## Dynamic analysis of dam-reservoir-intake tower considering sediments absorption

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#### Abstract

In this paper, dam-reservoir-intake tower system with interior water of tower are analyzed using finite element method considering sediments absorption. Reservoir is modeled by Lagrangian approach and effect of sediments absorption on responses of dam-reservoir-intake tower is considered. Three types of sediments and also distance between dam and tower are determined for parameter study. Dynamic analysis has been performed under horizontal and vertical excitation of Northridge and Tabas earthquakes. Results show that increasing distance between dam and tower, can increase frequencies of dam-reservoir-intake tower system. Also in models with rigid sediments, the frequencies of system are alittle more than model with two other kind of sediments. Dynamic responses show that increasing the distance between dam and tower responses. It is concluded that the maximum displacements of dam crest and principal stresses of dam heel increase of dam heel increase in rigid sediments but sediment type has fewer effects on tower responses specially on displacements.

Keywords: intake towers, lagrangian approach, sediments absorption, vertical excitation, reservoir.

#### **1. INTRODUCTION**

Seismic analysis of concrete dams and connected hydraulic structures like intake towers were studied by researchers because they are related to water supply systems and may influence human necessities in life. In primitive researchs basically solid-fluid interaction was study parameter where defined useful equations but in recent decades due to the computerized modeling, systems have been modeled and analyzed in three directions. Although precence of dam has been considered in new models but sediments in reservoir bottom didn't investigate as a parameter study. Because important role of reservoir interaction in analysis of dam-reservoir- intake tower system, in this paper reservoir bottom sediments are modeled for three absorption conditions and for three distance between dam and tower. In the following some important articles and their results about dynamic analyses of intake towers will express.

During 1974 to 1975 Liaw and Chopra [1,2] started extensive researchs on reservoir-intake tower interaction problems and studied sourronding water interaction by hydrodynamic added mass method on dynamic behavior of cantilever intake tower. They obtained dynamic responses of reservoir and tower by ignoring surface waves and compressibility of water in the hydrodynamic solutions. In 1989 Goyal and Chopra [3-5] by adding foundation to previous model, analyzed foundation-water-intake tower system under harmonic loading using hydrodynamic added mass method. They presented total system as four substructures: tower, sourronding water, contained water, and the foundation supported on flexible soil and defined frequency domain equations for these four substructures and foundation-water-intake tower system by analytical method. Daniell and Taylor [6] in 1993 conducted dynamic tests on a 50 meter high intake tower at Wimbleball dam in England and compared results of these tests with predictions from a corresponding numerical model. Their aim was to affirm the assumption that the compressibility of reservoir water is not important parameter in seismic analysis of intake towers. In the year 2009 for first time Millán and co-workers[7] added dam to reservoir-intake tower model and investigated dam effects on seismic responses of tower. They understood locating dam in proximity to the tower leads to a new resonance mode near the tower's second resonance frequency due to the dam-reservoir excitations. Alembagheri [8] in 2016 studied numerically dynamics of a coupled concrete gravity dam-foundation-reservoir-intake tower considering two hollow slender towers submerged in reservoir of gravity dam. He represented that presence of the dam significantly influence the seismic responses of the towers under both horizontal and vertical excitations;

however the dam didn't affected by the towers. It was concluded that when the dam was present in the model, the water contained inside the towers had different effects if the foundation was rigid, but it decreased the towers motion if the foundation was flexible.

#### 2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS OF SYSTEM

In this paper Lagrangian-Lagrangian formulation used for analyzing dam-reservoir-intake tower system with interior water of tower. Equilibrium dynamic motion equation for this system underground acceleration in terms of nodal displacements in finite element meshing is[9,10]:

