7.6.5 Remarks on the General Solution

The proposed general set of solutions may be summarized as follows: the density is represented in the form (7–26) with (7–27):

$$\rho(r, \theta, \lambda) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \sum_{k=0}^{N} x_{nmk} r^k Y_{nm}(\theta, \lambda) \quad , \tag{7–51}$$

where

$$x_{nmk} = x_{nmk}^{(1)} + x_{nmk}^{(2)} \quad , \tag{7–52}$$

$x^{(1)}$ corresponding to the solution (7–38) and $x^{(2)}$ to any solution of the homogeneous equation (7–48) as before. The coefficients a_{nmk} are given by (7–35):

$$a_{nmk} = \frac{4\pi G R^{n+k+3}}{(2n+1)(n+k+3)} \quad . \tag{7–53}$$

The set of solutions contains the following free parameters: an arbitrary positive definite symmetric matrix $[c_{ij}]$ in (7–38), different for each (m, n), and the "zero–potential–density vector" $x^{(2)}$ which is only subject to the condition that it satisfies (7–48). Evident restrictions such as the absence of the terms with $n = 1$ and of the terms $k = 0$ except for $n = 0$ have already been mentioned.

Now there comes a surprise (Fig. 7.7). Unless $b = V_{nm}$ is zero, the end point of the

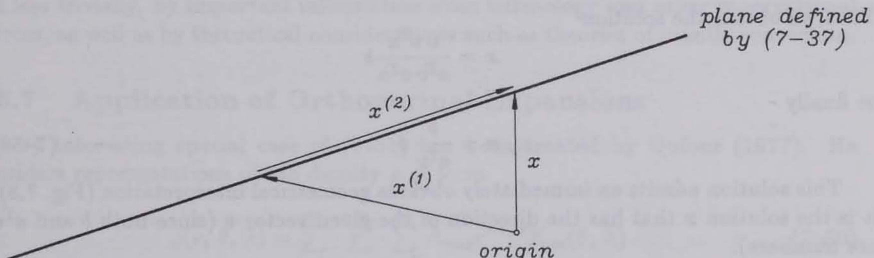

FIGURE 7.7: The sum $x = x^{(1)} + x^{(2)}$ again is of type $x^{(1)}$

vector x as given by (7–52) again lies in the hyperplane (7–37) and can therefore be represented in the form (7–38). Thus even the total solution (7–52), $x = x^{(1)} + x^{(2)}$, can be exclusively characterized by a certain matrix from our set of symmetric and positive definite matrices $[c_{ij}]$, so that we need only solutions of type $x^{(1)}$ as expressed by (7–38). Solutions of type $x^{(2)}$ are necessary only if $b = V_{nm} = 0$. Of course, on a closer look, this is not so surprising after all.

In statistical terms, $C = [c_{ij}]$ represents the covariance matrix of the vector x; in case it is given, (7–38) expresses a kind of least-squares (minimum norm) solution, by (7–42).