CHAPTER V.

MEASUREMENT AND ESTIMATION OF THE FLOW OF WATER.

Weight of Water—Units of Volume and Time—Discharge—Action of Gravity—Theoretical Velocity—Path traversed by a jet of Water issuing with a known Velocity—Orifices in thin Plates, or thin-edged Orifices—Coefficient of Velocity—True mean Velocity under small Charges—Contraction of the Fluid Vein—Coefficient of Discharge—Circular Orifices—Rectangular Orifices—True mean Velocity—Experiments by Poncelet and Lesbros—Notches and Weirs—Rectangular and Triangular Notches—Right-angled Triangular Notches—Experiments by Messrs. Blackwell and Simpson, and Boileau—Suppressed Contraction—Velocity of Approach—Separating Weirs—Submerged Orifices and Weirs—Adjutages: Cylindrical, Conically Converging, and Conically Diverging—Shoots—Discharge under a variable Head—Time of Emptying Prismatic and other Reservoirs—Discharge from one Vessel into another—Flow of Water through uniform Channels—Mean Velocity determined by Maximum Surface Velocity—Accelerating and Retarding Forces—Mean Velocity of Flow in Rivers and open Channels, and through long and short Pipes—Friction caused by Bends and sudden Enlargements—Total Loss of Head, and final Velocity—Determination of the Section when the Discharge and Head are given.

IN the following passages, water will be regarded as an inelastic fluid, it having been found (p. 18) that excessive pressure is required to effect even a very small diminution in bulk, inappreciable under ordinary practical circumstances.

The units which are adopted for the measurement of water are the cubic foot and the gallon. The weight of a cubic foot of water varies, of course, with the temperature—at its maximum density (39·1° Fahr.), it weighs 62·425 lbs. avoirdupois; at 62° Fahr. it weighs 62·355 lbs. The imperial gallon contains 10 lbs. avoirdupois of water (62° Fahr. and the barometer at 30 inches), so that a cubic foot of water contains 6·235 gallons. In practice it is usual to consider the cubic foot of water as weighing 62·5 lbs. and containing 6·25 gallons. Of the units of time, the second is coupled mostly with the cubic foot; the minute is frequently used for the discharge of streams; while the hour and day are employed with thousands or millions of gallons in speaking of the delivery of large quantities of water. The units of discharge, compounded from the units of volume and of time, are very numerous. Perhaps, on the whole, the cubic foot per second and the gallon per day are the most customary.

Discharge.—The discharge of a stream or current of water is the product of the sectional area of the stream, and the mean velocity with which the several 'threads' of water in that stream are flowing. Thus, if it be found by careful measurement that the section of a stream at right angles to its flow is 30 sq. feet, and also that its mean velocity is 2 feet per second, it will be shown that the discharge is 60 cubic feet per second. In the same way, the mean velocity may be found, if the discharge be divided by the area of the section. These two elements of the true section and true mean velocity are all that is essential for the calculation; and it is the determination of the values of the same under varying conditions which constitutes, in great part, the science of hydraulics.

The velocity of a current of water is due to the action of a force, mostly the force of gravity, but in any case a force of which gravity may be made a measure.

The *Theoretical Velocity*, or that due to the force of gravity, is given by the formula—

$$v = \sqrt{2 g H} \qquad (1)$$

which, for measure in feet, becomes

$$\left.\begin{aligned} v &= \sqrt{64{\cdot}4\,H} \\ \text{or } v &= 8{\cdot}025\sqrt{H} \end{aligned}\right\} \qquad (1\,A)$$

This is the velocity in feet per second* which a body would acquire upon falling in a vacuum through a height equal to H, and, but for the retarding effect of friction, to be hereafter mentioned, it would be the velocity which a stream of water would acquire upon flowing down a channel through a height equal to H; or the velocity with which a jet of water would issue from an orifice in the side of a reservoir, the head of water or 'charge' upon that orifice being equal to H. In the latter case, the velocity of issue would be the same as if the

* The value 64·4, or twice the measure of the force of gravity, varies slightly with the latitude, but not to an extent worth recognising in hydraulic formulæ.

total head or charge ($\text{H}=m+n$) consisted partly of the influence of a column of water of the height m, and partly of that of a loaded piston, the pressure upon which is equal to the weight of a column of water of the height n. On the other hand, if the stream were issuing from a closed vessel, in which a partial vacuum was maintained, the height of a column of water that would be a measure of the vacuum must be subtracted from the actual head of water or charge. In the following table are given some values of v for corresponding values of H. For measures in inches, $v=27{\cdot}8$.

