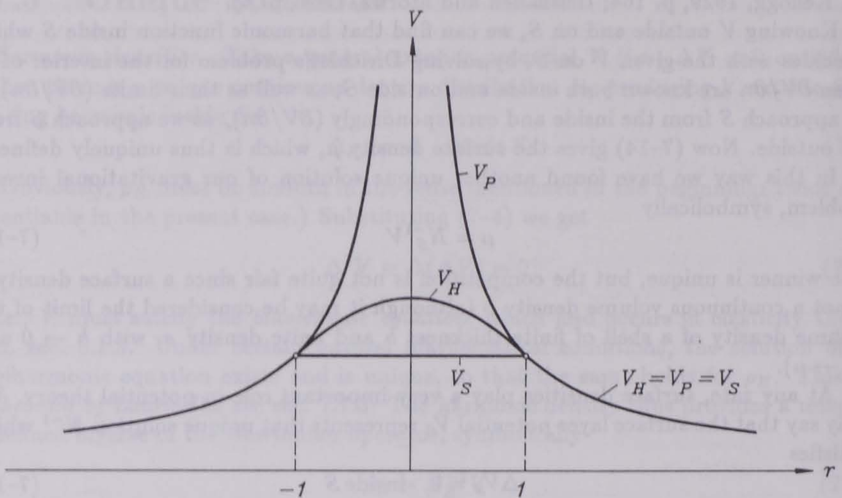


**FIGURE 7.2:** The potentials $V_P$, $V_H$ and $V_P$; negative arguments are for the symmetry of the figure only (negative $r$ are without geometric meaning!)

is continuous and differentiable everywhere, but it is not an analytic function in $\mathrm{R}^3$ because it is represented by two different analytic functions: by (7–19) for $r \leq 1$ and by (7–17) for $r \geq 1$; both functions are welded smoothly together at $r = 1$, so that their combination forms the nice bell-shaped curve for $V_H$ in Fig. 7.2. On the other hand, $V_S$ has a discontinuous derivative at $S$ ($r = 1$), which shows that it cannot be the potential of a volume distribution. At any rate, $V_H$ and $V_S$ "bridge", in different ways, the singularity of $V_P$ at the origin $r = 0$.

## 7.4 A "General" Solution

It is well known that the general solution of an inhomogeneous linear equation is obtained *as the sum of one particular solution of the inhomogeneous equation and the general solution of the corresponding homogeneous equation.* In our case, the particular solution is provided by the harmonic density described in the preceding section. The general solution of the homogeneous equation (7–7) (homogeneous means zero right-hand side) is the set of zero-potential densities forming the kernel of the Newtonian operator $N$.

Thus we find the general solution of the gravitational inverse problem by determining the uniquely defined harmonic density that corresponds to the given external potential, and adding any zero-potential density determined by the continuation method described in sec. 7.2; cf. also Fig. 7.1.

We may also proceed directly in the following way. We take the given harmonic function $V$ outside $S$ and continue it into the interior of $S$ in such a way that $V$ (including its continuation) is continuous and continuously differentiable throughout $R^3$ and that $\Delta V$ is piecewise continuous inside $S$. This is illustrated, for one dimension, in Fig. 7.3.

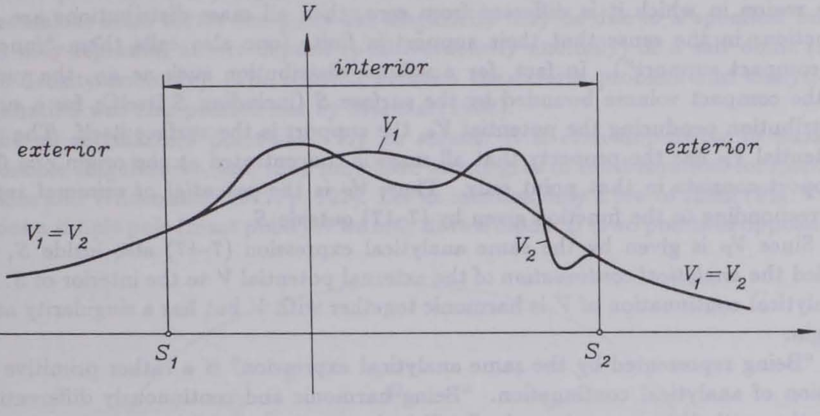


**FIGURE 7.3**: Two possible functions $V$ in one dimension

There is no doubt, however, that although we continue the external $V$ in the way described ($V_1$ or $V_2$ in Fig. 7.3), not only the external potential, but also *the mass $M$* and other *"Stokes constants"* (e.g., the spherical-harmonic coefficients) *remain the same*, because they are fully determined by the external potential (outside any sphere enclosing the body, cf. sec. 7.7.5). This is also expressed by the fact, mentioned in sec. 7.2, that the total mass of any zero-potential density is zero.

This is easy to understand in principle, but it is difficult to *really compute* or "construct" an smooth continuation in the way described. Therefore we have put the word "general" in the title of this section between quotation marks.

A constructive method can be obtained by superimposing the uniquely defined harmonic density $\rho_H$ and any zero-potential density $\rho_0$ according to Lauricella's integral (7–9); there follows the theorem, also due to Lauricella: *the Laplacian of the density of a body producing a given external potential can be arbitrarily assigned*, cf. sec. 7.7.3.

A general solution without smoothness assumptions can probably be found by the methods of modern potential theory, as a linear combination of "extremal measures", cf. (Anger, 1981, 1990), which make essential use of surface distributions $V_S$ and point masses $V_P$. However, this approach is mathematically very difficult, and solutions have been found so far for the simplest cases only.

We shall, therefore, try in sec. 7.6 a rather general and entirely elementary approach. It is limited to the sphere, but this anyway is the most interesting case for global geodesy and geophysics.