$$r^2 = x_P^2 + y_P^2 + z_P^2 \quad , \tag{7–116}$$
$$l^2 = (x - x_P)^2 + (y - y_P)^2 + (z - z_P)^2 \quad , \tag{7–117}$$
$$l'^2 = (x - x'_P)^2 + (y - y'_P)^2 + (z - z'_P)^2 \quad . \tag{7–118}$$

It is straightforward though somewhat cumbersome to compute

$$\Delta^2 H = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2}\right) \tag{7–119}$$

and to find that it is zero and regular even at P, so that H is indeed a regular solution of the biharmonic equation $\Delta^2 H = 0$.

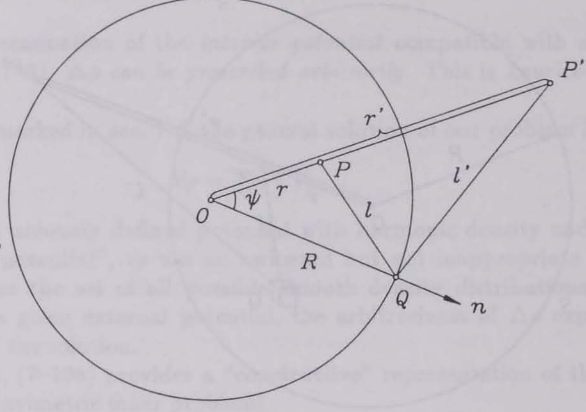

FIGURE 7.11: The point Q lies on the sphere S

There remains to verify the boundary conditions (7–96) on the sphere S. If Q lies on S, then (Fig. 7.11)

$$l^2 = r^2 + R^2 - 2rR\cos\psi \quad , \tag{7–120}$$
$$l'^2 = r'^2 + R^2 - 2r'R\cos\psi = \frac{R^4}{r^2} + R^2 - 2\frac{R^3}{r}\cos\psi$$
$$= \frac{R^2}{r^2}\, l^2 \quad , \tag{7–121}$$

so that by (7–112),

$$l_1 = \frac{r}{R}\, l' = \frac{r}{R}\frac{R}{r}\, l = l \quad \text{on} \quad S \quad . \tag{7–122}$$

Hence (7–113) gives

$$H = l \quad \text{on} \quad S \tag{7–123}$$

which is our first boundary condition.