 $Ma + Cv + Ku = P(s) \tag{1}$ 

Where *M*, *C* and *K* are mass, damping and stiffness matrixes and *a*, *v* and *u* are nodal dynamic accleration, velocity and displacement vectors of finite element meshing respectively and P(s) is nodal external forces vector. The total stiffness matrix of system has been obtained by assembling stiffness matrix of dam, reservoir, tower and interface elements, like this[11]:

$$K = K_D + K_R + K_T + K_{INT}$$
<sup>(2)</sup>

Where  $K_D$ ,  $K_R$  and  $K_T$  are stiffness matrix of dam, reservoir, tower respectively.  $K_{INT}$  is the stiffness matrix of interface elements which applied between solid and fluid leads to freedom sliding and water from dam and tower don't separate in connected boundaries. Stiffness matrix of dam and tower depends on D: elasticity matrix, B: shape function matrix,  $B^T$ : transposed of B and  $V_S$ : volume of integral range for each part of dam and tower[12].

$$K_D = K_T = \int_{VS} B^T D B dv \tag{3}$$

The total stiffness matrix of reservoir elements is achived by summation of  $S_f$ : stiffness matrix of surface waves and  $K_W$ : stiffness matrix of reservoir elements. In equation (4)  $\rho_W$  is water density, g is gravity acceleration, s is range area of reservoir,  $V_R$  is range volume of reservoir elements, N is shape function of fluid nodes and  $N^T$  is transposed of N [11]. In ANSYS software which we used in this article, by modeling surface of reservoir  $S_f$  creates and it is not need to apply boundary condition in that border.

$$K_R = K_W + S_f = \int_{VR} B^T \cdot C_f \cdot B \cdot dv + \rho_{W} \cdot g \int_{S} N^T \cdot N \cdot ds$$

$$\tag{4}$$

The total damping matrix of system is achived by summation of  $C_l$ : internal viscose damping matrix,  $C_R$ : damping matrix caused by wave propagation and  $C_{abs}$ : damping matrix of absorping waves by reservoir bottom sediments[12]:

$$C = C_I + C_R + C_{abs} \tag{5}$$

In equilibrium dynamic equation, internal viscose damping matrix is combination of mass matrix and stiffness matrix of system like this[11]:

$$C_I = \alpha M + \beta K \tag{6}$$

Where  $\alpha$  is mass matrix coefficient and  $\beta$  is stiffness matrix coefficient which computed by equations (7,8):

$$\alpha = 2\omega_1 \xi_1 - \omega_1^2 \tag{7}$$

$$\beta = 2(\omega_1 \xi_1 - \omega_2 \xi_2) / (\omega_1^2 - \omega_2^2) \tag{8}$$

in these two equations number of 1 and 2 are related to first and second structural modes,  $\omega$  is angular frequency of system and  $\xi$  is damping ratio which considered 0.05 for dam and intake tower.

Because sediments and masses of rock and soil in reservoir bottom have varied flexiblity, they have different absorption capability but rigid sediments in reservoir bottom can absorp all of the waves energy arrive them. In Table 1- bottom absorption coefficients of  $a_1$  and  $b_1$  for different kinds of reservoir bottom sediments define. For providing this condition we used equivalent dampers in three direction for 3D finite element model with this damping matrix [13]:

$$C_{abs} = b_{1} \rho_W . C_W \int_s N^T . N. ds \tag{9}$$

Where  $C_W$  is water wave velocity and relates to  $\rho_W$  and  $K_W$ : bulk modulus of water in this formulation:

$$C_W = (K_W / \rho_W)^{0.5}$$

(10)

Kinds of reservoir bottom	a1	<b>b</b> <sub>1</sub>	Result on waves
sediments			
Rigid	1	x	Waves completely come back
Stiff	0.6-0.8	4-9	Waves partially come back
Soft	0.5	3	Waves partially come back
Water	0	1	Waves propagate
Air	-1	0	Waves come back with reverse
			amplitude

 Table 1- Values of coefficients a1 and b1 for different reservoir bottom sediments

### 3. NUMERICAL MODELING OF SYSTEM

For determining dam-reservoir-intake tower interaction on dynamic responses of dam and tower, we modeled concrete gravity dam 21 meter high with 0.05 and 0.75 slope in upstream and downstream face of it, tower 20 meter high with length and width of 4 and 5 meters in plan and inside water and surrounding reservoir have 20 meter height. Longitudinal distance between tower and free upstream face of reservoir is 60 meter and lateral distance of them is 20 meter in each side. Dam-reservoir-intake tower system with interior water of tower has been modeled by considering three types of sediments with different absorption and distance between dam and tower are determined 10, 20 and 30 meters so we analyze 9 models. Figure 1. represent geometry of dam-reservoir-intake tower system with interior water of tower.



