which is in agreement with (Denis and Ibrahim, 1981, p. 189). Then

$$e_0'^2 = 0.00486 \quad . \tag{5-237}$$

For this we find

$$\begin{aligned} \rho_1 &= 6.7332 \quad , \\ a_0 &= 1.0049 \quad , \\ a_2 &= 0.0259 \quad , \\ a_4 &= -0.0241 \quad . \end{aligned} \tag{5-238}$$

Other values of f_0 such as 1/469 (Bullard, 1954, p. 96) will slightly change these values.

At any rate, the values (5-238) show that $F(u)$ as given by (5-221) is indeed close to unity.

5.9 Combined Density Models

According to the discussions of secs. 5.5 and 5.6, the density $\rho(u, \theta)$ of a mass distribution for the equipotential ellipsoid has been represented as follows

$$\rho(u, \theta) = \rho_0 + \bar{\rho}(u, \theta) + \Delta\rho(u, \theta) \quad . \tag{5-239}$$

The constant ρ_0 is the constant density of the homogeneous Maclaurin ellipsoid that corresponds to the given equipotential ellipsoid, the function $\bar{\rho}(u, \theta)$ is the "zero-potential density" that introduces the desired heterogeneity without changing the external gravity field of the Maclaurin ellipsoid, and $\Delta\rho(u, \theta)$ is the "deviatoric density" that changes the external field of the Maclaurin ellipsoid to the prescribed field of the original equipotential ellipsoid without changing appreciably (that is, by more than about 0.028 g/cm^3) the density distribution.

To present an example of a density distribution that arises in this way, we use a function $\Delta\rho(u, \theta)$ according to (5-156) and (5-165), and a function $\bar{\rho}(u,\theta)$ according to (5-184), the functions $A(u)$ and $B(u)$ being given by (5-203) and (5-222). We thus have

$$\begin{aligned} \rho(u, \theta) &= \rho_0 + \rho_1 - \left[b_0 + b_2\left(\frac{u}{b}\right)^2\right](x^2+y^2) - \\ &- \left[b_0 + b_2\left(\frac{u}{b}\right)^2\right]\left[a_0 + a_2\left(\frac{u}{b}\right)^2 + a_4\left(\frac{u}{b}\right)^4\right]z^2 + \\ &+ C\left(\frac{u}{b}\right)^4\left[1-\left(\frac{u}{b}\right)^2\right]\left(-1+\frac{u^2+E^2}{u^2+E^2\cos^2\theta}\right) \quad . \end{aligned} \tag{5-240}$$

The replacement of u by u/b in the polynomials representing $A(u)$ and $B(u)$ expresses the fact that we are no longer using b as a unit, but have returned to metric units.

Numerical values for the coefficients are given by (5–170), (5–171), (5–232), and (5–235).

The density model (5–240) is rigorous, simple, gives a concrete idea about possible density distributions, and is practically applicable. However, it is not very general because of the use of polynomial representations.

The general form of the density distributions discussed in the preceding section, by (5–239), (5–184), and (5–156) is

$$\rho(u, \theta) = \rho_0 + \rho_1 - (x^2 + y^2)A(u) - z^2 B(u) + \frac{E^2 \sin^2 \theta}{u^2 + E^2 \cos^2 \theta} h(u) \quad . \tag{5–241}$$

The functions $\alpha_n(u)$ that correspond to this distribution according to (5–86), are computed from (5–186), (5–187), (5–188), and (5–189), as well as from the auxiliary formula for $\sin^2 \theta$ given at the beginning of sec. 5.7:

$$\begin{aligned}
\frac{1}{4\pi}\,\alpha_0(u) &= \left(u^2 + \frac{1}{3}E^2\right)(\rho_0 + \rho_1) + \frac{2}{3}E^2 h(u) + \\
&+ \left(-\frac{2}{3}u^4 - \frac{4}{5}E^2u^2 - \frac{2}{15}E^4\right)A(u) + \left(-\frac{1}{3}u^4 - \frac{1}{5}E^2u^2\right)B(u), \\
\frac{1}{4\pi}\,\alpha_2(u) &= \frac{2}{3}E^2(\rho_0 + \rho_1) - \frac{2}{3}E^2 h(u) + \\
&+ \left(\frac{2}{3}u^4 + \frac{4}{7}E^2u^2 - \frac{2}{21}E^4\right)A(u) + \left(-\frac{2}{3}u^4 - \frac{4}{7}E^2u^2\right)B(u), \\
\frac{1}{4\pi}\,\alpha_4(u) &= \frac{8}{35}\left[(E^2u^2 + E^4)A(u) - E^2u^2B(u)\right] \quad ; \\
\alpha_n(u) &\equiv 0 \quad \text{if} \quad n > 4 \quad .
\end{aligned} \tag{5–242}$$

The functions $h(u)$, $A(u)$, and $B(u)$ are rather arbitrary; they must only satisfy the conditions (5–162) and (5–198), together with the constant ρ_1. The "Maclaurin density" ρ_0 follows from (5–164).

We clearly see that the present model is not of the simple form (5–121) but implies a nonzero $\alpha_4(u)$. We also remark that the function $h(u)$ introduced in sec. 5.3 by (5–113), is of very general significance and also enters in (5–242), whereas the other auxiliary function $g(u)$ introduced in (5–112) was of more limited applicability: it was used in sec. 5.3.1 and, as the constant Maclaurin density (5–133), has played a basic role in sec. 5.4; $g(u)$ was also still used in sec. 5.6 but later on it lost its significance together with (5–178).

5.10 Numerical Considerations and Problems

In this section we shall work with the density function (5–241). The unit of length will again be chosen equal to the semiminor axis b of the reference ellipsoid:

$$b = 1 \quad . \tag{5–243}$$