The comparison of (8–95) and (8–96) gives (8–94).

Substituting (8–93) into (8–91) we find

$$B_C = 2\pi G\rho h_m \quad , \tag{8–97}$$

so that by (8–92),

$$A_C = 2\pi G\rho h_m + \frac{1}{2R} V_C \quad . \tag{8–98}$$

According to our model, assuming crust and mantle to be homogeneous, the gravity anomaly Δg is caused only by the combined effect of topography and compensation:

$$\Delta g = A - A_C \quad , \tag{8–99}$$

where A is the attraction of topography. Substituting (8–79) and (8–98) we thus have

$$\Delta g = 2\pi G\rho(h_P - h_m) - C + \frac{1}{2R}(V - V_C) \quad . \tag{8–100}$$

The last term, which is very small (of order 1 mgal) because V and V_C are almost equal, will be neglected, and there remains (on omitting the subscript P)

$$\Delta g = 2\pi G\rho(h - h_m) - C \quad . \tag{8–101}$$

This equation expresses the "free-air" gravity anomaly Δg (see below) corresponding to our model. We clearly see the linear correlation with elevation, and we see at once that *the linear correlation should be even more pronounced if the terrain correction C is added to Δg* because

$$\Delta g + C = 2\pi G\rho(h - h_m) \quad . \tag{8–102}$$

The Bouguer anomaly is generally defined as

$$\Delta g_B = \Delta g - 2\pi G\rho h + C \quad , \tag{8–103}$$

by (8–36) and (8–38) with $g - \gamma = \Delta g$; thus in our model (homogeneous crust and mantle!) we simply have

$$\Delta g_B = -2\pi G\rho h_m \quad . \tag{8–104}$$

The isostatic anomaly is obviously zero for the model:

$$\Delta g_I = 0 \quad . \tag{8–105}$$

8.2.5 Conclusions Regarding Gravity Anomalies

Thus our model gives a reasonably realistic interpretation of the following empirical facts (Heiskanen and Moritz, 1967, pp. 281–285):

1. The free-air anomalies (see below) fluctuate around zero but are linearly correlated with elevation.

2. The Bouguer anomalies in mountain areas are systematically negative and increase in magnitude by

$$2\pi G\rho \doteq 100\,\text{mgals} \tag{8–106}$$

per km of mean elevation h_m.

These facts, which are well known from observation to hold quite generally and of which one is a consequence of the other, can be explained by isostatic compensation as we shall discuss now in more detail.

Correlation with elevation. The *free-air anomaly* is defined by

$$\Delta g = g_P + F - \gamma \quad ; \tag{8–107}$$

cf. sec. 8.1.5 (only the free-air reduction F is applied) and (Heiskanen and Moritz, 1967, pp. 146 and 293). Empirically, free-air anomalies are *linearly correlated with elevation*, that is, approximately they satisfy a linear relation

$$\Delta g = a + bh \quad , \tag{8–108}$$

where a and b are more or less constants.

On disregarding the terrain correction C, eq. (8–101) becomes

$$\Delta g = 2\pi G\rho(h - h_m) \quad . \tag{8–109}$$

The comparison with (8–108) shows that

$$b = 2\pi G\rho \tag{8–110}$$

and that

$$a = -2\pi G\rho h_m \tag{8–111}$$

essentially is nothing else than the Bouguer anomaly (8–104).

Linear correlation means that a linear functional relation is satisfied, not exactly but on the average. Fluctuations occur for three main reasons:

1. Density anomalies in the crust and the mantle and, possibly, in the core have been disregarded.

2. Isostatic equilibrium is not exact: local deviations from equilibrium occur. These are the main reasons.

3. The terrain correction C has been disregarded. This indicates that the "modified free-air anomaly" $\Delta g + C$ should exhibit this correlation even better than Δg itself, according to (8–102).

It is also clear that the parameter b in (8–108) is, for constant density ρ, really a constant; cf. (8–110). The parameter a, however, is essentially the Bouguer anomaly, by (8–104) and (8–111), and is therefore at best a "regional constant", that is, it varies, but much more slowly than Δg.

Thus an expression such as (8–111) explains the facts we have mentioned at the beginning of this section: the Bouguer anomalies in mountain areas are essentially negative and approximately proportional to a mean elevation h_m in such a way that a change in h_m of 1000 meters corresponds to a change in the Bouguer anomaly of about 100 mgals; for an application see (Heiskanen and Moritz, 1967, p. 328).

On the other hand, a look on (8–109) explains why the free-air anomaly exhibits no systematic tendency to either positive or negative (such a tendency is removed by h_m being subtracted from h) although it is approximately a linear function of h.