### Figure 1. Geometry of dam-reservoir-intake tower system with interior water of tower

Mechanical properties of materials applied in modeling are defined in Table 2- where E is elasticity modulus, v is poisson coefficient and  $\rho$  is density. Volumetric percents of reinforcement bars applied in finite element model are 0.0213 in X axis, 0.0223 in Y axis and 0.573 in Z axis.

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Materials applied	E , N/m <sup>2</sup>	υ	$\rho$ , Kg/m <sup>3</sup>
Concrete of dam and tower	$2.5 \times 10^{10}$	0.17	2400
Water	2×10 <sup>9</sup>	0	1000
Reinforcement bars	$2.1 \times 10^{11}$	0.3	7800

Table 2- Mechanical proper	rties of materia	als applied in fi	nite element model
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ANSYS was used for modeling dam-reservoir-intake tower system and 3D eight nodes elements Solid65 for concrete and Lagrangian 3D eight nodes elements Fluid80 for water were applied in finite element model of system. Solid-fluid interaction faces are coupled in contact direction and in free faces of reservoir linear elements Combination14 with distinct damping and stiffness were used in finite element model and also in reservoir bottom these elements applied with distinct damping and stiffness for each kind of sediments as an equivalent dampers in three directions.

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## 4. **RESULTS OF MODAL ANALYSES**

For obtaining angular frequencies of system  $\omega$ , modal analyses performed on finite element models and their values for first and second structural modes are represented in Table 3- also  $\alpha$  as mass matrix coefficient and  $\beta$  as stiffness matrix coefficient which computed by equations (7) and (8) are defined in this table. Table 3represent that by increasing distance between dam and tower, frequencies of this system become greater. Also, in models with rigid sediments, the frequencies are a little more than model with two other kind of sediments, which can be due to greater stiffness.

## Table 3- Results of modal analyses for different types of sediments and distance between dam and tower

Types of sediments			Soft		Stiff			Rigid		
	tance between and tower, m	10	20	30	10	20	30	10	20	30
ω,	First mode	36.72	39.26	39.36	37.37	39.14	40.09	37.61	39.86	40.36
rad/s	Second mode	47.31	42.97	46.41	47.77	46.84	50.01	47.93	48.30	50.89
	α	2.07	2.05	2.13	2.10	2.15	2.23	2.11	2.18	2.25
β		0.00119	0.00122	0.00117	0.00117	0.00116	0.00111	0.00117	0.0011 3	0.00110

## 5. **RESULTS OF DYNAMIC ANALYSES**

Dynamic analyses has been performed under horizontal, lateral and vertical excitations of Northridge and Tabas earthquakes which their properties are showed in Table 4-.

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Earthquke	Time occured	Station	Magnitude	Distance from epicenter, Km	Recorded component	PGA (g)				
Northridge	1994/1/17	Arleta CDMG	6.9	11.79	Horizantal	0.308				
Norundge	1774/1/1/17	Station 24087	0.7	11.77	Lateral	0.344				
					Vertical	0.438				
					Horizantal	0.852				
Tabas	1978/9/16	Tabas	7.4	74.8	Lateral	0.868				
					Vertical	0.702				

## Table 4- Properties of earthquakes used for dynamic analysis

After analyzing system, displacements of dam and tower crest and principal stresses of dam heel and tower bottom have been extracted for each model and their maximum positive and negative values define in Table 5,6- for Northridge and in Table 7,8- for Tabas earthquakes. Where  $u_x$  is horizontal displacement,  $u_y$  is lateral displacement,  $u_z$  is vertical displacement and  $\sigma$  is principal stress.

