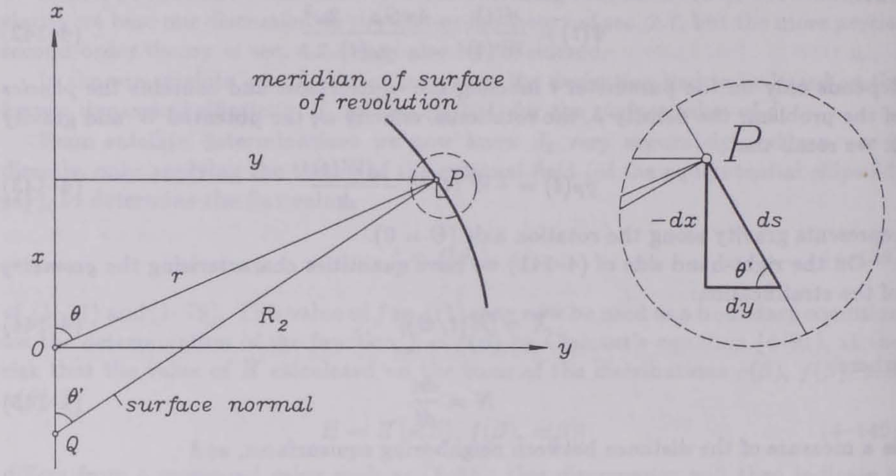

FIGURE 4.7: The normal radius of curvature

order to have $x = r\cos\theta$, $y = r\sin\theta$ as usual for plane polar coordinates). This holds not only for the ellipsoid, but also for an arbitrary surface of revolution; cf. sec. 1.4.

From Fig. 4.7 we read

$$y = r\sin\theta = R_2\sin\theta' \quad ,$$

whence

$$R_2 = r\frac{\sin\theta}{\sin\theta'} \quad . \tag{4–150}$$

The elementary triangle at P, shown in a magnified manner next to the main diagram (Fig. 4.7), gives

$$\sin\theta' = -\frac{dx}{ds} \quad . \tag{4–151}$$

Differentiating $x = r\cos\theta$ we have

$$dx = dr\cos\theta - r\sin\theta d\theta \quad . \tag{4–152}$$

Furthermore,

$$ds^2 = dr^2 + r^2 d\theta^2 \quad . \tag{4–153}$$

In both formulas we put

$$dr = r_\theta d\theta \tag{4–154}$$

by (4–149); in fact, by (4–147), r depends on θ only, so that here

$$r_\theta = \frac{\partial r}{\partial\theta} = \frac{dr}{d\theta} \tag{4–155}$$