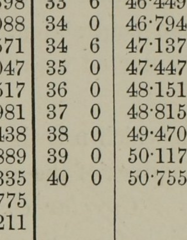
Head		Theoretical velocity in feet per second	Head		Theoretical velocity in feet per second	Head		Theoretical velocity in feet per second	Head		Theoretical velocity in feet per second	Head	Theoretical velocity in feet per second	Head	Theoretical velocity in feet per second
Feet and inches	Feet		Feet and inches	Feet		Feet and inches	Feet		Feet and inches	Feet		Feet and inches		Feet and inches	
0 $0\frac{1}{8}$	·0104	·819	0 $3\frac{1}{2}$	·2916	4·334	0 $9\frac{1}{2}$	·7916	7·140	2 9	2·7500	13·308	9 6	24·735	22 0	37·641
0 $0\frac{3}{16}$	·0156	1·003	0 $3\frac{5}{8}$	·3020	4·410	0 $9\frac{3}{4}$	·8125	7·233	2 $10\frac{1}{2}$	2·8749	13·607	9 9	25·058	22 6	38·066
0 $0\frac{1}{4}$	·0208	1·158	0 $3\frac{3}{4}$	·3125	4·486	0 10	·8333	7·325	3 0	3·0000	13·990	10 0	25·377	23 0	38·487
0 $0\frac{5}{16}$	·0260	1·295	0 $3\frac{7}{8}$	·3229	4·560	0 $10\frac{1}{4}$	·8541	7·417	3 $1\frac{1}{2}$	3·1249	14·186	10 3	25·693	23 6	38·903
0 $0\frac{3}{8}$	·0312	1·418	0 4	·3333	4·633	0 $10\frac{1}{2}$	·8749	7·506	3 3	3·2500	14·467	10 6	26·004	24 0	39·315
0 $0\frac{7}{16}$	·0364	1·532	0 $4\frac{1}{8}$	·3437	4·705	0 $10\frac{3}{4}$	·8958	7·595	3 $4\frac{1}{2}$	3·3749	14·743	10 9	26·312	24 6	39·722
0 $0\frac{1}{2}$	·0416	1·638	0 $4\frac{1}{4}$	·3541	4·775	0 11	·9166	7·683	3 6	3·5000	15·013	11 0	26·616	25 0	40·125
0 $0\frac{9}{16}$	·0468	1·737	0 $4\frac{3}{8}$	·3645	4·845	0 $11\frac{1}{4}$	·9374	7·770	3 $7\frac{1}{2}$	3·6249	15·279	11 3	26·917	25 6	40·525
0 $0\frac{5}{8}$	·0520	1·831	0 $4\frac{1}{2}$	·3749	4·914	0 $11\frac{1}{2}$	·9582	7·856	3 9	3·7500	15·540	11 6	27·214	26 0	40·920
0 $0\frac{11}{16}$	·0572	1·920	0 $4\frac{5}{8}$	·3853	4·982	0 $11\frac{3}{4}$	·9791	7·941	3 $10\frac{1}{2}$	3·8749	15·797	11 9	27·501	26 6	41·312
0 $0\frac{3}{4}$	·0625	2·006	0 $4\frac{3}{4}$	·3958	5·049	1 0	1·0000	8·025	4 0	4·0000	16·050	12 0	27·800	27 0	41·700
0 $0\frac{13}{16}$	·0677	2·088	0 $4\frac{7}{8}$	·4062	5·114	1 $0\frac{1}{2}$	1·0416	8·190	4 2	4·1666	16·381	12 3	28·088	27 6	42·084
0 $0\frac{7}{8}$	·0729	2·167	0 5	·4166	5·180	1 1	1·0833	8·352	4 4	4·3333	16·705	12 6	28·373	28 0	42·465
0 $0\frac{15}{16}$	·0781	2·243	0 $5\frac{1}{8}$	·4270	5·244	1 $1\frac{1}{2}$	1·1249	8·512	4 6	4·5000	17·023	12 9	28·655	28 6	42·842
0 1	·0833	2·316	0 $5\frac{1}{4}$	·4374	5·308	1 2	1·1666	8·668	4 8	4·6666	17·336	13 0	28·935	29 0	43·216
0 $1\frac{1}{8}$	·0937	2·457	0 $5\frac{3}{8}$	·4478	5·371	1 $2\frac{1}{2}$	1·2082	8·821	4 10	4·8333	17·643	13 3	29·212	29 6	43·587
0 $1\frac{1}{4}$	·1041	2·590	0 $5\frac{1}{2}$	·4582	5·433	1 3	1·2500	8·972	5 0		17·944	13 6	29·486	30 0	43·955
0 $1\frac{3}{8}$	·1145	2·716	0 $5\frac{5}{8}$	·4686	5·494	1 $3\frac{1}{2}$	1·2916	9·120	5 3		18·388	13 9	29·758	30 6	44·320
0 $1\frac{1}{2}$	·1250	2·837	0 $5\frac{3}{4}$	·4791	5·555	1 4	1·3333	9·266	5 6		18·820	14 0	30·027	31 0	44·682
0 $1\frac{5}{8}$	·1353	2·953	0 $5\frac{7}{8}$	·4895	5·615	1 $4\frac{1}{2}$	1·3749	9·410	5 9		19·243	14 6	30·558	31 6	45·041
0 $1\frac{3}{4}$	·1458	3·064	0 6	·5000	5·674	1 5	1·4166	9·551	6 0		19·657	15 0	31·081	32 0	45·397
0 $1\frac{7}{8}$	·1562	3·172	0 $6\frac{1}{4}$	·5208	5·791	1 $5\frac{1}{2}$	1·4582	9·791	6 3		20·063	15 6	31·595	32 6	45·750
0 2	·1666	3·276	0 $6\frac{1}{2}$	·5416	5·906	1 6	1·5000	9·828	6 6		20·460	16 0	32·100	33 0	46·101
0 $2\frac{1}{8}$	·1770	3·377	0 $6\frac{3}{4}$	·5625	6·018	1 7	1·5833	10·098	6 9		20·850	16 6	32·598	33 6	46·449
0 $2\frac{1}{4}$	·1874	3·475	0 7	·5833	6·129	1 8	1·6666	10·360	7 0		21·232	17 0	33·088	34 0	46·794
0 $2\frac{3}{8}$	·1978	3·570	0 $7\frac{1}{4}$	·6041	6·237	1 9	1·7500	10·616	7 3		21·608	17 6	33·571	34 6	47·137
0 $2\frac{1}{2}$	·2082	3·663	0 $7\frac{1}{2}$	·6249	6·344	1 10	1·8333	10·866	7 6		21·977	18 0	34·047	35 0	47·447
0 $2\frac{5}{8}$	·2186	3·753	0 $7\frac{3}{4}$	·6458	6·449	1 11	1·9166	11·110	7 9		22·341	18 6	34·517	36 0	48·151
0 $2\frac{3}{4}$	·2291	3·841	0 8	·6666	6·552	2 0	2·0000	11·349	8 0		22·698	19 0	34·981	37 0	48·815
0 $2\frac{7}{8}$	·2395	3·928	0 $8\frac{1}{4}$	·6874	6·654	2 $1\frac{1}{2}$	2·1249	11·698	8 3		23·050	19 6	35·438	38 0	49·470
0 3	·2500	4·012	0 $8\frac{1}{2}$	·7082	6·754	2 3	2·2500	12·037	8 6		23·397	20 0	35·889	39 0	50·117
0 $3\frac{1}{8}$	·2604	4·095	0 $8\frac{3}{4}$	·7291	6·852	2 $4\frac{1}{2}$	2·3749	12·367	8 9		23·738	20 6	36·335	40 0	50·755
0 $3\frac{1}{4}$	·2708	4·176	0 9	·7500	6·950	2 6	2·5000	12·688	9 0		24·075	21 0	36·775		
0 $3\frac{3}{8}$	·2812	4·256	0 $9\frac{1}{4}$	·7708	7·045	2 $7\frac{1}{2}$	2·6249	13·002	9 3		24·407	21 6	37·211		

The velocity of a jet of water being known, the path it will follow may be readily traced; for it may be shown to be a parabola whose parameter is equal to four times the height due to the velocity of projection. If the body be projected in the direction A Y (fig. 19) with a velocity due to the height of h, then

$$y^2=4\,h\,x \quad . \quad . \quad . \quad . \quad (2)$$

from which expression any value may be determined, when the other two are known.

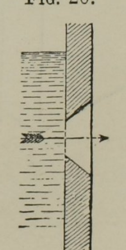
FIG. 19.



DISCHARGE THROUGH ORIFICES AND OVER NOTCHES AND WEIRS.

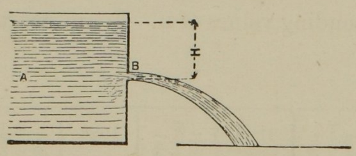
Orifices in thin Plates, or thin-edged Orifices.—It is necessary here to define what is meant by a thin-edged orifice, as mistakes often arise on this point. The thin edge should be formed on the inner side of the plate, as in fig. 20, so that for all practical purposes the orifice shall, as far as the current of water is concerned, be the same as if it were formed in a very thin plate. Let A (fig. 21) be a reservoir in which the level of the water is maintained constant, and let an orifice, the area of which is known, be perforated in the vertical side of the reservoir at B. From what has already been said, it might be inferred that, in calculating the discharge from the orifice B, the following process only would suffice. Ascertain the velocity due to the head from the level of still water to the centre of the orifice, regarding it as the mean of the velocity of the several threads, and multiply this by the area of the orifice. This is sometimes called, although not with strict accuracy, the theoretical discharge; and it is in excess of the actual discharge

FIG. 20.



from two causes, which are, first, the friction of the water against the sides of the orifice, and, second, a diminution in the actual section of the current of water, termed the 'contraction.'

FIG. 21.

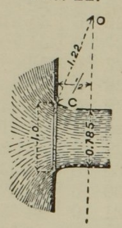


The friction diminishes the velocity of the current, and $v' \div v = m$, wherein v' is the actual, and v the theoretical, velocity—is the coefficient of velocity—which has to be determined by experiment. It is found that the velocity is proportional to the square root of the head or charge, the coefficient remaining practically constant at about $m = \cdot 975$.

When the head or charge is greater than about three or four times the height of the orifice, it is sufficiently accurate to regard the mean theoretical velocity as that due to the height from the surface of the still water to the centre of gravity of the orifice. It may be shown* that, for heads less than this, the greatest error cannot exceed four per cent. in the case of circular orifices, and six per cent. in the case of rectangular ones, in excess of the values given by formulæ mathematically correct, even when the upper side of the orifice is on the level of still water, the orifice thus becoming a 'notch.'

The contraction of the fluid vein is caused by the convergence of the fluid threads towards the centre of the orifice, as shown in fig. 22. If the orifice be circular and in the thin vertical side of a reservoir, the maximum contractions will occur at a distance from the orifice equal to half its diameter. If the jet issue downwards, it will be greater, and if upwards at a less distance than this. With rectangular orifices, the section of the vein varies continually. With circular ones, the form of sections is preserved, but its dimensions are gradually reduced, until at the point of maximum contraction, as above, the diameter is only ·785 of the original diameter, and, in consequence, the area is diminished from 1 to $\cdot 785^2$, or from 1 to ·616. It may be shown that in fig. 22 the radius O C is equal to 1·22. The amount of contraction is influenced by the position of the orifice with regard to the sides of the reservoir, being least when the orifice is near the upper surface of the water, and near a side or bottom of the reservoir, and greatest when most distant from the same. Generally, the coefficients for friction and contraction are combined into a 'coefficient of discharge,' being the ratio of the actual to the theoretical discharge. This will be theoretically the product of the coefficients of velocity and contraction. Numerous experiments, details of which will be found in treatises on Hydraulics, have been conducted with a view to determine practically the value of this coefficient c in the equation

FIG. 22.



$$D = c\,A\sqrt{2\,g\,H} \qquad (3)$$

in which D is the discharge, and A the area of the orifice. As might be expected, from the irregularities in the conduct of the experiments, the coefficients are very variable.

