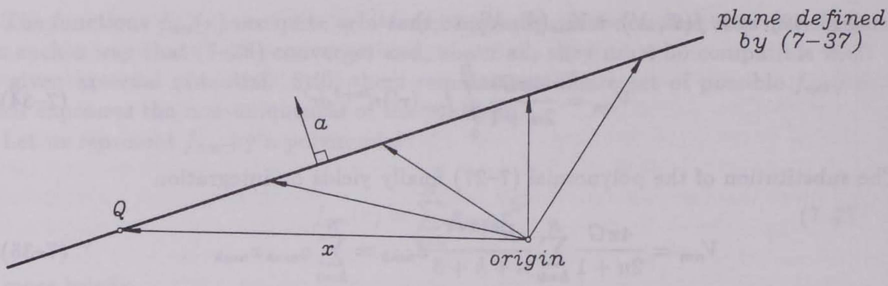

FIGURE 7.5: Possible choices of the vector x

The form (7–38) is motivated by the theory of generalized matrix inverses: if

$$Ax = b \tag{7–39}$$

is an underdetermined system of equations, the solution is formally given by

$$x = A^{-1}b \quad , \tag{7–40}$$

where the generalized inverse has the form (T denotes the transpose)

$$A^{-1} = CA^T(ACA^T)^{-1} \quad , \tag{7–41}$$

with any positive-definite symmetric square matrix C of appropriate dimension (cf. Bjerhammar, 1973, p. 110; Moritz, 1980, p. 164). Clearly (7–37) and (7–38) are special cases of (7–39) and (7–40) with (7–41).

The solution (7–38) satisfies the minimum condition

$$x^T P x = \text{minimum} \quad , \tag{7–42}$$

where $P = C^{-1}$. This means that x represents the "shortest" distance of the plane (7–37) from the origin, but of course in a non-orthogonal coordinate system whose metric tensor is P. That any point in the plane can be reached by a suitable choice of P can be seen in the following way (Krarup, 1972).

As we have mentioned, eq. (7–37) defines an N-dimensional hyperplane in our $(N+1)$-dimensional space (Fig. 7.5). Choose, for the first N base vectors, any set of N mutually orthogonal unit vectors (in the Euclidean sense) spanning the hyperplane. For the remaining $(N+1)$st base vector simply take the vector x leading from the origin to the desired point Q in the plane (Fig. 7.5). It is "orthogonal" to the hyperplane in the sense of the metric tensor P (though not in the Euclidean sense!) by the very condition (7–42), and its length is arbitrarily taken as unity.

Now we have found a set of $N+1$ linearly independent non-orthogonal vectors, and we must determine the metric tensor P for which they constitute an "orthonormal" set of base vectors. Let A now be the $(N+1)\times(N+1)$ matrix having as column vectors the