3.2.1 Stratification of Equisurfaces

Let $S(t)$ denote the set of equisurfaces (surfaces of constant density and of constant potential), as a function of a parameter t (there is no danger of confusing it with time!). The parameter t thus "labels" the individual equisurfaces and could, in principle, be selected in many ways. Formerly, we have labeled the equisurface by its *mean radius* q, but in Wavre's theory it is more convenient instead to take the parameter t as the *semiminor axis* of the spheroidal equisurface under consideration. (This is well known since the ellipsoidal coordinate u also has this character, cf. sec. 5.1. For the limiting ("free") surface S we take $t = 1$, so that $S = S(1)$.

We again assume rotational symmetry about the z-axis, knowing already that the stratification must also be symmetric with respect to the equatorial plane (invariance for $z \to -z$). Thus we have no dependence on longitude λ; as latitudinal coordinate we take a parameter Θ that labels the plumb lines as indicated in Fig. 3.2.

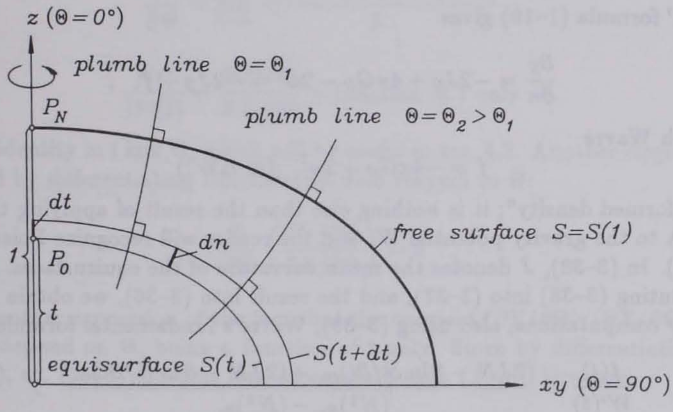

FIGURE 3.2: The geometry of stratification

Since the equisurfaces t = const. are not parallel, their infinitesimal distance dn differs, in general, from dt. We put

$$\frac{dn}{dt} = N(t, \Theta) \quad , \tag{3–32}$$

where the function N is unknown *a priori*. Note that N is always positive (from geometry), dimensionless (by our choice of units) and equals 1 on the rotation axis $\Theta = 0$. (The symbol N has also been used for the geoidal height and the ellipsoidal normal radius of curvature!)

Since, by definition, the potential W depends on t only, we have for gravity

$$g = -\frac{\partial W}{\partial n} = -\frac{dW}{dt} \bigg/ \frac{dn}{dt} = -\frac{1}{N}\frac{dW}{dt} \quad . \tag{3–33}$$