(Fig. 4.1). The radius vector of the equi-axial ellipsoid is given by (1–73), so that κ represents the *deviation* of the equilibrium spheroid from the ellipsoid. The spheroid is found to lie below the ellipsoid, the maximum depression at 45° being about 4.3 meters.

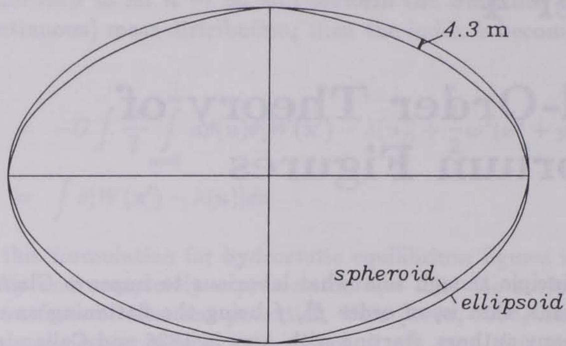

FIGURE 4.1: Ellipsoid and equilibrium spheroid

An analogous representation holds for all equisurfaces. Thus we may represent f and κ as functions of q, the mean radius of the equisurface as usual. For $f = f(q)$ we have *Clairaut's equation* (2–114) supplemented by second-order terms:

$$q^2 \frac{d^2 f}{dq^2} + 6\frac{\rho}{D}\, q\, \frac{df}{dq} + \left(-6 + 6\frac{\rho}{D}\right) f + O_1(f^2) = 0 \quad , \qquad (4\text{–}4)$$

and for $\kappa = \kappa(q)$ we get a quite similar differential equation, *Darwin's equation*:

$$q^2 \frac{d^2 \kappa}{dq^2} + 6\frac{\rho}{D}\, q\, \frac{d\kappa}{dq} + \left(-20 + 6\frac{\rho}{D}\right) \kappa = O_2(f^2) \quad , \qquad (4\text{–}5)$$

where $O_1(f^2)$ and $O_2(f^2)$ denote terms of second order to be computed in this chapter.

A theoretical difficulty lies in the use of Legendre's series (1–53) for $1/l$, which may not be convergent, as we have already noticed in sec. 2.4. This has lead Wavre (1932) to devise an ingenious method (*procédé uniforme*) that works with convergent series only. This complicated procedure is not really necessary, as we shall see below. Our approach, based on recent progress in understanding analytical continuation (Moritz, 1980, secs. 6 and 7) is extremely simple and may be new.

A thorough check is provided by Wavre's geometric theory (sec. 3.2, completely different from his "procédé uniforme"!) to the present problem. In this way the fundamental equations (4–4) and (4–5) can be obtained without using any spherical harmonics.