huge literature on this subject; we can only mention a recent textbook (Tarantola, 1987) but cannot help quoting the fundamental paper (Backus, 1970).

7.2 Zero-Potential Densities

Since N^{-1} is non-unique, it is fundamental to investigate the *kernel* (or *nullspace*) of the operator N: the set of all density distributions ρ_0 within S that produce zero external potential:

$$N\rho_0 = 0 \quad \text{outside } S \quad . \tag{7-7}$$

Such density distributions ρ_0 will be called zero-potential densities. We repeat: *the set of all possible zero-potential densities forms the kernel of the Newtonian operator* N, symbolized by $\ker(N) = N^{-1}(0)$.

Clearly, ρ_0 must be alternatively positive and negative, so that the total mass is zero; otherwise (7–7) would be impossible. Contrary to the usage of much of standard potential theory, we do not require ρ to be positive now. In fact, in practical applications, V will represent *potential anomalies* rather than potentials, and the corresponding ρ will be *density anomalies* which may be positive or negative.

It is extremely easy to find a rather general method of determining $\ker(N)$. Take any function V_0 that is zero outside S and continued in a continuous and differentiable manner to the inside of S in such a way that it is also twice piecewise differentiable within S. This is illustrated in Fig. 7.1 for one instead of three dimensions; then the

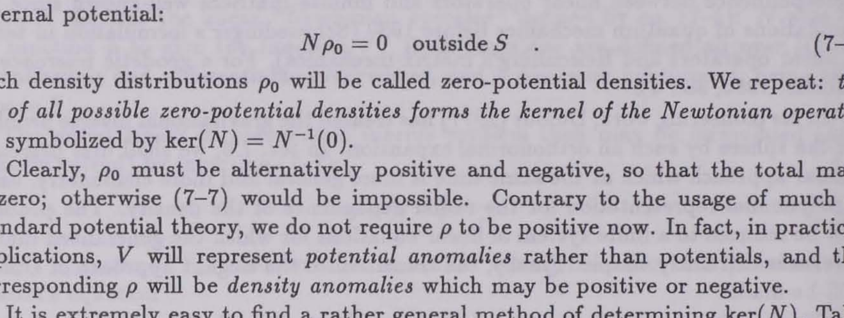

FIGURE 7.1: Two possible functions V_0 in one dimension

boundary S consists of two points S_1 and S_2.

Return to R^3. Since after continuation to the inside of S, V_0 is now defined throughout R^3, the corresponding density ρ_0 is given by (7–4):

$$\rho_0 = -\frac{1}{4\pi G}\,\Delta V_0 \quad . \tag{7-8}$$

Outside S this gives $\rho_0 = 0$ as it should, and inside, the zero potential density ρ_0 is piecewise continuous according to our differentiability assumptions concerning V_0.

Taking the set of all such functions V_0 we obtain the set of all piecewise continuous zero-potential densities, which do not span the complete nullspace $\ker(N)$ but are at least dense in it.

Note that because of the continuity of the gradient vector $\mathbf{g}_0 = \text{grad}V_0$ on S, this vector must be zero on S as well as V itself, which is expressed in Fig. 7.1 by the fact that both possible curves for V_0 are tangent to the x-axis at S_1 and S_2. For a more rigorous treatment cf. (Schulze and Wildenhain, 1977, p. 102), but don't think this book is easy!

A first note on the Green's function method. The approach just described is easy to understand, but it does not provide a method for explicitly constructing V inside S and hence the zero-potential density ρ_0: there is no straightforward prescription for continuing the zero function outside S into its interior in such a way that it is continuous and twice piecewise differentiable.

An explicit representation of zero-potential densities was given by Lauricella (1911, 1912): he showed that for an *arbitrary* smooth function ρ_0, the integral

$$V = \iiint_v G_2 \Delta \rho_0 dv \tag{7–9}$$

gives zero external potential, where Δ is Laplace's operator and G_2 is a special kind of Green's function to be described in sec. 7.7.

Sphere and ellipsoid. It is well known that concentric mass shells, of equal mass M, have the same external potential (2–31), in the exterior of the larger of the two shells. Thus the configuration consisting of both shells but with densities of opposite sign has zero-potential density. More generally, *redistributions of mass that preserve concentric spherical stratification do not change the external potential.*

In the *ellipsoidal case* we have remarked in sec. 5.6 that redistributions of mass that preserve *confocal* ellipsoidal stratification do not change the external potential.

We are almost exclusively considering the external potential here. It is not without interest, however, to compare this situation with the case of the *interior* potential inside a *hollow* sphere or ellipsoid. Inside a spherical shell, the interior potential is constant, $V_i = C$, by (2–34). This is the equivalent to zero external potential since for $V \equiv C$, the gravitational attraction $\text{grad}V \equiv 0$, so that we have zero attraction. From the considerations of sec. 2.2 it follows that redistributions of mass within the shell that preserve concentric spherical stratification do not change internal gravity. The ellipsoidal analogue is provided by *Newton's theorem* (sec. 3.2.6): an ellipsoidal shell bounded by two similar ellipsoids has constant potential or zero attraction in its interior. From this it follows that redistribution of masses within an ellipsoidal shell does not change the gravitational attraction, provided this redistribution preserves the *similar* stratification. Note the difference: confocal stratification for the external case and similar (homothetic) stratification in the internal case.