Our model corresponds to complete isostatic compensation but the manner of compensation is quite unrealistic: we have assumed the compensating masses forming a surface layer situated at a constant depth T below sea level. The purpose of this model, however, was only to furnish the simplest mathematical description of the surface gravity field, and as such it is quite adequate. If a more realistic model, for instance of Airy, Pratt, or even Vening Meinesz type, is considered, then the definition (8–93) of h_m will be replaced by a more complicated one, but this is rather the only change. The relevant formulas, such as (8–101), will still be valid, with h_m being still some sort of a mean elevation, but with different weighting. The only essential prerequisite is *that the compensating masses produce approximately the same potential and the same attraction at the corresponding points P and P_0* (Fig. 8.13). If the major part of the compensating masses is sufficiently deep, this will certainly be true. The validity of our results is thus far wider than the rather special model would indicate.

The reason may be summarized as: equation (8–101) is valid in any isostatic model if h_m is suitably defined; and the succeeding argument is based only on (8–101) and on the prerequisite just mentioned.

The dipole character of isostasy is particularly evident from equations such as (8–109).

A remark on the Bouguer reduction. As we have seen (eq. (8–71)), the attraction of a spherical Bouguer plate is $4\pi G\rho h$ and not $2\pi G\rho h$. Thus, strictly speaking, it is wrong to consider the term (8–39) as the attraction of an "infinite Bouguer plate". In fact, eq. (8–84) indicates that $2\pi G\rho h$ is in reality related to the discontinuity $2\pi G\kappa$ of the attraction of an *arbitrary* surface layer rather than to the attraction of a *plane* plate.

Thus, so to speak, the term $2\pi G\rho h$ represents the "local" effect of the Bouguer plate, and this is exactly what we want. Standing at a point of elevation h_P, it would be grossly unrealistic to assume that the actual earth's surface can be approximated by a "spherical Bouguer plate" extending with constant elevation h_P all around the earth! The major part of the earth is covered by the oceans for which $h = 0$, so that we can operate with a Bouguer plate only locally, and this local effect is $2\pi G\rho h_P$ even for the sphere. This justifies the conventional way of computing Bouguer anomalies. A further justification is provided by the fact that Bouguer anomalies usually are not an end in themselves, but that they are, e.g., a means for computing isostatic anomalies, for which

$$A - A_C \doteq B - B_C \tag{8–112}$$

by (8–65) and (8–92), since $V \doteq V_C$ and hence $(V - V_C)/2R$ is very nearly zero; and B is associated with the factor 2π and not 4π, as (8–76) shows.

8.3 Inverse Problems in Isostasy

Consider Pratt's model (sec. 8.1.1). The compensation takes place along vertical columns; this is *local compensation.* There is a *variable* density contrast $\Delta\rho$ given in terms of elevation h by (8–3). The corresponding isostatic gravity anomaly Δg_I (8–37) will in general not be zero, partly because of imperfections in the model. The inverse problem consists in trying to make

$$\Delta g_I \equiv 0 \tag{8–113}$$

by *determining a suitable distribution* $\Delta\rho(z)$ *of the density anomaly* in each vertical column.

On the other hand, consider isostatic models of Airy and Vening Meinesz type. Here the density contrast $\Delta\rho$ is *constant*, but the Moho depth T is variable, depending on the topography locally (Airy) or regionally (Vening Meinesz) in a prescribed way (now T and T_0 are again used in the sense of sec. 8.1!). Here the inverse problem would consist in making Δg_I zero by *determining a suitable variable Moho depth* T for a prescribed constant density contrast $\Delta\rho$, which need not be $0.6\,\mathrm{g/cm^3}$ but can be any given value between 0 and $0.7\,\mathrm{g/cm^3}$ (say).

Rather than making Δg_I zero, we may also prescribe the Bouguer anomaly field. This amounts to the same since by (8–37), $\Delta g_I = 0$ implies

$$A_C = -\Delta g_B \quad . \tag{8–114}$$

So the problem is in fact: given A_C, to determine the compensating masses that produce it. In the inverse Pratt problem this is done by seeking an appropriate density contrast $\Delta\rho$, in the inverse Vening Meinesz problem this is achieved by suitably selecting the Moho depth T. Thus we have genuine inverse problems (with given constraints) in the sense of Chapter 7 (cf. also Barzaghi and Sansò, 1986).

8.3.1 The Inverse Pratt Problem

The basic paper is (Dorman and Lewis, 1970). Consider a column defined by fixing the spherical coordinates (θ, λ); the column extends from the earth's surface radially to the earth's center (theoretically: this corresponds to $D = R$ in sec. 8.1.1). In each column $\Delta\rho$ is a function of the radius vector r (or of depth), which accounts for the functional dependence

$$\Delta\rho = \Delta\rho(r, \theta, \lambda) \quad . \tag{8–115}$$

One assumes $\Delta\rho$ to be linearly related to the topography (height h) by a *"convolution"*

$$\Delta\rho(r', \theta', \lambda') = \iint_\sigma h(\theta'', \lambda'') K(r', \psi') d\sigma \quad , \tag{8–116}$$