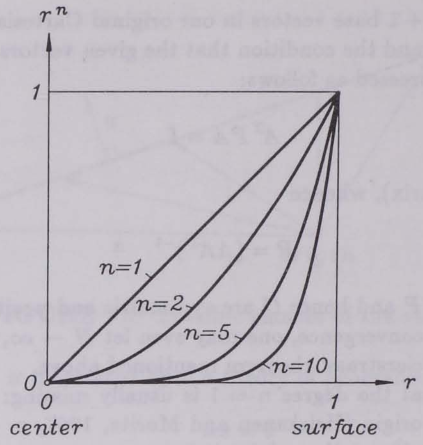

FIGURE 7.6: The powers r^n $(0 \leq r \leq 1)$

7.6.4 Zero-Potential Densities

The solution (7–38) gives $x_k = 0$ if the right-hand side of (7–37), $b = V_{nm}$, is zero. This is the case of the homogeneous equation corresponding to (7–36),

$$\sum_{k=0}^{N} a_{nmk} x_{nmk} = 0 \quad , \tag{7–48}$$

or briefly, corresponding to (7–37),

$$\sum_{k=0}^{N} a_k x_k = 0 \quad , \tag{7–49}$$

which represents the case of a mass distribution that produces zero external potential.

These are the "zero-potential densities" (sec. 7.2), forming the kernel of the Newtonian operator, for the present case. It is very easy to find non-zero solutions of (7–48) or (7-49): eq. (7–49) means simply that the vector x is normal to the given vector a (in the usual Euclidean metric)! Thus any vector x in the plane normal to a is admissible.

Finally we mention that the set of solutions of (7–49), forming *the vector* $x^{(2)}$ in (7–52) *is "orthogonal" to the vector* (7–38), denoted in (7–52) by $x^{(1)}$, if we again take P as metric tensor. This is geometrically evident and is also immediately verified by direct computation: using (7–38) in matrix notation, we have

$$x^{(1)T} P x^{(2)} = \frac{b}{a^T C a} a^T C P x^{(2)} = 0 \tag{7-50}$$

since $CP = I$ (unit matrix) and $a^T x^{(2)} = 0$ by (7–49).