	Northridge earthquake for different models									
Structure	Distance between dam and tower, m	Types of sediments	$u_x, m$	$u_y$ , m	$u_z, m$	$\sigma$ , $N\!/m^2$				
		Soft	0.00087	0.00025	0.00020	6.6×10 <sup>5</sup>				
	10	Stiff	0.00089	0.00025	0.00020	6.7×10 <sup>5</sup>				
		Rigid	0.00091	0.00026	0.00021	6.8×10 <sup>5</sup>				
Dam		Soft	0.00103	0.00028	0.00029	7.2×10 <sup>5</sup>				
	20	Stiff	0.00104	0.00028	0.00029	7.3×10 <sup>5</sup>				
		Rigid	0.00106	0.00028	0.00030	7.4×10 <sup>5</sup>				
		Soft	0.00108	0.00033	0.00029	7.6×10 <sup>5</sup>				
	30	Stiff	0.00109	0.00033	0.00029	7.7×10 <sup>5</sup>				
		Rigid	0.00110	0.00034	0.00030	7.8×10 <sup>5</sup>				
		Soft	0.00530	0.00356	0.00091	$2.24 \times 10^{6}$				
	10	Stiff	0.00537	0.00354	0.00092	$2.26 \times 10^{6}$				
		Rigid	0.00548	0.00350	0.00093	2.31×10 <sup>6</sup>				
Tower		Soft	0.00699	0.00361	0.00086	$2.38 \times 10^{6}$				
	20	Stiff	0.00706	0.00359	0.00087	$2.41 \times 10^{6}$				
		Rigid	0.00713	0.00355	0.00088	$2.46 \times 10^{6}$				
	20	Soft	0.00704	0.00405	0.00112	$2.66 \times 10^{6}$				
	30	Stiff	0.00708	0.00403	0.00112	2.67×10 <sup>6</sup>				
		Rigid	0.00713	0.00401	0.00113	$2.69 \times 10^{6}$				

Table 5- Maximum positive values of dam and tower Dynamic responses under
Northridge earthquake for different models

## Table 6- Maximum negative values of dam and tower Dynamic responses under Northridge earthquake for different models

Structure	Distance between dam	Types of sediments	u <sub>x</sub> , m	u <sub>y</sub> , m	$u_z, m$	$\sigma$ , $N\!/m^2$
	and tower, m					
		Soft	0.00089	0.00033	0.00019	$5.9 \times 10^{5}$
	10	Stiff	0.00090	0.00033	0.00019	6×10 <sup>5</sup>
		Rigid	0.00092	0.00034	0.00020	6.1×10 <sup>5</sup>
Dam		Soft	0.00091	0.00035	0.00023	6.2×10 <sup>5</sup>
	20	Stiff	0.00092	0.00035	0.00023	6.2×10 <sup>5</sup>
		Rigid	0.00094	0.00036	0.00023	6.3×10 <sup>5</sup>
		Soft	0.00104	0.00042	0.00026	6.6×10 <sup>5</sup>
	30	Stiff	0.00104	0.00042	0.00026	6.6×10 <sup>5</sup>
		Rigid	0.00105	0.00043	0.00026	$6.7 \times 10^{5}$
		Soft	0.00558	0.00403	0.00080	3.7×10 <sup>5</sup>
	10	Stiff	0.00565	0.00400	0.00081	3.7×10 <sup>5</sup>
		Rigid	0.00577	0.00396	0.00082	3.8×10 <sup>5</sup>
Tower		Soft	0.00569	0.00492	0.00101	5.1×10 <sup>5</sup>
	20	Stiff	0.00575	0.00490	0.00102	5.2×10 <sup>5</sup>
		Rigid	0.00581	0.00485	0.00103	5.3×10 <sup>5</sup>
		Soft	0.00658	0.00498	0.00106	5×10 <sup>5</sup>
	30	Stiff	0.00662	0.00495	0.00107	5×10 <sup>5</sup>
		Rigid	0.00667	0.00493	0.00107	5.1×10 <sup>5</sup>

