In view of (5-252) this gives

$$
\begin{equation*}
\zeta=-\frac{\Delta}{\sigma^{\prime}} . \tag{5-255}
\end{equation*}
$$

The flattening of the surfaces of constant density may then be expressed by

$$
\begin{equation*}
f=\frac{\left(r_{0}+\zeta_{a}\right)-\left(r_{0}+\zeta_{b}\right)}{r_{0}+\zeta_{a}} \doteq \frac{\zeta_{a}-\zeta_{b}}{r_{0}} \tag{5-256}
\end{equation*}
$$

where $\zeta_{a}$ and $\zeta_{b}$ are the values of $\zeta$ at the equator and at the poles, respectively. By (5-255) this becomes (with $r_{0}$ replaced by $r$ )

$$
\begin{equation*}
f=-\frac{\Delta_{a}-\Delta_{b}}{r \sigma^{\prime}}=\frac{\Delta_{a}-\Delta_{b}}{r^{2}\left(2 b_{0}+4 b_{2} r^{2}\right)} \tag{5-257}
\end{equation*}
$$

The singularity in ellipsoidal coordinates. Before studying (5-257) further, we must consider a strange singularity of the ellipsoidal coordinate system. The equatorial plane, coinciding with the $x y$-plane in Fig. 5.4 , is given by two equations: outside


FIGURE 5.4: The "focal disc singularity" in the ellipsoidal coordinate system; $\epsilon$ is an arbitrary small number (dimensionless if $b=1$ )
the "focal disc" obtained by rotating $O F_{2}$ around the $z$-axis and indicated by the segment $F_{1} F_{2}$ in Fig. 5.4, we have (e.g., for point $A$ ):

$$
\begin{equation*}
\theta=90^{\circ} \tag{5-258}
\end{equation*}
$$

