The continuous analogue of (3–95) is

$$E_V = \frac{1}{2} G \iiint_v \iiint_v \frac{\rho(\mathbf{x})\rho(\mathbf{x}')}{||\mathbf{x}-\mathbf{x}'||} dv dv' \tag{3–96}$$

with obvious notations: $\mathbf{x}$, $\mathbf{x}'$ position vectors; v volume of the body; dv, dv' volume elements; and $l = ||\mathbf{x}-\mathbf{x}'||$. Another form of (3–96) is

$$E_V = \frac{1}{2} \iiint_v V \rho dv \quad , \tag{3–97}$$

where V is the usual gravitational potential. Comparing with (3–93) note the factor 1/2, reflecting the fact that E_V is produced by an *internal field* created by the mass elements $dm = \rho dv$ themselves.

For the centrifugal part we have

$$E_\Phi = \sum m_i \Phi_i = \iiint_v \Phi \rho dv \quad , \tag{3–98}$$

in agreement with (3–93), since the centrifugal potential Φ acts as an *external* field.

The potential energy of the gravity potential $W = V + \Phi$ thus is the sum of (3–97) and (3–98):

$$E_W = \int \left(\frac{1}{2} V + \Phi\right) \rho dv \quad , \tag{3–99}$$

using only a simple integral sign for notational convenience.

3.3.2 Dirac's and Heaviside's Functions

We recall the basic definition of *Dirac's delta function* (cf. Moritz, 1980, pp. 28–30):

$$\begin{aligned} \delta(x) &= 0 \quad && \text{except for } x = 0 \quad , \\ \delta(0) &= \infty && \text{in such a way that} \end{aligned} \tag{3–100}$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \quad . \tag{3–101}$$

It is a somewhat strange "function" but is extremely useful and popular in physics.

Its integral is Heaviside's step function:

$$\theta(x) = \int_{-\infty}^{x} \delta(x') dx' \quad . \tag{3–102}$$

From (3–100) and (3–101) it immediately follows that

$$\theta(x) = \begin{cases} 0 & \text{for } x < 0 \quad , \\ 1 & \text{for } x > 0 \quad . \end{cases} \tag{3–103}$$

For $\theta(0)$ we may take the value 1/2.

From (3–102) there follows the basic relation

$$\delta(x) = \frac{d\theta(x)}{dx} = \theta'(x) \quad . \tag{3–104}$$