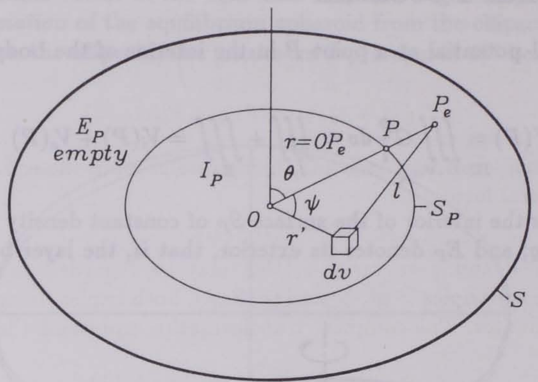

FIGURE 4.3: Illustrating the computation of V_i

The trick is to leave I_P but to calculate V first at a point P_e which lies on the radius vector of P but outside S_P in such a way that $r' < r$ is always satisfied (Fig. 4.3). Thus we compute

$$V_i(P_e) = G \iiint_{I_P} \frac{\rho}{l}\, dv = \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \cdot G \iiint_{I_P} \rho r'^n P_n(\cos\psi) dv \tag{4–9}$$

(the interchange of sum and integral offers no problem because of the absolute convergence of the integrand series). Since $V_i(P_e)$ is harmonic, the shell between S_P and S being disregarded for the time being, and because of rotational symmetry, (4–9) must necessarily have the form (1–37) with zonal harmonics only:

$$V_i(P_e) = \sum_{n=0}^{\infty} \frac{K_n}{r^{n+1}} P_n(\cos\theta)$$

or

$$V_i(P_e) = \frac{K_0(q)}{r} + \frac{K_2(q)}{r^3} P_2(\cos\theta) + \frac{K_4(q)}{r^5} P_4(\cos\theta) \quad , \tag{4–10}$$

neglecting higher-order terms. Here r, θ, λ are the spherical coordinates of P_e as usual; because of rotational symmetry there is no explicit dependence on longitude λ (no tesseral terms); and there are only even-degree zonal terms because of symmetry with respect to the equatorial plane. The coefficients K_n evidently depend on S_P and hence on its label q.

4.1.2 Change of Variable

The equation of any surface of constant density may be written as