Circular Orifices.—Michelotti determined, from orifices of 1 to 3 inches in diameter, a coefficient of ·614; while from Bossut's experiments with smaller orifices, a mean of ·62 is obtained. Rennie's experiments give even larger coefficients; here, however, ·62 will be considered a fair average; so that

$$D = \cdot 62 \times A\sqrt{2\,g\,H}$$

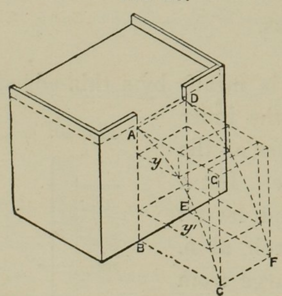
which, for cubic feet per second, becomes

$$\left.\begin{aligned} D &= 5\,A\sqrt{H} \text{ nearly} \\ &= 3\cdot 908\,d^2\sqrt{H} \end{aligned}\right\} \quad (4)$$

d being the diameter of the orifice in feet.

Rectangular Orifices.—It has been seen that the velocity of any horizontal layer of water will vary as $\sqrt{H}$. From this, it may be shown that if the horizontal distances y, y' (fig. 23) be drawn, representing this velocity, due to the several heads, the curve A C thus determined will be a parabola, with its vertex at A; and the volume of water discharged will be the prism A B C DE F, whose base is the parabolic segment A B C, and height the width A D of the stream of water. From a well-known property of the parabola, the segment A B C is $\frac{2}{3}$ the rectangle A G C B; but $B C = \sqrt{2\,g\,(A B)} = \sqrt{2\,g\,h}$, so that, calling l the width of the stream, we have for the volume discharged—

FIG. 23.



$$D = \tfrac{2}{3} \times l \times A B\sqrt{2\,g\,(A B)}$$

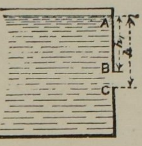
or, to introduce the coefficient of discharge, and adopt the usual form and notation,

$$D = c \times \tfrac{2}{3} \times l\sqrt{2\,g} \times h\sqrt{h} \qquad (5)$$

* See Neville's Hydraulics.

But the discharge from a rectangular orifice B C (fig. 24) will be that due to the height A C, minus that due to the height A B; or, symbolically,

Fig. 24.



$$D = c \times \tfrac{2}{3} \times l \sqrt{2g}\left(h\sqrt{h} - h_1\sqrt{h_1}\right) \quad . \quad (6)$$

in which h is the head to the bottom, and h_1 to the top of the orifice. If for the height of the orifice $(h - h_1)$, d be substituted, A for $l \times d$, and the head be measured to the centre of the orifice, the discharge will become very nearly

$$D = c\left(1 - \frac{d^2}{96\,H^2}\right) A \sqrt{2 g H} \quad . \quad . \quad . \quad (7)$$

The formula commonly used, however, is

$$D = c A \sqrt{2 g H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

the coefficient c including an approximate correction for the incompleteness of the remainder of the expression.

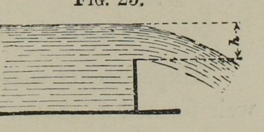
With a view to determine the coefficients of discharge c, for rectangular orifices, a valuable series of experiments was conducted by Poncelet and Lesbros, at Metz. The apertures were about 8 inches wide, and of varying heights, while the heads or charges vary from less than half an inch to nearly 10 feet. It would appear from the experiments, that, for smaller and more oblong orifices, the coefficient increases as the head diminishes, while the reverse is the case with orifices which are larger and of proportions nearer a square. The following table is founded upon these experiments.

Head of water ÷ depth of orifice	Height of orifice ÷ breadth						Head of water ÷ depth of orifice	Height of orifice ÷ breadth					
	1	0·5	0·25	0·15	0·1	0·05		1	0·5	0·25	0·15	0·1	0·05
·05	—	—	—	—	—	·709	2·50	·602	·617	·631	·630	·640	·643
·10	—	—	—	—	·660	·698	3·00	·6035	·616	·630	·629	·6385	·640
·15	—	—	—	·638	·660	·691	3·50	·604	.616	·629	·629	·637	·638
·20	—	—	·612	·640	·659	·685	4·00	·604	·615	·629	·628	·6355	·634
·25	—	—	·617	·640	·659	·682	4·50	·6045	·615	·628	·628	·634	·631
·30	—	·590	·622	·640	·658	·678	5·00	·605	·615	·627	·627	·632	·627
·35	—	·595	·624	·639	·658	·674	5·50	·6045	·614	·626	·626	·630	·625
·40	—	·600	·626	·639	·657	·671	6·00	·604	·614	·624	·624	·6275	·623
·45	—	·602	·627	·638	·656	·669	6·50	·604	·613	·623	·623	·625	·621
·50	—	·605	·628	·638	·655	·667	7·00	·6035	·613	·622	·622	·623	·620
·55	—	·607	·629	·637	·655	·665	7·50	·603	·612	·621	·621	·621	·618
·60	·572	·609	·630	·637	·654	·664	8·00	·602	·611	·619	·619	·618	·616
·65	·578	·609	·630	·637	·654	·662	8·50	·602	·610	·618	·617	·6165	·615
·70	·582	·610	·631	·636	.653	·661	9·00	·6015	·609	·616	·616	·6155	·615
·75	·585	·611	·631	·636	·653	·660	9·50	·6015	·608	·614	·614	·614	·614
·80	·587	·611	·632	·635	·652	·659	10·00	·601	·607	·613	·613	·613	·613
·85	·589	·611	·632	·635	·652	·658	11·00	·601	·606	·611	·611	·6115	·612
·90	·591	·612	·633	·634	·651	·657	12·00	·601	·605	·609	·610	·611	·611
·95	·592	·612	·633	·634	·651	·656	13·00	·601	·604	·608	·609	·6095	·610
1·00	·592	·613	·634	·634	·650	·655	14·00	·601	·604	·607	·608	·609	·609
1·50	·598	·616	·632	·632	·645	·650	15·00	·601	·603	·606	·607	·608	·609
2·00	·600	·617	·631	·631	·642	·647							

With heads of less than from three to five times the height of the orifice, there is a perceptible depression of the water-line at the plate: the heads given in the table are measured to the level of still water above this depression. The coefficients include a correction for measuring the head from the centre of the orifice, instead of from the point where the mean velocity occurs, which is a little above the centre.

Notches and Weirs.—The formulæ given above will apply to notches and weirs, if the orifice be regarded as extending up to the level of the surface. Thus, if in equation (6), $h_1 = o$, we shall have equation (5), which really gives the discharge over a weir, where h is the difference of level between the thin horizontal edge of the weir board and the still water, and l the length of the overfall.

Fig. 25.

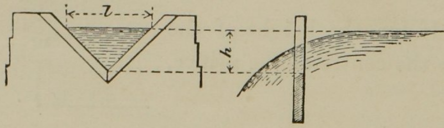


$$\text{But} \quad l \times h = A$$

$$\text{therefore} \quad D = c \times \tfrac{2}{3} \times A \sqrt{2 g h} \quad . \quad . \quad (9)$$

With a triangular notch, the discharge is

FIG. 26.



$$D = c \times \tfrac{4}{15}\, l h \sqrt{2gh} \quad . \quad . \quad . \quad . \quad (10)$$

in which l is the width of the notch at the level of still water, and h the distance of the apex below the same. In a notch of any given angle the proportion of height to base remains constant: for a right-angled triangular notch (fig. 26)

$$D = c \times \tfrac{8}{15}\, h^2 \sqrt{2gh} \quad . \quad . \quad . \quad . \quad (11)$$

The observation of the true amount of head demands the exercise of great care, as the surface of the water is curved for some distance above the overfall. (*See* fig. 25.) Mr. Neville gives for the difference between the thickness of the sheet of water passing over the crest, and the head (h) measured to the level of still water,

$$h - h_w = \cdot 14 \sqrt{h} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

for measures in feet. The difference, except for very small heads, will be found to vary from one-tenth to one-quarter of the true head.*

The coefficients derived from direct experiments with notches and weirs are very variable, perhaps on account of some of the modifying causes to be hereafter mentioned. In the present instance, we shall class the coefficients for *thin-edged* weirs as follows:—

When the width of the weir is about one-fourth of that of the canal itself . $c = \cdot 600$

When the width of the weir is equal to the total width of the canal . . $c = \cdot 665$

Between the above limits (b = width of the canal, and b' that of the weir) . $c = \cdot 57 + \dfrac{b'}{10\,b}$

For a right-angled triangular notch . . . . $c = \cdot 617$

The coefficients for rectangular notches decrease as the depth of water flowing over is greater in proportion to the length of the notch. The coefficients for triangular notches vary with the form of the triangle; but when the form of the triangle is constant, it is probable that the coefficient will remain the same, whatever be the depth flowing over the notch.

