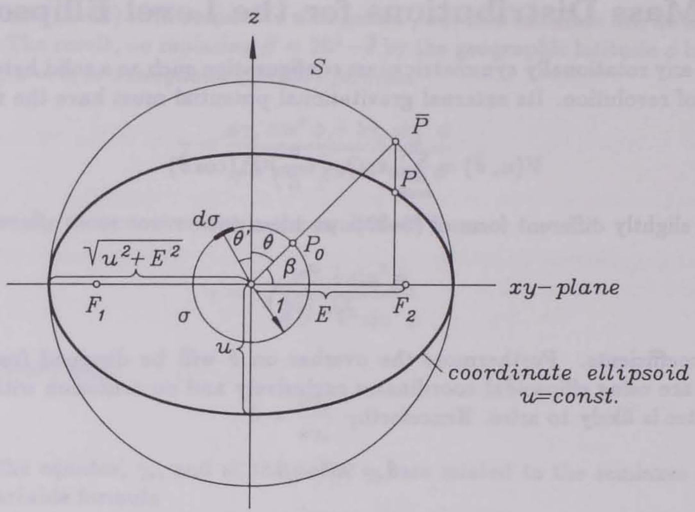

FIGURE 5.3: A coordinate ellipsoid u = const. and the auxiliary spheres S and σ

with

$$d\sigma = \sin\theta' d\theta' d\lambda' \tag{5–82}$$

denoting the element of solid angle as usual; more precisely, it is the surface element of the auxiliary unit sphere σ on which the point P_0 in Fig. 5.3 is situated. The primes indicate that dv refers to the integration point (u', θ', λ'). For $E \to 0$, eq. (5–80) reduces to the usual expression for the volume element in spherical coordinates.

At this point it is appropriate to use Fig. 5.3 to recall the geometric situation and make it completely clear. Take an arbitrary point P (u, β, λ) in space and pass the appropriate coordinate ellipsoid u = const. through it. Its semiaxes are u and $\sqrt{u^2+E^2}$. The auxiliary "affine" sphere S thus has the radius $\sqrt{u^2+E^2}$. For the reduced latitude β or its complement θ we have the familiar construction $P \to \bar{P}$; θ is the polar distance, not of P, but of the auxiliary point $\bar{P}$. As we have seen, we also need the concentric unit sphere σ; to P there corresponds the auxiliary point P_0 on σ.

Repeat the same construction for the point Q (u', θ', λ') which carries the volume element dv, but note that the coordinate ellipsoid u' = const. and the auxiliary sphere S will be different! The concentric unit sphere σ, however, remains of course the same. In this way, to Q there corresponds on σ the auxiliary point Q_0 which carries the surface element $d\sigma$. The coordinate ellipsoid u' = const. and the details of the construction $Q \to \bar{Q} \to Q_0$ are not shown in order not to overload the figure.

Orthogonality relations such as (1–41) will be used later; the corresponding inte-