To repeat, this extremely simple solution is not the equation of the actual bending curve (8–20) but gives an excellent qualitative picture. This can be seen by drawing the graph of (8–27), with $x$ replaced by $-x$ for negative values of $x$: a central depression surrounded by very small waves of decreasing amplitude.

### 8.1.4 Attraction of the Compensating Masses

As a preparatory step for computing isostatic reductions, to be discussed in sec. 8.1.5, we need the attraction of the compensating masses. For simplicity we consider the problem in the usual local plane approximation, replacing the geoid by its tangential plane. The spherical approximation will be used later (sec. 8.2).

We shall assume a basic definition concerning our three-dimensional local Cartesian coordinate system (Fig. 8.6): The $xy$-plane represents sea level, the $z$-axis points

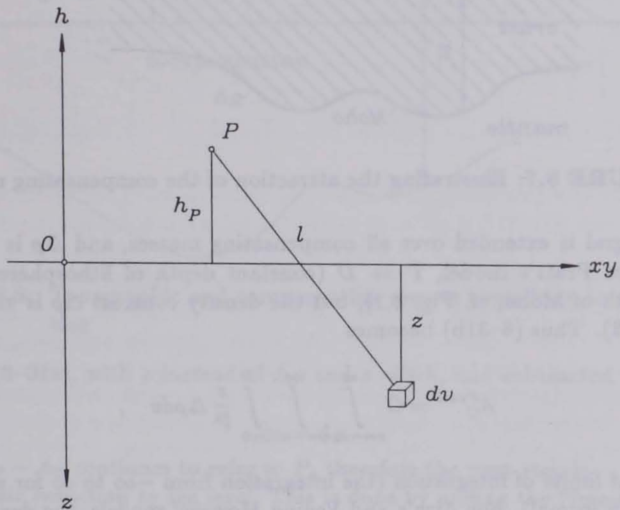


**FIGURE 8.6**: The basic coordinate systems $xyz$ and $xyh$

vertically *downwards*, whereas the $h$-axis points vertically *upwards*, so that, for an arbitrary point,

$$z = -h \quad . \tag{8–28}$$

Keeping this definition in mind, the distance $l$ between the computation point $P$ and the volume element $dv$ becomes

$$l^2 = (z + h_P)^2 + (x - x_P)^2 + (y - y_P)^2 \quad . \tag{8–29}$$

The potential $V_C$ of the compensating masses thus is

$$V_C = G \iiint \frac{\Delta\rho}{l}\, dv \qquad (8\text{–}30)$$

and their attraction (positive downward)

$$A_C = -\frac{\partial V_C}{\partial h_P} = G \iiint \frac{h_P + z}{l^3}\, \Delta\rho dv \qquad (8\text{–}31a)$$

with $\partial l^{-1}/\partial h_P$ by (8–29). For a point at sea level ($h_P = 0$) this reduces to

$$A_C = G \iiint \frac{z}{l^3}\, \Delta\rho dv \quad . \qquad (8\text{–}31b)$$

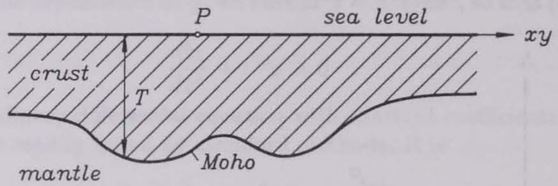


**FIGURE 8.7**: Illustrating the attraction of the compensating masses

The integral is extended over all compensating masses, and $\Delta\rho$ is their density contrast. For Pratt's model, $T \Rightarrow D$ (constant depth of lithosphere rather than variable depth of Moho, cf. Fig. 8.1), but the density contrast $\Delta\rho$ is variable, being given by (8–3). Thus (8–31b) becomes

$$A_C^{Pratt} = G \int\limits_{x=-\infty}^{\infty} \int\limits_{y=-\infty}^{\infty} \int\limits_{z=0}^{D} \frac{z}{l^3}\, \Delta\rho dv \quad , \qquad (8\text{–}32a)$$

with constant limits of integration (the integration from $-\infty$ to $\infty$ for $x$ and $y$ is, of course, purely formal). For Airy's and Vening Meinesz' models, the density contrast $\Delta\rho = \rho_1 - \rho_0$ is constant ($0.6\,\mathrm{g/cm^3}$, say), but the Moho depth $T$ is variable (Fig. 8.7), so that for these models,

$$A_C = G\Delta\rho \iint\limits_{-\infty}^{\infty} \int\limits_{z=T_0}^{T} \frac{z}{l^3}\, dv \quad . \qquad (8\text{–}32b)$$

The integrals are to be evaluated by numerical integration, using standard methods (cf. Heiskanen and Moritz, 1967, pp. 117–118; Forsberg, 1984).

Very similar integrals hold, of course, for the attraction of the topography, as we shall see in what follows.