The following table, which is from a valuable series of experiments by Mr. T. E. Blackwell, will show the effects of substituting for thin edges various broad crests of different inclinations. From the circumstances under which the experiments were conducted, it is probable that the coefficients are somewhat lower for the larger heads than what should be considered fair averages.

COEFFICIENTS OF DISCHARGE FROM WEIRS, FROM EXPERIMENTS BY MR. T. E. BLACKWELL.

Heads in inches, measured from still water in the reservoirs	Thin plates, $\frac{1}{16}$ inch		Planks 2 inches thick, square on crest				Crests 3 feet wide					
	3 feet long	10 feet long	3 feet long	6 feet long	10 feet long	10 ft. long wing boards converging at an angle of 64°	3 feet long, level	6 feet long, level	10 feet long, level	3 feet long, fall 1 in 18	10 feet long, fall 1 in 18	3 feet long, fall 1 in 12
1	·677	·808	·467	·459	·435	·754	·452	—	·381	·545	·467	·467
2	·675	·802	·509	·561	·585	·675	·482	—	·479	·546	·495	·533
3	·630	·642	·563	·597	·569	—	·441	·492	—	·537	—	·539
4	·617	·655	·549	·575	·602	·656	·419	·497	—	·431	·515	·455
5	·601	·649	·588	·601	·609	·671	·479	—	·518	·516	—	—
6	·592	—	·593	·608	·576	—	·501	—	·513	—	·543	·531
7	—	—	·616	·608	·576	—	·488	·497	—	·513	—	·527
8	—	·581	·606	·590	·548	—	·470	—	·468	·491	·507	—
9	—	·530	·600	·569	·558	—	·476	·480	·486	·492	—	·498
10	—	—	·614	·539	—	—	—	·465	·455	—	—	—
12	—	—	—	·525	—	—	—	·467	—	—	—	—
14	—	—	—	·549	—	—	—	—	—	—	—	—
Mean	632	667	570	565	562	689	467	483	471	508	505	507

Experiments were conducted by Messrs. Blackwell and Simpson, at Chew Magna, in Somerset, with a 10-feet weir formed as shown in figs. 27 and 28; the cill was a cast-iron plate, two inches thick, with a square top. In

* Neville's Hydraulics.

the plan, fig. 27, A B is the overfall, to which the water was conducted by a channel of equal width. On the whole, it may be seen that the coefficients increase as the head is greater; but this is to be accounted for by the

Fig. 27.

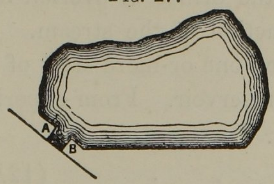
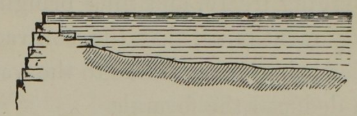


Fig. 28.



fact that with the larger heads the *velocity of approach* (see p. 74) was considerable, but was nevertheless omitted from the calculations by which the coefficients were ascertained.

Coefficients of Discharge.

Experiments by Messrs. Simpson and Blackwell.

Head in feet	Coefficients	Head in feet	Coefficients	Head in feet	Coefficients
·083 to ·073	·591	·3437	·743	·5	·749
·083 to ·088	·626	·3594	·760	·5156	·748
·182 to ·187	·682	·3646	·741	·5156 to ·521	·747
·229	·665	·3610	·750	·5781	·772
·2435	·670	·375	·725	·639	·717
·2396	·655	·416	·780	·6666	·802
·2422	·653	·4227	·781	·66 to ·734	·737
·2448	·654	·4505	·749	·7448	·750
·25 to ·253	·725	·453 to ·456	·751	·75	·781
·3333	·745	·4948	·728	Mean.	·723

The following are the results of some experiments carried on by Boileau, at Metz, in 1854, with a vertical plank weir extending from side to side of the supplying channel:—

Head of weir above bottom of channel	Head	Mean coefficient
Feet	Feet	
3	·2 to ·6	·645
1·3	·16 to ·5	·622
·6	·15 to ·25	·625

When the water in the lower channel rose to the level of the weir board, the results were as follow:—

Head of weir above bottom of channel	Head	Mean coefficient
Feet	Feet	
2	1 to 1·6	·694
1·3	·6 to 1·8	·690
·6	·36 to 1·3	·675

With a plank weir 1·5 feet in height, leaning up stream four inches in a foot, the mean value of the coefficient was ·620, the heads varying from about 3 to 6 inches. When the weir board was still inclined, and the tail-water rose to the crest, the latter being rounded to a semi-circle, the values of c were ·696 and ·843, with heads of about 3 and 6 inches respectively.*

Suppressed Contraction.—In all the cases treated above, except where otherwise specified, it has been supposed that the water has had the opportunity of flowing towards the orifice or overfall from all directions, the fluid threads converging freely, and thus bringing about the contraction of the stream. It frequently occurs, however, as already mentioned, that the contraction is suppressed on one or more sides of the opening, in consequence of the orifice being formed close to the walls or bottom of the reservoir. From experiments on rectangular orifices, Weisbach deduced the formula

$$c' = c\left(1 + \cdot 132\frac{n}{p}\right) \quad . \quad . \quad . \qquad (12\text{A})$$

in which p is the perimeter of the orifice, n that part of it where the contraction is suppressed, c the coefficient of free contraction, as before, and c' the coefficient of partial contraction. In a similar equation, M. Bidone gives ·152, instead of ·132; so that, adopting a mean value for c, we may consider approximately

$$c' = c + \cdot 09\frac{n}{p} \quad . \quad . \quad . \quad . \qquad (13)$$

Velocity of Approach.—When the discharge through an orifice or over a weir is from a channel in which there is a sensible velocity of approach, let v' be that velocity in feet per second; then the head due to that velocity is, from (1),

$$h' = v'^2 \div 64\cdot 4$$

and the discharge will be that due to the head ($H + h'$). Thus, the head being measured from the centre of the orifice,

$$\left.\begin{aligned} D &= c\,A\sqrt{64\cdot 4\left(H + \frac{v^2}{64\cdot 4}\right)} \quad . \quad . \quad . \\ &= c\,A\sqrt{64\cdot 4\,H + v^2} \quad . \quad . \quad . \quad . \end{aligned}\right\} (14)$$

The following, however, is a more correct formula for rectangular orifices, the true mean velocity of discharge being regarded:—

$$D = \tfrac{2}{3}\,c\,l\sqrt{2\,g}\left\{(h + h')^{\frac{3}{2}} - (h_1 + h')^{\frac{3}{2}}\right\} \quad . \quad . \quad . \qquad (15)$$

in which h and h_1 are the heads, measured from the bottom and top of the orifice respectively. For a notch or weir, h_1 vanishes, and formula (15) becomes

$$D = \tfrac{2}{3}\,c\,l\sqrt{2\,g}\left\{(h + h')^{\frac{3}{2}} - h'^{\frac{3}{2}}\right\} \quad . \quad . \quad . \qquad (16)$$

If A be the area of an orifice, and A_1 the sectional area of the supplying canal, taken at right angles to the current,

$$\frac{A}{A_1} = \frac{v'}{v}$$

v and v' being the *mean* velocities in the orifice and canal respectively. The head due to the velocity of approach ($v\,A \div A_1$) will be

$$h' = \frac{1}{2\,g}\left(\frac{v\,A}{A_1}\right)^2 \quad . \quad . \quad . \quad . \qquad (17)$$

But $D = v \times A$; therefore

$$h' = \frac{D^2}{2\,g\,.\,A_1^{\,2}} \quad . \quad . \quad . \quad . \quad . \qquad (17\text{A})$$

An approximate value for the velocity of approach having been ascertained, the height h' due to it is to be inserted in formula (15) or (16), and an approximate discharge computed. A new and closer value of h' may then be obtained from (17) or (17A); and thus by continued substitution of the new values, any required degree of accuracy may be obtained. For general purposes, a mean velocity of approach, ascertained by one or other of the usual methods, will suffice for the determination of the discharge.

