This follows at once from the fact that A and B differ only by $V/2R$ and that as a linear approximation $V = V_S$. Thus as a linear approximation, *the potentials of the original and of the condensed topography are equal, but the attractions differ by the terrain correction.*

8.2.4 Effect of Compensation

We shall now consider a crustal density model by which the linear correlation of the free-air gravity anomalies with elevation can be explained and which at the same time is simple. Obviously, isostatic compensation must in some way be taken into account.

If we look at the Airy–Heiskanen isostatic model, we see that the compensation is given by the mountain roots which are some 30 km below sea level. The effect of this type of compensation on the earth's surface is thus quite similar as that of a surface layer of density $(-\rho h)$ on the sphere of radius $R - T$, where T may be identified with the normal thickness of the earth's crust of about 30 km, formerly denoted by T_0; see Fig. 8.13 and Fig. 8.10 above. The idea of regarding, for mathematical simplicity,

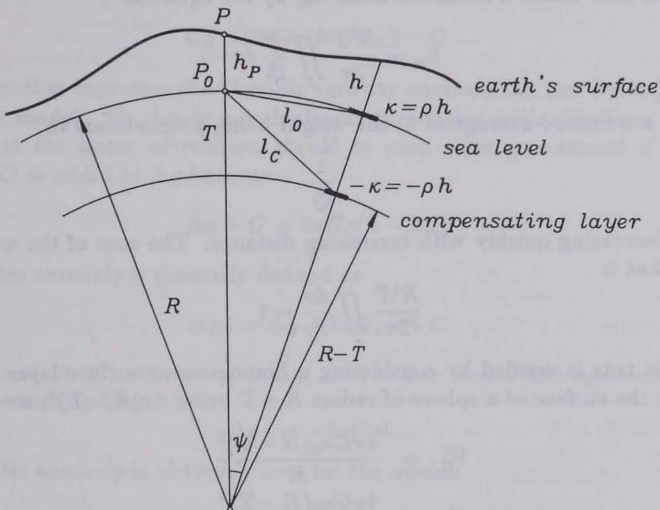

FIGURE 8.13: Spherical equivalent of Fig. 8.10; note again the dipole character

the isostatic compensation as a surface layer on a sphere concentric to the terrestrial sphere, was also used by Jung (1956, p. 590); we are following (Moritz, 1968c).

Let us now consider potential V_C and attraction A_C of this compensation layer. Since $h << T$, these quantities are almost the same whether referred to P or to P_0

(Fig. 8.13). We thus refer to P_0 and have

$$V_C = G\rho R^2 \iint_\sigma \frac{h}{l_C} d\sigma \quad , \tag{8-90}$$

$$B_C = G\rho R^2 T \iint_\sigma \frac{h}{l_C^3} d\sigma \quad . \tag{8-91}$$

The quantity B_C is defined in analogy to (8–65) as

$$B_C = A_C - \frac{1}{2R} V_C \tag{8-92}$$

and is expressed by an appropriate modification of (8–67): the mass element $\rho d\sigma d\eta$ in (8–67) is replaced by the mass element $\kappa d\sigma = \rho h d\sigma$ for a surface potential, and

$$\eta = -T \; , \quad h_P = 0 \quad .$$

With these changes, and on replacing the triple (volume) integral by a double (surface) integral, (8–67) indeed reduces to (8–91).

We shall now define a mean elevation h_m by the equation

$$h_m = \frac{R^2 T}{2\pi} \iint_\sigma \frac{h}{l_C^3} d\sigma \quad ; \tag{8-93}$$

h_m is thus a weighted average of h, the weight being proportional to

$$\frac{1}{l_C^3}$$

and thus decreasing quickly with increasing distance. The sum of the weights must be unity, that is

$$\frac{R^2 T}{2\pi} \iint_\sigma \frac{d\sigma}{l_C^3} = 1 \quad . \tag{8-94}$$

That this is true is verified by considering a homogeneous surface layer of constant density κ_0; the surface of a sphere of radius $R - T$ being $4\pi(R-T)^2$, we then have

$$V'_C = \frac{4\pi G\kappa_0 (R-T)^2}{R} \quad ,$$

$$A'_C = \frac{4\pi G\kappa_0 (R-T)^2}{R^2}$$

and thus, by (8–92),

$$B'_C = 2\pi G\kappa_0 \frac{(R-T)^2}{R^2} \doteq 2\pi G\kappa_0 \tag{8-95}$$

with a relative error of about 1%. On the other hand, from (8–91),

$$B'_C = G\kappa_0 R^2 T \iint_\sigma \frac{d\sigma}{l_C^3} \quad . \tag{8-96}$$

The comparison of (8–95) and (8–96) gives (8–94).

Substituting (8–93) into (8–91) we find

$$B_C = 2\pi G\rho h_m \quad , \tag{8–97}$$

so that by (8–92),

$$A_C = 2\pi G\rho h_m + \frac{1}{2R} V_C \quad . \tag{8–98}$$

According to our model, assuming crust and mantle to be homogeneous, the gravity anomaly Δg is caused only by the combined effect of topography and compensation:

$$\Delta g = A - A_C \quad , \tag{8–99}$$

where A is the attraction of topography. Substituting (8–79) and (8–98) we thus have

$$\Delta g = 2\pi G\rho(h_P - h_m) - C + \frac{1}{2R}(V - V_C) \quad . \tag{8–100}$$

The last term, which is very small (of order 1 mgal) because V and V_C are almost equal, will be neglected, and there remains (on omitting the subscript P)

$$\Delta g = 2\pi G\rho(h - h_m) - C \quad . \tag{8–101}$$

This equation expresses the "free-air" gravity anomaly Δg (see below) corresponding to our model. We clearly see the linear correlation with elevation, and we see at once that *the linear correlation should be even more pronounced if the terrain correction C is added to Δg* because

$$\Delta g + C = 2\pi G\rho(h - h_m) \quad . \tag{8–102}$$

The Bouguer anomaly is generally defined as

$$\Delta g_B = \Delta g - 2\pi G\rho h + C \quad , \tag{8–103}$$

by (8–36) and (8–38) with $g - \gamma = \Delta g$; thus in our model (homogeneous crust and mantle!) we simply have

$$\Delta g_B = -2\pi G\rho h_m \quad . \tag{8–104}$$

The isostatic anomaly is obviously zero for the model:

$$\Delta g_I = 0 \quad . \tag{8–105}$$

8.2.5 Conclusions Regarding Gravity Anomalies

Thus our model gives a reasonably realistic interpretation of the following empirical facts (Heiskanen and Moritz, 1967, pp. 281–285):

1. The free-air anomalies (see below) fluctuate around zero but are linearly correlated with elevation.