## 7.5 ANALYTICAL CONTINUATION

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found in (Moritz, 1980, secs. 6 to 8). What follows is heuristic "small talk" about a fascinating and both theoretically and practically important subject.

The practical relevance is known from geophysical prospecting. If the analytical continuation of V (or of gravity anomalies  $\Delta g$ ) into the interior of the earth is found to have a point-like (say) singularity at some interior point, then one may look for an anomalous mass there: the point-like singularity may be due to a spherical body which may represent an ore deposit (positive density anomaly) or a salt dome (negative density anomaly). The relation between the inverse problem and analytical continuation was also pointed out by Marussi (1982).

Are all singularities point-like? By no means: it is even impossible to classify all possible singularities, and they may have any degree of mathematical complexity (Schulze and Wildenhain, 1977, p. 121). Let us mention only a few of them (Fig. 7.4). Besides a simple pole (mass point) A we may have a dipole B (two points of oppositely



FIGURE 7.4: Possible types of singularities of the analytical continuation

equal mass "infinitely close" to each other), a quadrupole C or any higher multipole, a piece of line D or a "dipole line" E, a surface piece F, a piece of a "double layer surface" G which is some kind of "dipole surface", (cf. Heiskanen and Moritz, 1967, pp. 7 to 8), and so on to arbitrary complexity.

All these singularities have one property in common: they are not only of zero measure but also of zero capacity. Measure is a mathematical precision of the notion of volume, and capacity is not only a physical notion familiar from electrostatics but also a mathematical concept fundamental in modern potential theory. Also, if the masses are concentrated on sets of capacity zero, the energy becomes infinite (for  $V_S$ , the energy was minimum!) (cf. Schulze and Wildenhain, 1977, p. 122). On sets of capacity zero, V also becomes infinite.

A closed surface S is not a set of minimal support in the present sense: the potential does not become infinite on it and S, though being a set of zero measure, is not a set of zero capacity. An *open* surface, however, is a singularity in the present