The foregoing is on the supposition that the whole of the discharge suffers a contraction whose coefficient is c. If, however, that part of the discharge which is due to the velocity of approach suffer no contraction, the head required to produce that velocity in the orifice, with contraction, will be

$$h' = \frac{v^2}{2\,g\,.\,c^2}\cdot\frac{A^2}{A_1^{\,2}} \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

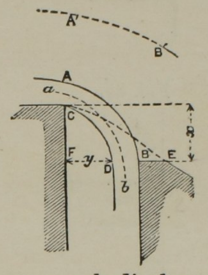
or, from equation (17A),

$$h' = \frac{D^2}{2\,g\,c\,A_1^{\,2}} \quad . \quad . \quad . \quad . \quad . \qquad (18\text{A})$$

* P. Boileau, Traité de la Mesure des Eaux Courantes, etc. Paris, 1854.

Separating Weirs.—It has been seen (p. 69) that a jet of water issuing with a certain velocity describes a parabola whose parameter is four times the height to which that velocity is due. In a stream of considerable depth passing over a weir the various fluid threads will have velocities depending upon their depths below the surface. It will be sufficiently accurate for all practical purposes, however, to suppose that the stream will advance in a curved sheet A B C D (fig. 29), parallel on its upper and lower surfaces with the curve due to the mean velocity. The fluid layer having the mean velocity is that, *a b*, which is at four-ninths of the depth of the stream, measured from the level of still water, and the mean velocity is two-thirds of that due to the head, measured from the weir crest to the level of still water. In fig. 29 are shown two streams; the one in full lines (A B, C D) has such a mean velocity that it will just fall within the distance F B; and the other in dotted lines (A′ B′, C E), being due to a much greater head, is carried beyond the distance F E. The utility of the arrangement consists in separating the clear water of streams in their normal condition from the turbid water which rushes down in the times of floods; and in order that the weir may be properly adjusted, it is necessary to gauge the stream at such times as it commences to be turbid, that the flow of water may be known. The head above the weir due to such discharge will be given by the value of h in equation (9), and the corresponding parabolas may then be determined. From (2) we have

Fig. 29.



$$y = 2\sqrt{h\,x}$$

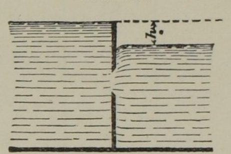
but the mean velocity of the sheet of water being two-thirds the velocity due to the head h above the weir, the horizontal distance y to which the cascade will leap in the height x will be

$$y = \tfrac{4}{3}\sqrt{h\,x} \quad . \quad . \quad . \quad . \quad (19)$$

in which h is the height from the weir crest to the level of still water.

Submerged Orifices and Weirs.—The case represented by fig. 30 is known as a submerged or drowned orifice; and it is evident that from all parts of the orifice the stream will issue with a velocity due to the head caused by the difference between the levels of still water in the upper and lower reservoir; thus

Fig. 30.

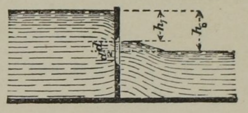


$$\text{D} = c\,\text{A}\sqrt{2\,g\,h_o} \quad . \quad . \quad . \quad (20)$$

The coefficient of discharge c in equation (20) has been found to have a value of about ·5.

When the orifice is only partially submerged (fig. 31) it may be considered divided into two parts—d_1, that below the level of the water in the lower reservoir, as a submerged orifice, and the remaining or upper part, d, as a free orifice; the total discharge will then be

Fig. 31.



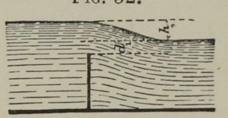
$$\text{D} = l\sqrt{2\,g}\left\{c\,d_1\sqrt{h_o} + \tfrac{2}{3}\,c\left(h_o\sqrt{h_o} - h_1\sqrt{h_1}\right)\right\} \quad (21)$$

If the water in the reservoir has a determined velocity of approach, the head h', due to that velocity, must be added to h_o and h_1 above, and the new values substituted.

The case of a drowned weir (fig. 32) may be regarded as consisting of an ordinary free notch, with a head equal to h_o, and a submerged orifice whose height is d_1, the head being also h_o; so that

$$\text{D} = l\sqrt{2\,g}\left(\tfrac{2}{3}\,c\,h_o\sqrt{h_o} + c\,d_1\sqrt{h_o}\right) \quad . \quad (22)$$

Fig. 32.



which, simplified, becomes

$$\text{D} = l\sqrt{2\,g\,h_o}\left(\tfrac{2}{3}\,c\,h_o + c\,d_1\right) \quad . \quad (22\text{A})$$

Where there is a velocity of approach due to a head h', then h_o becomes $(h_o + h')$; and, from (21), we have

$$\text{D} = l\sqrt{2\,g}\left[c\,d_1\sqrt{h_o + h'} + \tfrac{2}{3}\left\{(h_o + h')^{\frac{3}{2}} - h'^{\frac{3}{2}}\right\}\right] \quad (23)$$

The coefficient of discharge for the submerged sections of drowned weirs and partially submerged orifices may be taken as about the same as that already given for a completely submerged orifice, namely, ·5. Series of careful experiments with drowned weirs and partially submerged orifices are much required.

Adjutages.—In the experiments hitherto referred to it has been supposed, except where otherwise stated, that the orifices and notches were formed either in thin plates or with a thin edge on the up-stream side. If the orifice be placed in the side of a vessel of a thickness large in proportion to the dimensions of the orifice, the coefficient is considerably influenced, whilst similar effect is produced by adjutages or mouth-pieces consisting of short tubes, which may be of various forms and dimensions.

Experiments by Bossut on cylindrical tubes 1 inch in diameter and 2 inches long gave coefficients varying from ·818 to ·803, with heads of from 1 to 15 feet. Michelotti derived a mean coefficient of ·814 with tubes of from $1\frac{1}{2}$ to 3 diameters in length, and heads of from 3 to 20 feet. Having regard also to other experiments, ·815 may be taken as a fair average.

Fig. 33.

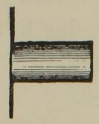
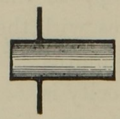


Fig. 34.



If the tube project within the side of the reservoir (fig. 34), the coefficient will be reduced to ·715.

If the inner end of the adjutage be rounded to the form of the contracted vein (figs. 22 and 35), the coefficient will be increased. Weisbach's experiments give ·958, ·969, ·975 for heads of 1, 5, and 10 feet respectively, the tube being 9 inches in diameter and 1·5 inches long. A variation in the form of adjutage from that of the contracted vein will of course result in a reduction of the coefficient.

Fig. 35.

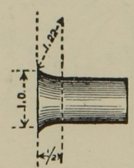
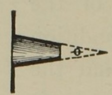


Fig. 36.



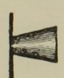
Conical convergent adjutages present some curious features. The velocity of the jets of water and the discharge vary with the angle of convergence of the sides, as will be seen from the following table, founded by Mr. Neville upon experiments by D'Aubuisson and Castel.*

Conical Convergent Tubes.

Converging Angle	Coefficient of Discharge	Coefficient of Velocity	Converging Angle	Coefficient of Discharge	Coefficient of Velocity	Converging Angle	Coefficient of Discharge	Coefficient of Velocity
1°	·858	·858	8°	·931	·933	20°	·922	·971
2°	·873	·873	10°	·937	·950	22°	·917	·973
3°	·908	·908	12°	·942	·955	26°	·904	·975
4°	·910	·909	14°	·943	·964	30°	·895	·976
5°	·920	·916	16°	.937	·970	40°	·869	·980
6°	·925	·923	18°	·931	·971	50°	·844	·985

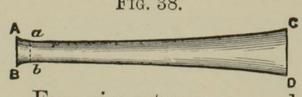
The experiments were made with tubes of ·61 inches in diameter at the smaller end, and 1·57 inches long. It will be seen that the coefficient of discharge starts at ·829, the tube being then cylindrical, and gradually increases until it attains the maximum, at an angle of about $13\frac{1}{2}$° or 14°; it then diminishes, the angle still increasing, until the latter attains its maximum, or 180°, when the orifice would be virtually in a plane plate. The coefficients of velocity increase with the angle. It must be understood that the smaller diameter is used in determining the coefficient, and not the larger or inner one. It is found with conical convergent adjutages, as with cylindrical ones, that the most favourable results are obtained when the length is about $2\frac{1}{2}$ times the diameter.

