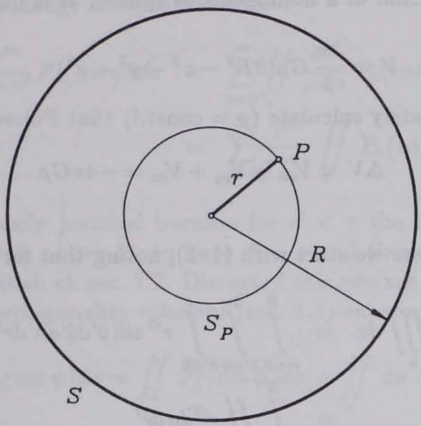

FIGURE 2.2: Computation point P inside the sphere

where

$$M_P = \frac{4\pi}{3} r^3 \rho \tag{2–38}$$

denotes the mass of the part enclosed by S_P; ρ is the constant density. ("Core" is meant in a figurative sense and has, of course, nothing to do with the actual earth's core!)

Thus

$$g_P = g_1 + g_2 = g_2 = \frac{4\pi G}{3} r\rho \quad , \tag{2–39}$$

by (2–36), (2–37), and (2–38).

In order to find the potential V, we integrate (2–33) in our case,

$$\frac{dV}{dr} = -g = -\frac{4\pi G}{3} \rho r \quad , \tag{2–40}$$

which gives

$$V = -\frac{2\pi G}{3} \rho r^2 + C_1 \quad . \tag{2–41}$$

The integration constant C_1 is determined such that, at the outer surface $r = R$, (2–41) must yield the same result as (2–31):

$$V(R) = -\frac{2\pi G}{3} \rho R^2 + C_1 = \frac{GM}{R} = \frac{4\pi G}{3} R^3 \rho \frac{1}{R} \quad , \tag{2–42}$$

whence $C_1 = 2\pi G \rho R^2$, and

$$V_i = V(r) = 2\pi G \rho \left(R^2 - \frac{1}{3} r^2 \right) \tag{2–43}$$