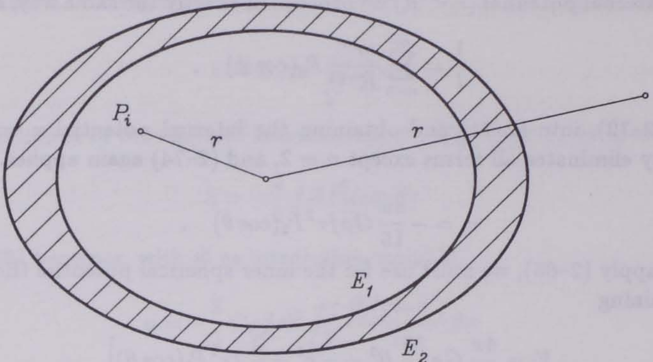

FIGURE 2.4: Ellipsoidal shell

remove the inner solid ellipsoid bounded by E_1; the result obviously is the ellipsoidal shell of Fig. 2.4. Thus the potential of the shell is the difference of the potentials of two homogeneous ellipsoids! This is why we have needed the theory of the homogeneous ellipsoid (sec. 2.3).

Thus, for the external potential, at some point P_e (Fig. 2.4), we have

$$dV_e = V_{shell} = V_e(E_2) - V_e(E_1) = \frac{dV_e}{dq}\,dq \quad , \tag{2–84}$$

since everything depends on q. Now V_e is given by (2–76), with R replaced by q, so that

$$dV_e = \frac{4\pi}{3} G\rho \frac{d}{dq}\left[\frac{q^3}{r} - \frac{2}{5}\frac{q^5}{r^3} f P_2(\cos\theta)\right] dq \quad . \tag{2–85}$$

Note that f, through (2–80), depends on q, but r, being the radius vector of P_e, is to be considered constant with respect to the differentiation. Thus, with

$$\frac{d}{dq}(q^5 f)dq = d(q^5 f) \quad ,$$

(2–85) becomes

$$dV_e = 4\pi G\rho\left[\frac{q^2}{r}\,dq - \frac{2}{15}\frac{P_2(\cos\theta)}{r^3}\,d(q^5 f)\right] \tag{2–86}$$

as the external potential of our thin ellipsoidal shell.

Similarly (2–79) gives

$$dV_i = \frac{4\pi}{3} G\rho \frac{d}{dq}\left[\frac{3}{2}q^2 - \frac{1}{2}r^2 - \frac{2}{5} f r^2 P_2(\cos\theta)\right] dq$$

or

$$dV_i = 4\pi G\rho\left[q dq - \frac{2}{15} r^2 P_2(\cos\theta) df\right] \tag{2–87}$$