7.7.4 Green's Function for the Sphere

It is easy to give Green's function G (7–99) if the boundary surface S is a sphere.

Submit the point P (to which V_P refers) to a *Kelvin transformation*, or *inversion in a sphere*. Cf. (Kellogg, 1929, pp. 231–223); for a different application see (Heiskanen and Moritz, 1967, pp. 143–144).

Fig. 7.10 shows the geometric situation. The inversion in the sphere transforms P into a point P' on the same radius as P, such that

$$rr' = R^2 \quad . \tag{7–111}$$

Define a function l_1 by

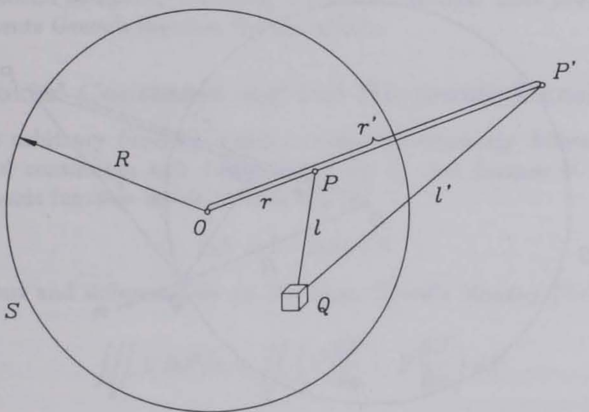

FIGURE 7.10: Kelvin transformation as an inversion in the sphere

$$l_1 = \frac{r}{R} l' \quad . \tag{7–112}$$

Then the auxiliary function H in (7–99) simply is

$$H = \frac{1}{2}\frac{l^2}{l_1} + \frac{1}{2} l_1 \quad , \tag{7–113}$$

so that Green's function (7–99) becomes

$$G = l - \frac{1}{2}\frac{l^2}{l_1} - \frac{1}{2} l_1 \tag{7–114}$$

(Marcolongo, 1901).

With coordinates for $P(x_P, y_P, z_P)$, $P'(x'_P, y'_P, z'_P)$ and $Q(x, y, z)$ we thus have

$$x'_P = \frac{R^2}{r^2} x_P \ , \quad y'_P = \frac{R^2}{r^2} y_P \ , \quad z'_P = \frac{R^2}{r^2} z_P \ , \tag{7–115}$$