that are not an appropriate for such systems.