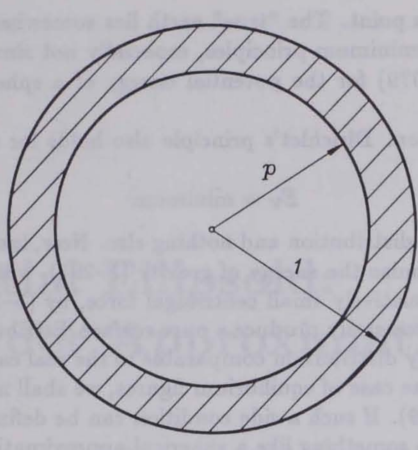

FIGURE 5.5: A spherical shell

for the potential energy of a *surface layer* on the sphere. For $p < 1$ we always have

$$E > E_{\text{min}} \quad . \tag{5–312}$$

This is not surprising after all: *Dirichlet's principle* (cf. Kellogg, 1929, p. 279) explicitly states that E *is minimized if the masses are concentrated on the boundary* and the interior is empty!

For the homogeneous sphere we have by (5–309)

$$\frac{E_{\text{hom}}}{E_{\text{min}}} = \frac{6}{5} = 1.2 \quad , \tag{5–313}$$

which certainly is > 1. For the actual earth we get approximately (we may use a Roche-type polynomial)

$$\frac{E_{\text{earth}}}{E_{\text{min}}} \doteq 1.3 \quad . \tag{5–314}$$

Further, if we let the core radius go to zero, always keeping the total mass constant and the mantle density zero, we get

$$E \to \infty \; ! \tag{5–315}$$

This is clear because, if the mass is concentrated at a point, we have

$$V = \frac{GM}{r} \tag{5–316}$$

and (5–305) becomes infinite (verify)!

This minimum and maximum potential energy (if we consider $E = \infty$ as some kind of maximum) correspond to physically (for the earth) meaningless cases: a surface