Nor is it difficult to integrate (8–70) with respect to η if l is expressed by (8–54). The result is

$$B = 2\pi G\rho h_P + G\rho R^2 \iint_\sigma \left(\frac{1}{l_1} - \frac{1}{l_0}\right) d\sigma \quad , \tag{8–78}$$

where l_0 and l_1 are given by (8–55) and (8–54) with $\eta = h$. This was already found by Pellinen (1962).

Now it is easy to obtain the attraction A. Combining (8–65), with $r_P \doteq R$, and (8–76) we have

$$A = 2\pi G\rho h_P - C + \frac{1}{2R} V \quad . \tag{8–79}$$

We finally note that B has to A the same relation as the gravity anomaly Δg to the gravity disturbance δg: compare (8–65) with eq. (2–151e) of (Heiskanen and Moritz, 1967).

8.2.3 Condensation on Sea Level

The linear approximation (8–61) admits of a simple interpretation. We consider a layer of surface density

$$\kappa = \rho h \tag{8–80}$$

on the mean terrestrial sphere $r = R$ which represents the sea level. The potential of this surface layer at a point P_0 of the surface is given by

$$V_S = G \iint_\sigma \frac{\kappa}{l_0} R^2 d\sigma = G\rho R^2 \iint_\sigma \frac{h}{l_0} d\sigma \quad . \tag{8–81}$$

This can be transformed as

$$V_S = G\rho R^2 h_P \iint_\sigma \frac{d\sigma}{l_0} + G\rho R^2 \iint_\sigma \frac{h - h_P}{l_0} d\sigma \quad . \tag{8–82}$$

The first term on the right-hand side is the potential of a homogeneous spherical surface layer, which is given by the same formula (8–50) as the potential of a homogeneous sphere or of a spherical shell. Since even (8–51) holds for our surface layer (now $r_P = R$ exactly), the first term of (8–82) is given by (8–52), and we have

$$V_S = 4\pi G\rho h_P R + G\rho R^2 \iint_\sigma \frac{h - h_P}{l_0} d\sigma \quad . \tag{8–83}$$

This formula, which is rigorously valid for a spherical surface layer of density (8–80), is seen to agree with the linear approximation (8–61) to the potential of the topographic masses.

This immediately suggests a relation to the well-known condensation reduction of Helmert (Heiskanen and Moritz, 1967, p. 145), in which the topographic masses are compressed into a surface layer of density (8–80) on the geoid. We thus see that the change of potential because of the condensation, $V - V_S$, is a small quantity of second order, because as a linear approximation V agrees with V_S. Here we have assumed that the point P, originally situated on the earth's surface, goes over into the corresponding point P_0 at sea level after condensation.

Thus, if we limit ourselves to the linear approximation which is often sufficient, we may regard the potential V as being generated by a spherical surface layer, the points P or P_0 being assumed to lie in both cases on the boundary of the attracting masses.

We shall now further investigate this surface layer. Let us first consider the attraction A and the auxiliary quantity B introduced in sec. 8.2.2. The point P is situated on the spherical surface, but at the outer boundary of the attracting masses. Thus A_S, the attraction of the surface layer at P, is given by the negative *external* derivative of V_S, e.g., expressed by equation (1–17a) of (Heiskanen and Moritz, 1967, p. 6). Thus we have

$$A_S = 2\pi G\kappa - G \iint_\sigma \kappa \frac{\partial}{\partial r_P} \left(\frac{1}{l}\right) R^2 d\sigma \quad . \tag{8–84}$$

To get the integrand, we must put $r = R = r_P$ in (8–64). We then obtain

$$A_S = 2\pi G\kappa + \frac{1}{2} GR \iint_\sigma \frac{\kappa}{l_0} d\sigma$$

and, with (8–80) and (8–81),

$$A_S = 2\pi G\rho h_P + \frac{1}{2R} V_S \quad . \tag{8–85}$$

We now consider the auxiliary quantity B_S defined in analogy to (8–65) as

$$B_S = A_S - \frac{1}{2R} V_S \quad . \tag{8–86}$$

We see that simply

$$B_S = 2\pi G\rho h_P \quad , \tag{8–87}$$

which is formally identical with the attraction of a "plane Bouguer plate". Equation (8–84) indicates, however, that the quantity B is in reality related to the discontinuity $2\pi G\kappa$ of the normal derivative of the surface potential on an *arbitrary* surface rather than to the attraction of a *plane* plate.

Let us now compare the quantities B for the actual topography and B_S for the surface layer. From (8–76) and (8–87) we obtain immediately

$$B = B_S - C \quad . \tag{8–88}$$

This means that these two quantities differ by the terrain correction C.

This has a consequence which will be of basic significance. As a linear approximation, also the attractions A and A_S differ by C,

$$A = A_S - C \quad . \tag{8–89}$$

This follows at once from the fact that A and B differ only by $V/2R$ and that as a linear approximation $V = V_S$. Thus as a linear approximation, *the potentials of the original and of the condensed topography are equal, but the attractions differ by the terrain correction.*

8.2.4 Effect of Compensation

We shall now consider a crustal density model by which the linear correlation of the free-air gravity anomalies with elevation can be explained and which at the same time is simple. Obviously, isostatic compensation must in some way be taken into account.

If we look at the Airy–Heiskanen isostatic model, we see that the compensation is given by the mountain roots which are some 30 km below sea level. The effect of this type of compensation on the earth's surface is thus quite similar as that of a surface layer of density $(-\rho h)$ on the sphere of radius $R - T$, where T may be identified with the normal thickness of the earth's crust of about 30 km, formerly denoted by T_0; see Fig. 8.13 and Fig. 8.10 above. The idea of regarding, for mathematical simplicity,

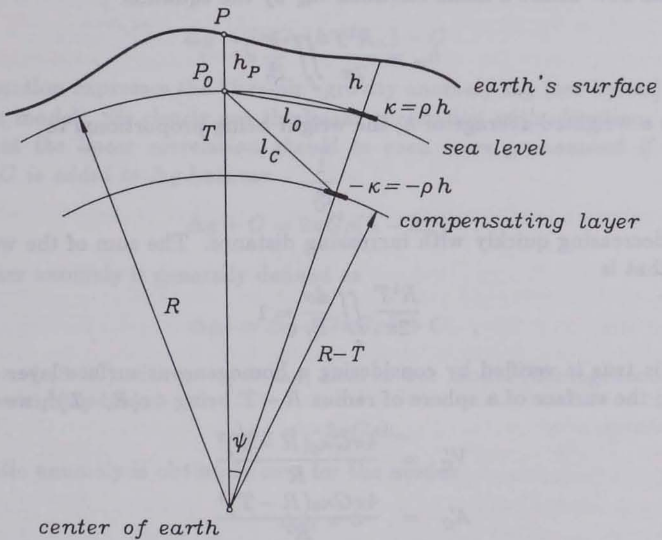

FIGURE 8.13: Spherical equivalent of Fig. 8.10; note again the dipole character

the isostatic compensation as a surface layer on a sphere concentric to the terrestrial sphere, was also used by Jung (1956, p. 590); we are following (Moritz, 1968c).

Let us now consider potential V_C and attraction A_C of this compensation layer. Since $h << T$, these quantities are almost the same whether referred to P or to P_0