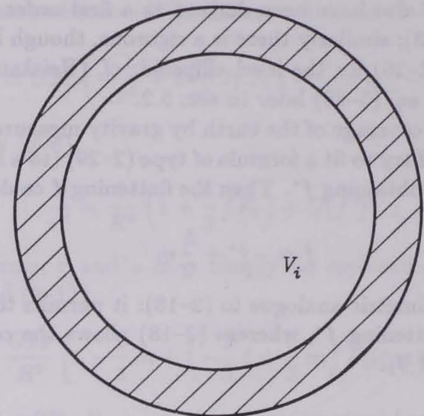

FIGURE 2.1: A spherical shell

The *internal* potential is more complicated. First we consider the potential in the interior of a hollow spherical shell (Fig. 2.1). It is easily seen to be constant:

$$V_i = C = \text{const.} \tag{2–34}$$

In fact, the potential V_i is a harmonic function, satisfying Laplace's equation $\Delta V = 0$, in the interior of the shell, and must therefore admit a spherical-harmonic expansion

$$V_i = \sum_{n=0}^{\infty} r^n Y_n(\theta, \lambda) = Y_0 + \sum_{n=1}^{\infty} r^n Y_n(\theta, \lambda) \quad , \tag{2–35}$$

analogous to (2–32), but with the outer harmonics (1–35b) replaced by the inner harmonics (1–35a). Repeating the previous argument considering spherical symmetry, only the term Y_0 can survive in (2–35), and setting $Y_0 = C$ proves (2–34).

It is clear that the structure of the shell has no influence as long as it is spherically symmetric: it may be homogeneous or layered (stratified).

Since the potential is identically constant inside the shell, the force vanishes there:

$$\mathbf{g} = \operatorname{grad} V_i = 0 \tag{2–36}$$

inside the shell.

Homogeneous sphere. The gravity (gravitational attraction) of a homogeneous sphere at an internal point P is found by a simple but very useful trick (this trick is one reason for treating the physically rather uninteresting homogeneous case here). Consider the sphere S_P passing through P (Fig. 2.2). Then *gravity g_1 due to the shell between S and S_P, is zero* by (2–36). The gravity g_2 due to the "core" bounded by S_P is then given by the "external" formula (2–33):

$$g_2 = \frac{GM_P}{r^2} \quad , \tag{2–37}$$