Fig. 37.



The discharge from conical divergent tubes (fig. 37), when running full, is greater than that from convergent tubes. It was found by Venturi, from his experiments, that a discharge 1·46 times the theoretic discharge from the smaller diameter *a b*, fig. 38, might be obtained with a tube of 9 times the smaller diameter in length, diverging at an angle of 5° 6′. If the mouth-piece be curved, as in fig. 38, the inner end being of the form of the contracted vein (fig. 22), *a* C being 9 times *a b*, and C D 1·8 times *a b*, the coefficient will rise to 1·57; so that the discharge will be 1·57 ÷ ·62 = 2·53 times that through a thin-edged orifice of the diameter of *a b*. If A B and *a b* be correctly proportioned, the discharge through adjutages thus formed will be about equal to the theoretic discharge from an orifice of the diameter A B.

Fig. 38.



Experiments were conducted by Mr. Bateman, at Manchester, with rectangular orifices, sections of which are given on plate 26, figs. 20, 21, and 22. The coefficients derived from the experiments were ·697, ·872, and ·947 respectively, with heads of from 1 to 4 feet above the centre of the openings.

Shoots.—When channels, open at the top, are attached to orifices, there is a diminution in the discharge, which is less as the discharge is greater; and when the charges are from 2 to $2\frac{1}{2}$ times greater than the height of the orifice itself, the effect of the addition of the shoot is inconsiderable; with very small heads, however, the discharge is diminished a fourth or more. Similar effects are produced when channels are attached to weirs or

* Neville's Hydraulics.

overfalls, as the following table will show. The experiments were by Poncelet and Lesbros; the channel was 9·84 feet long, ·656 feet wide—the same width as the overfall—and adjusted so as to be horizontal.

Head	Coefficient		Loss per cent.
	Without channel	With channel	
Feet			
0·675	0·582	0·479	18
0·475	0·590	0·471	20
0·337	0·591	0·457	23
0·196	0·599	0·425	29
0·147	0·609	0·407	33
0·091	0·622	0·340	45

Castel experimented on overfalls 8 inches wide and 8 inches long, inclined 4° 18′, or 1 in 13·3: the reservoir itself was 2 feet 3 inches wide. With heads varying from ·36 foot to ·16 foot, the coefficient was found to vary only from ·526 to ·530.

Discharge under a Variable Head.—It may be shown from the fundamental laws of mechanics, that the time occupied by the complete discharge from a prismatic vessel is twice that in which the same volume would flow out under a constant head equal to that at the commencement of the flow. If A = the area of the vessel on plan, a the area of the orifice, and H the head at the commencement of the discharge, the above theorem may be expressed by the equation

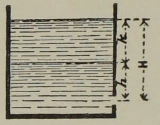
$$T = 2 \times \frac{A\sqrt{H}}{c\,a\sqrt{2g}} \quad . \quad . \quad . \quad . \qquad (24)$$

c being the coefficient of discharge, as before.

The time which will be occupied in discharging from a prismatic vessel a given quantity whose depth is k (fig. 39) will obviously be the difference between the times occupied in discharging from the heights H and h. Whence

$$T = \frac{2A}{c\,a\sqrt{2g}}(\sqrt{H} - \sqrt{h}) \quad . \quad . \quad . \qquad (25)$$

Fig. 39.



The discharge (D) for a given time is

$$D = T\,c\,a\sqrt{2g}\left(\sqrt{H} - \frac{T\,c\,a\sqrt{2g}}{4A}\right) \quad . \quad . \qquad (26)$$

The following formula gives the time of discharge when a constant stream is flowing into the reservoir, at the rate of q cubic feet per second:—

$$T = \frac{2A}{(c\,a\sqrt{2g})^2}\left\{c\,a\sqrt{2g}\,(\sqrt{H} - \sqrt{h}) + q \text{ hyp.log.} \frac{c\,a\sqrt{2gH} - q}{c\,a\sqrt{2gH} - q}\right\} \qquad (27)$$

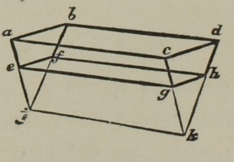
Hyp. log. = common log. × 2·30258.

If the time (T) be given, the value of h will give the level to which the water in the reservoir will have descended at the end of the time, under the same circumstances.

If the water in the reservoir be discharged over a weir, there being no influx into the basin, the time occupied in lowering the water from a head H to a head h will be

$$T = \frac{3A}{c\,l\sqrt{2g}}\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{H}}\right) \quad . \quad . \qquad (28)$$

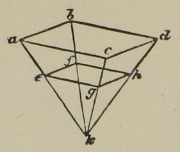
Fig. 40.



For wedged-shaped reservoirs ($a\,b\,d\,c\,j\,k$, fig. 40), the time of complete discharge will be $1\frac{1}{3}$ that of the same volume discharged under the initial head; while for pyramidal reservoirs ($a\,b\,d\,c\,k$, fig. 41), the time of complete discharge is to that of the same volume under the initial head as $1\frac{1}{5}$ to 1.

The time required to discharge a reservoir with sloping sides and vertical ends (as $a\,b\,d\,c\,e\,f\,h\,g$, fig. 40), or a reservoir with all its sides sloping equally (as *a b d c e f h g*, fig. 41), may be found by an obviously simple process of subtraction. Many reservoirs of comparatively irregular form will be capable of subdivision into frustra of wedges and pyramids, so that the principles given above will apply.

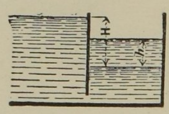
Fig. 41.



Vessels or reservoirs which cannot be subdivided thus it is necessary to regard as divided into a series of horizontal layers whose areas are known; these may be considered approximately as prismatic, and the discharge from them can be ascertained by the formulæ already given.

Let it now be supposed that a prismatic vessel is to be supplied by an orifice at its base from a reservoir whose surface remains at a constant level (fig. 42). If the level of the water in the lower reservoir or vessel also remain constant, and the orifice be submerged, the discharge will be simply that due to a head equal to the difference of level of the water-surfaces in the two reservoirs. If the water in the vessel rise as the flow proceeds, the discharge will be due to a head continually diminishing, so that the time occcupied in raising the water a given height will be twice that which would be occupied in discharging the same volume through a free orifice, or

Fig. 42.



$$\text{T} = \frac{2\,\text{A}}{c\,a\sqrt{2\,g}}\left(\sqrt{\text{H}} - \sqrt{h}\right)$$

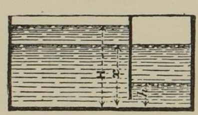
as in equation (25), in which A is the section on plan of the receiving vessel, and a the area of the orifice. If the lower vessel be filled to the level of the water in the upper vessel, then the formula will become

$$\text{T} = \frac{2\,\text{A}\sqrt{\text{H}}}{c\,a\sqrt{2\,g}}$$

as in equation (24).

Next let it be supposed that the upper or supplying reservoir is prismatic and of known capacity (fig. 43), and that the discharge takes place from the one vessel to the other, the total quantity of water in the two vessels remaining constant. Let H and h be the heads of water, above the orifice or other communication, in the upper and lower vessels respectively, before the flow commences; x the height above the orifice of the water-surface in the upper reservoir after the flow has been proceeding during the time t; A and B the sections (on plan) of the upper and lower reservoirs respectively; a the section of the passage of communication; and c the coefficient of discharge through the same; then

Fig. 43.



$$t = \frac{2\,\text{A}\sqrt{\text{B}}}{c\,a\sqrt{2\,g}\,(\text{A} + \text{B})}\left\{\sqrt{\text{B}\,(\text{H} - h)} - \sqrt{(\text{A} + \text{B})\,x - \text{A}\,h - \text{B}\,h}\right\} \quad . \quad (29)$$

The time which would be occupied in bringing the two surfaces to the same level is given by the formula

$$t = \frac{2\,\text{A}\,\text{B}\sqrt{\text{H} - h}}{c\,a\sqrt{2\,g}\,(\text{A} + \text{B})} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Flow of Water through Uniform Channels.

