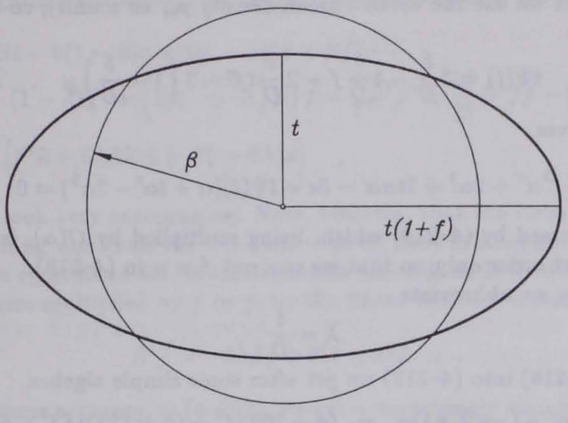

FIGURE 4.10: Polar radius t and mean radius β

is the geometric mean of all three axes (Fig. 4.10). (In a more familiar notation this is $R = \sqrt[3]{a^2 b}$, the sphere being defined as having the same volume as the ellipsoid.) In view of the smallness of f, (4–210) reduces in the linear approximation to

$$\beta = t\left(1 + \frac{2}{3} f\right) \quad , \tag{4–211}$$

$$t = \beta\left(1 - \frac{2}{3} f\right) \quad . \tag{4–212}$$

Hence,

$$W'(t) = \frac{dW}{dt} = \frac{dW}{d\beta}\frac{d\beta}{dt} = \frac{dW}{d\beta}\left(1 + \frac{2}{3} f + \frac{2}{3} t f'\right) \quad . \tag{4–213}$$

Using (4–209) with (4–211), this gives

$$W'(t) = -\frac{4\pi G}{3} D t\left(1 + \frac{2}{3} f\right)\left(1 + \frac{2}{3} f + \frac{2}{3} t f'\right) + \frac{2}{3}\omega^2 t \tag{4–214}$$

(since $\omega^2 = O(f)$, we have been able simply to replace β by t in the last term).

Introducing the dimensionless quantity (4–66), in the present units

$$\mu = \frac{3}{4\pi G}\frac{\omega^2}{D} \quad , \tag{4–215}$$

which is $O(f)$, we thus have to $O(f)$

$$g_P = -W'(t) = \frac{4\pi G}{3} D t\left(1 + \frac{4}{3} f + \frac{2}{3} t f' - \frac{2}{3}\mu\right) \quad . \tag{4–216}$$

Now

$$\frac{4\pi G\rho - 2\omega^2}{\frac{4\pi}{3} G t D} = \frac{3}{t}\frac{\delta}{D} - \frac{2}{t}\mu \tag{4–217}$$