

nic function  $V$  outside  $S$  and continue it into the interior of  $S$  in such a way that  $V$  (including its continuation) is continuous and continuously differentiable throughout  $\mathbb{R}^3$  and that  $\Delta V$  is piecewise continuous inside  $S$ . This is illustrated, for one dimension, in Fig. 7.3.

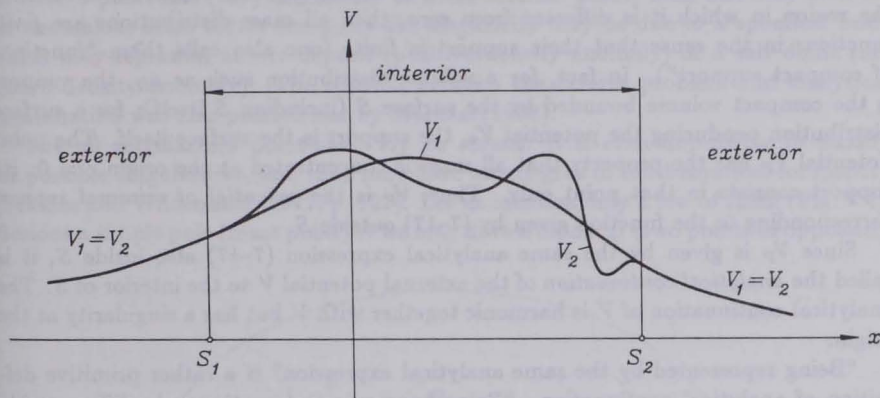


FIGURE 7.3: Two possible functions  $V$  in one dimension

There is no doubt, however, that although we continue the external  $V$  in the way described ( $V_1$  or  $V_2$  in Fig. 7.3), not only the external potential, but also the mass  $M$  and other "Stokes constants" (e.g., the spherical-harmonic coefficients) remain the same, because they are fully determined by the external potential (outside any sphere enclosing the body, cf. sec. 7.7.5). This is also expressed by the fact, mentioned in sec. 7.2, that the total mass of any zero-potential density is zero.

This is easy to understand in principle, but it is difficult to really compute or "construct" an smooth continuation in the way described. Therefore we have put the word "general" in the title of this section between quotation marks.

A constructive method can be obtained by superimposing the uniquely defined harmonic density  $\rho_H$  and any zero-potential density  $\rho_0$  according to Lauricella's integral (7-9); there follows the theorem, also due to Lauricella: *the Laplacian of the density of a body producing a given external potential can be arbitrarily assigned*, cf. sec. 7.7.3.

A general solution without smoothness assumptions can probably be found by the methods of modern potential theory, as a linear combination of "extremal measures", cf. (Anger, 1981, 1990), which make essential use of surface distributions  $V_S$  and point masses  $V_P$ . However, this approach is mathematically very difficult, and solutions have been found so far for the simplest cases only.

We shall, therefore, try in sec. 7.6 a rather general and entirely elementary approach. It is limited to the sphere, but this anyway is the most interesting case for global geodesy and geophysics.