Mean Velocity.—In open channels the mean velocity (v) may be ascertained from the maximum or mean surface velocities. The following is an adaptation of Prony's formula to measures in English feet, V being the *maximum surface velocity* :—

$$v = \left(\frac{7{\cdot}783 + \text{V}}{10{\cdot}345 + \text{V}}\right)\text{V} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

This formula was derived from experiments in small channels. For large channels,

$$v = {\cdot}835\ \text{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Accelerating and retarding forces.—Water in flowing down a uniform channel is acted on by the force of gravity, which gives rise to the motion, and by certain resistances, commonly known as friction, tending to counteract or retard that motion. The velocity of the stream is at first gradually accelerated, but soon the maximum velocity is attained, and the channel is said to be 'in train,' the retarding forces being then equal to the accelerating forces, and the velocity becoming in consequence uniform.

We have seen that the velocity is proportionate to the square root of the height. The laws of the friction of water may be stated as follows : (1) It is independent of the pressure. (2) It is proportionate to the surface in contact with the flowing water. (3) It is inversely proportionate to the area of the cross-section of the stream. (4) It is proportionate to the square of the velocity *nearly*. Experiment has shown that the resistance does not increase quite so rapidly as the square of the velocity, but that it would be more nearly given as proportionate to

$$(a\,v + b\,v^2)$$

in which a and b are constants.

Equating the accelerating and retarding forces, we have

$$2\,g\,h = (a\,v + b\,v^2) \times l \times \frac{\text{P}}{\text{S}} \qquad (33)$$

in which S is the section of the stream, and p the wetted perimeter or border. The value S ÷ P = R is known as the *mean radius* or *hydraulic mean depth*. Omitting $2\,g$, as its value is constant and may therefore be embodied with the coefficient, we have

$$\text{R}\frac{h}{l} = (a\,v + b\,v^2) \qquad (34)$$

from which,

$$v = \sqrt{\frac{r\,h}{l\,b} + \frac{a^2}{4\,b^2}} - \frac{a}{2\,b} \qquad (35)$$

Different experimenters have assigned different values to the coefficients a and b, and the following are some of the resulting equations.

From Eytelwein's experiments with rivers, we have the general formula

$$\left.\begin{aligned} v &= \sqrt{8975{\cdot}4\,\text{R}\frac{h}{l} + {\cdot}011886} - {\cdot}109 \\ &= 94{\cdot}5\sqrt{\text{R}\frac{h}{l}} - {\cdot}11 \text{ nearly} \end{aligned}\right\} \quad (36)$$

From experiments on canals in which the velocities did not exceed three feet per second, Prony derived coefficients which give

$$\left.\begin{aligned} v &= \sqrt{10607\,\text{R}\frac{h}{l} + {\cdot}0556} - {\cdot}236 \\ &= 10\sqrt{\text{R}\frac{h}{l}} - {\cdot}24 \text{ nearly} \end{aligned}\right\} \quad (37)$$

An allowance should be made in the value of R when aquatic plants, reeds, &c. interfere with the progress of the water. This is sometimes provided for by multiplying the wetted perimeter (or dividing R, which is the same thing) by 1·7. No definite value, however, can be given when the conditions are liable to such extreme variations. Allowance must be made according to the judgment of experience, as, for instance, in the case of small water-courses pitched with materials of which the irregularities are comparatively large in proportion to the hydraulic mean depth.

For the coefficients a and b in (35), Mr. Neville gives for clear straight rivers

$$a = {\cdot}0000035 \qquad b = {\cdot}000115$$

from which

$$\left.\begin{aligned} v &= \sqrt{8695{\cdot}6\,\text{R}\frac{h}{l} + {\cdot}00023} - {\cdot}0152 \\ &= 93\sqrt{\text{R}\frac{h}{l}} - {\cdot}02 \end{aligned}\right\} \quad (38)$$

Du Buat's well-known formula for rivers, pipes, and channels, was determined after a most careful study of the results of numerous experiments. For measures in feet, it is as follows:—

$$v = \frac{88{\cdot}5(\sqrt{\text{R}} - {\cdot}03}{\sqrt{\frac{l}{h}} - hyp.\ log.\ \sqrt{\left(\frac{l}{h} + 1{\cdot}6\right)}} - {\cdot}084\,(\sqrt{\text{R}} - {\cdot}03) \qquad (39)$$

Mr. Neville gives the following general formula for pipes and channels:—

$$v = 140\,\sqrt{\text{R}\,s} - 11\,\sqrt[3]{\text{R}\,s} \qquad (40)$$

in which $s = h \div l$.

For pipes, Prony's coefficients, deduced from experiments by Du Buat, Bossut, and Couplet, give

$$\left.\begin{aligned} v &= \sqrt{9419{\cdot}7\,\text{R}\frac{h}{l} + {\cdot}00665} - {\cdot}0816 \\ &= 97\sqrt{\text{R}\frac{h}{l}} - {\cdot}08 \text{ nearly.} \end{aligned}\right\} \quad . \quad . \quad . \quad (41)$$

The pipes he experimented upon were from 1 to 5 inches in diameter, 30 to 7,000 feet long, and one 19-inch pipe 4,000 feet long.

Eytelwein's coefficients, derived from the same experiments, give

$$\left.\begin{aligned} v &= \sqrt{11704\,\text{R}\frac{h}{l} + {\cdot}01698} - {\cdot}13 \\ &= 108\sqrt{\text{R}\frac{h}{l}} - {\cdot}13 \text{ nearly} \end{aligned}\right\} \quad . \quad . \quad . \quad (42)$$

If in (33) we substitute ($c_f\ v^2$) for ($a\,v + b\,v^2$), and solve for v, we shall have

$$v = \sqrt{\frac{2\,g\,\text{R}\,h}{c_f\,l}} \quad . \quad . \quad . \quad . \quad . \quad (43)$$

c_f being the coefficient of friction, to which Weisbach has assigned the value

$$c_f = \left({\cdot}0036 + \frac{{\cdot}0043}{\sqrt{v}}\right) \quad . \quad . \quad . \quad . \quad (44)$$

thus recognising the principle that the friction diminishes somewhat as the velocity increases, and giving results for high velocities much nearer the truth.

In using (43) with Weisbach's coefficient (44), it is necessary first to obtain an approximate value for v, and for this either (41) or (42) may be used. An approximate value for c_f being then obtained from (44), it should be introduced into (43), from which the mean velocity, near enough for all practical purposes, may then be derived. Greater accuracy will, if required, be given by continued approximations, the new value for v being introduced into (44), and the process repeated.

Mr. Neville gives the following formula for pipes, recognising the principle above mentioned, and at the same time allowing the velocity to be computed at one operation:—

$$v = 140\sqrt{\text{R}\,s} - 11\sqrt[3]{\text{R}\,s} \quad . \quad . \quad . \quad . \quad (45)$$

in which $s = h \div l$, as before. It may be remarked that this formula fails when R $s = {\cdot}000000235$; but this does not affect its practical value.

M. Darcy, from a series of nearly two hundred experiments on pipes varying from half an inch to twenty inches in diameter, and with velocities of from about 1 inch to nearly 20 feet per second, derived a coefficient, which, reduced to English measures, is

$$c_f = {\cdot}005\left\{1 + \frac{1}{\text{dia. in inches}}\right\} \quad . \quad . \quad . \quad (46)$$

It has been found from observations on long pipe conduits of large diameter, that the formulæ in most general use—such as Du Buat's (39), Weisbach's (43 and 44), and others—give velocities considerably below those found to obtain in the cases referred to, and it has become the practice to make an addition—on an average, about 25 per cent.—to the velocities and discharges which these formulæ give. Darcy's expression for the coefficient (46) will, under certain conditions of velocity, give results nearer the truth; thus, with a 48-inch cast-iron pipe in the Loch Katrine Works, having an inclination of 1 in 1056, or 5 feet per mile, the actual velocity was found to be 3·46 feet per second, and Darcy's formula gives practically the same result, against about 3 feet for the common formulæ. Darcy's formula, however, inasmuch as it makes the coefficient depend only upon the hydraulic mean depth, does not accord with the received opinions on this subject.

