

(for the sake of generality, we keep the notation  $\partial r / \partial \theta$  because later on  $r$  will depend on  $t$  as well).

In view of (4-154) we may write (4-152) and (4-153) as

$$\begin{aligned} -dx &= r \sin \theta \left( 1 - \frac{r_\theta}{r} \cot \theta \right) d\theta, \\ ds &= \sqrt{r^2 + r_\theta^2} d\theta, \end{aligned} \quad (4-156)$$

and substitute into (4-151) and then into (4-150). The result is

$$\frac{1}{R_2} = \frac{1}{\sqrt{r^2 + r_\theta^2}} \left( 1 - \frac{r_\theta}{r} \cot \theta \right). \quad (4-157)$$

Combining (4-148) and (4-157) we thus have for the mean curvature (1-20)

$$J = \frac{1}{2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{2\sqrt{r^2 + r_\theta^2}} \left( 2 - \frac{r_\theta}{r} \cot \theta + \frac{r_\theta^2 - r r_{\theta\theta}}{r^2 + r_\theta^2} \right). \quad (4-158)$$

Consider now Wavre's function (4-145),

$$N = \frac{dn}{dt},$$

using Fig. 4.8. Along the straight line  $OP'$  we obviously have  $\theta = \text{const.}$ , so that

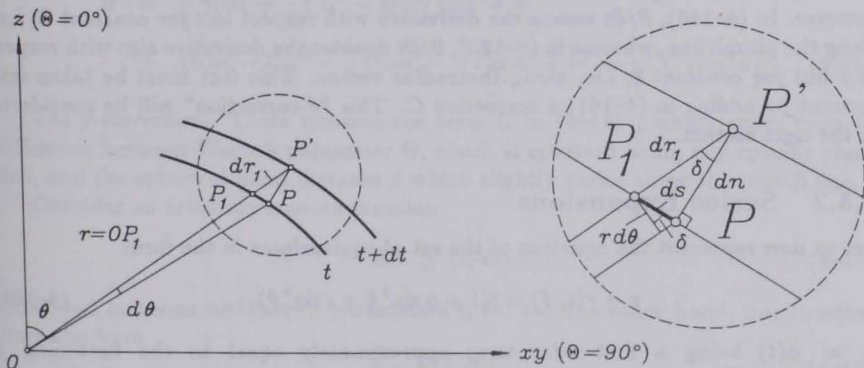


FIGURE 4.8: The distance between two neighboring equisurfaces

$$dr_1 = r_t dt = \frac{\partial r}{\partial t} dt,$$

which is the change of  $r$  because of  $t$  only. From the enlarged part of Fig. 4.8 we read

$$dn = dr_1 \cos \delta = r_t dt \cos \delta,$$