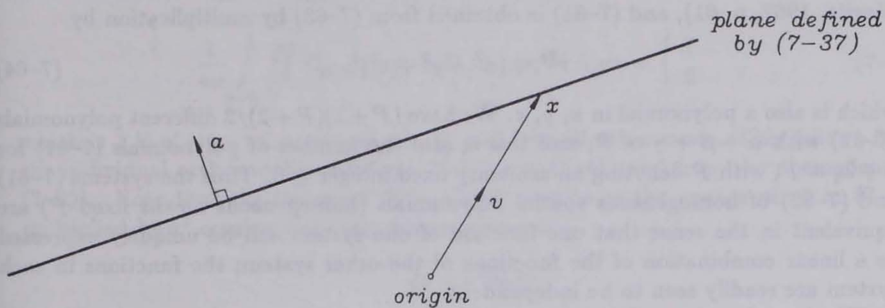

FIGURE 7.8: Representation of the solution x by an arbitrary vector v

Our set of density distributions comprises densities that are partly negative. As we have seen, this is not unphysical if V is regarded as a disturbing potential and ρ as a density anomaly, with respect to an underlying reference density model such as PREM. In fact, as mentioned before, this interpretation is of practical relevance if for V we take one of the global spherical harmonic expansions as discussed, e.g., in (Rapp, 1986).

The set of possible solutions can then be suitably restricted: by the obvious condition that the total density (reference density plus density anomaly) must be positive, and less trivially, by important information from seismology and other observational sources, as well as by theoretical considerations such as theories of mantle convection.

7.6.7 Application of Orthonormal Expansions

A very interesting special case of (7–51) has been treated by Dufour (1977). He considers representations of the density ρ of form

$$\rho(r,\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{q=0}^{Q}\beta_{nmq}r^{n+2q}Y_{nm}(\theta,\lambda) \quad . \tag{7–60}$$

A first glance shows that (7–60) is less general than (7–51) because the powers r^k for $k<n$ are missing, as well as the powers r^{n+1}, r^{n+3}, r^{n+5}, ... However, the base functions

$$r^{n+2q}Y_{nm}(\theta,\lambda)=r^nY_{nm}(\theta,\lambda)\cdot r^{2q} \tag{7–61}$$

are easily seen to be polynomials in the Cartesian coordinates x, y, z of form

$$x^\alpha\, y^\beta\, z^\gamma \quad , \tag{7–62}$$

α, β, γ being integers ≥ 0.

In fact, the solid harmonics (1–35a), or

$$r^nY_n(\theta,\lambda) \quad , \tag{7–63}$$