Mr. Hawksley gives for pipes a formula which, reduced to measures in feet, gives

$$v = 48\sqrt{\frac{d\,\text{H}}{l + 54\,d}} \quad . \quad . \quad . \quad . \quad (47)$$

This formula includes an allowance for the resistance at the orifice of entry, and is therefore applicable approximately, without modification, to short pipes. In all the formulæ for pipes and channels before given, h is the loss of head due to the friction in the pipe; and in long straight pipes this is the only loss of head that need be regarded. But in short pipes the loss of head from other causes is too large a percentage of the whole to be disregarded; so that before applying equations (39 to 46) to short pipes, we must deduct from h the several other losses of head. Thus there is the head due to the velocity in the pipe—

$$h = \frac{v^2}{2g}$$

and then there is the head due to the resistance at the orifice of entry—

$$h = c_r \frac{v^2}{2g}$$

c_r being the ratio which this head has to that due to the velocity in the pipe. These together, or

$$h = (1 + c_r)\frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

may be shown to be the same as

$$h = \frac{1}{c_d^2} \times \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

in which c_d may be either of the coefficients given for the cases represented in figs. 33, 34, and 35. The loss of head due to bends and other resistances, if any, should also be deducted from h in the several formulæ given for the velocity before applying them to cases of short pipes, and indeed when applying them to long pipes if these resistances are such as together to demand a large proportionate loss of head.

From (43) we shall have, for short straight pipes—

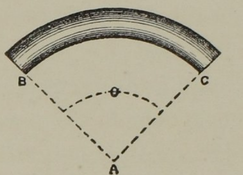
$$v = \sqrt{\frac{2gh}{\frac{1}{c_d^2} + c_f \frac{l}{R}}} \quad . \quad . \quad . \quad . \quad . \quad (50)$$

in which $1 \div c_d^2$ may be taken as ·664, ·511, and ·95 for orifices of entry corresponding to figs. 33, 34, and 35 respectively.

For the resistance due to bends and curves, the following are Weisbach's formulæ for the coefficients for circular tubes:—

$$c_b = \frac{\theta}{180°} \times \left\{ ·131 + 1·847 \left(\frac{d}{2r}\right)^{\frac{7}{2}} \right\} \quad . \quad . \quad . \quad (51)$$

Fig 44.



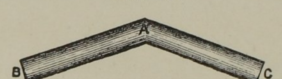
and for rectangular tubes

$$c_b = \frac{\theta}{180°} \times \left\{ ·124 + 3·104 \left(\frac{d}{2r}\right)^{\frac{7}{2}} \right\} \quad . \quad . \quad . \quad (52)$$

in which r is the radius of curvature of the pipe at the bend; θ, the angle B A C (fig. 44), through which it is bent, and d the diameter, all in feet.

For angular bends or elbows in pipes, the coefficient of friction is given as

Fig. 45.



$$c_a = ·946 \sin.^2 \frac{\theta}{2} + 2·05 \sin.^4 \frac{\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad (53)$$

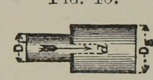
in which θ is the angle B A C (fig. 45) made by the two parts of the pipe.

For the friction of diaphragms, and at sudden contractions and enlargements, let A_1 and A_2 (fig. 46) be the sectional areas of the channel in the two parts respectively, between which there is a diaphragm reducing the area to a.

Professor Rankine gives the following formula:—

$$c_k = (r - 1)^2 \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Fig. 46.



in which

$$v = \frac{A_2}{a}\sqrt{2·618 - 1·618 \frac{a^2}{A_1^2}} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

In the above cases the loss of head due to the co-efficient c_b, c_a, or c_k, will be $H_b = c_b V^2 \div 2g$; $h_a = c_a V^2 \div 2g$; and $H_k = c_k V^2 \div 2g$. We have therefore for the total loss of head from all causes,

$$h = \left(\frac{1}{c_d} + c_f \frac{l}{R} + c_b + c_a + c_k\right)\frac{v^2}{2g} \quad . \quad . \quad . \quad (56)$$

and therefore

$$V = \sqrt{\frac{2gH}{\frac{1}{c_d^2} + c_f \frac{l}{R} + c_b + c_a + c_k}} \quad . \quad . \quad (57)$$

in which c_e is the co-efficient for the orifice of entry (figs. 33, 34, 35), c_f that of the friction in the pipe, and c_b, c_a, and c_k, the co-efficients for bends, enlargements, &c. as first given. In most cases of practice all the co-efficients, except c_f may be disregarded, as their values will generally be comparatively inconsiderable.

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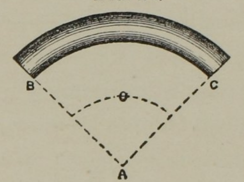

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$$c_b = \frac{\theta}{180^\circ} \times \left\{ ·124 + 3·104 \left(\frac{d}{2r}\right)^{\frac{7}{2}} \right\} \quad . \quad . \quad . \quad (52)$$

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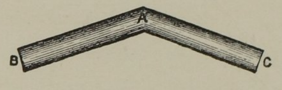

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For the friction of diaphragms, and at sudden contractions and enlargements, let A_1 and A_2 (fig. 46) be the sectional areas of the channel in the two parts respectively, between which there is a diaphragm reducing the area to a.

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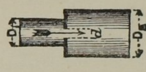

$$c_k = (r - 1)^2 \quad . \quad . \quad . \quad . \quad . \quad (54)$$

in which

$$v = \frac{A_2}{a} \sqrt{2·618 - 1·618 \frac{a^2}{A_1^2}} \quad . \quad . \quad . \quad . \quad . \quad (55)$$

In the above cases the loss of head due to the co-efficient C_b, C_a, or C_k, will be $H_b = C_b V^2 \div 2g$; $h_a = C_a V^2 \div 2g$; and $H_k = C_k V^2 \div 2g$. We have therefore for the total loss of head from all causes,

$$h = \left(\frac{1}{C_d} + C_f \frac{l}{R} + C_b + C_a + C_k\right) \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad (56)$$

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$$V = \sqrt{\frac{2gH}{\frac{1}{C_d^2} + C_f \frac{l}{R} + C_b + C_a + C_k}} \quad . \quad . \quad (57)$$

in which C_e is the co-efficient for the orifice of entry (figs. 33, 34, 35), C_f that of the friction in the pipe, and C_b, C_a, and C_k, the co-efficients for bends, enlargements, &c. as first given. In most cases of practice all the co-efficients, except C_f may be disregarded, as their values will generally be comparatively inconsiderable.

The formulæ that have been given are mostly for finding the mean velocity, when the loss of head, or virtual fall, is known ; and the discharge may be computed by multiplying the mean velocity into the sectional area of the stream.

The loss of head due to several causes is given by 56, or by transposition and reduction from any of the formulæ for notches, weirs, pipes, or channels in which it is involved. A well-known and very useful table of the loss of head due to friction in pipes running full has been calculated by Messrs. Thomson and Fuller of Belfast, and will be found in the 'Engineer's, Architect's, and Contractor's Pocket-Book.'*

When the discharge and fall are given, to ascertain therefrom the dimensions of a required channel, it is necessary first to assume the dimensions of a channel of exactly similar form, and compute the discharge from it. We have seen the mean velocity to vary nearly as $\sqrt{R\,s}$; in channels of similar sections R will vary with the linear dimensions λ, so that when s is constant the mean velocity will vary as $\sqrt{\lambda}$. The discharge will depend on the mean velocity and the section of the channel; in similar channels the sections will be as the squares of the linear dimensions (λ^2), so that the discharge will vary as $\lambda^2 \sqrt{\lambda}, = \sqrt{\lambda^5}$. Therefore the square root of the fifth power of the linear dimensions of the required channel is to that of the linear dimensions of the assumed channel as the required discharge is to that from the assumed channel or

$$\sqrt{\lambda^5} : \sqrt{\lambda_1^5} :: D : D_1, \quad . \quad . \quad (58)$$

With the assistance of Neville's table (in Appendix), the required dimensions of the new channel may be readily ascertained.