Structure	Distance between dam and tower , m	Types of sediments	u <sub>x</sub> , m	u <sub>y</sub> , m	$u_z, m$	$\sigma$ , $N\!/m^2$
		Soft	0.00168	0.00052	0.00032	2.08×10 <sup>6</sup>
	10	Stiff	0.00172	0.00053	0.00033	2.12×10 <sup>6</sup>
		Rigid	0.00179	0.00055	0.00034	2.17×10 <sup>6</sup>
		Soft	0.00192	0.00055	0.00034	2.21×10 <sup>6</sup>
	20	Stiff	0.00198	0.00056	0.00034	2.24×10 <sup>6</sup>
D	20	Rigid	0.00224	0.00058	0.00035	2.29×10 <sup>6</sup>
Dam		Soft	0.00219	0.00067	0.00041	2.33×10 <sup>6</sup>
	30	Stiff	0.00222	0.00067	0.00041	2.34×106
		Rigid	0.00234	0.00067	0.00042	2.36×10 <sup>6</sup>
		Soft	0.01116	0.01303	0.00110	3.92×10 <sup>6</sup>
	10	Stiff	0.01123	0.01293	0.00110	4.26×10 <sup>6</sup>
	10	Rigid	0.01134	0.01279	0.00111	$4.45 \times 10^{6}$
		Soft	0.01121	0.01524	0.00122	4.01×10 <sup>6</sup>
	20	Stiff	0.01147	0.01510	0.00121	4.28×10 <sup>6</sup>
Tower -	20	Rigid	0.01168	0.01480	0.00122	4.51×10 <sup>6</sup>
		Soft	0.01317	0.01506	0.00162	4.32×10 <sup>6</sup>
	30	Stiff	0.01323	0.01505	0.00163	4.42×10 <sup>6</sup>
	50	Rigid	0.01337	0.01502	0.00164	4.80×10 <sup>6</sup>

# Table 7- Maximum positive values of dam and tower Dynamic responses under Tabas earthquake for different models

# Table 8- Maximum negative values of dam and tower Dynamic responses under Tabas earthquake for different models

Structure	Distance between dam	Types of sediments	u <sub>x</sub> , m	u <sub>y</sub> , m	$u_z, m$	$\sigma$ , $N\!/m^2$
	and tower, m					
		Soft	0.00189	0.00061	0.00029	$2.7 \times 10^{5}$
	10	Stiff	0.00193	0.00062	0.00029	2.8×10 <sup>5</sup>
		Rigid	0.00201	0.00065	0.00030	2.9×10 <sup>5</sup>
Dam		Soft	0.00217	0.00066	0.00037	2.9×10 <sup>5</sup>
	20	Stiff	0.00224	0.00068	0.00037	2.9×10 <sup>5</sup>
		Rigid	0.00231	0.00070	0.00038	3×10 <sup>5</sup>
		Soft	0.00226	0.00073	0.00040	3×10 <sup>5</sup>
	30	Stiff	0.00229	0.00073	0.00040	3×10 <sup>5</sup>
		Rigid	0.00231	0.00074	0.00041	3.1×10 <sup>5</sup>
		Soft	0.00965	0.01303	0.00096	6.5×10 <sup>5</sup>
	10	Stiff	0.00971	0.01293	0.00097	7.1×10 <sup>5</sup>
		Rigid	0.00977	0.01281	0.00098	7.4×10 <sup>5</sup>
Tower		Soft	0.01096	0.01339	0.00124	7×10 <sup>5</sup>
	20	Stiff	0.01117	0.01326	0.00128	7.5×10 <sup>5</sup>
		Rigid	0.01142	0.01301	0.00134	7.9×10 <sup>5</sup>
		Soft	0.01303	0.01793	0.00136	7×10 <sup>5</sup>
	30	Stiff	0.01309	0.01791	0.00137	7.1×10 <sup>5</sup>
		Rigid	0.01322	0.01788	0.00138	7.7×10 <sup>5</sup>

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As it is seen from the above tables, due to the hardness and thickness of dam and the enclosure of the tower with the reservoir, the dam responses are far less than the tower's response. Dynamic responses in the above tables show that increasing the distance between dam and tower can disorder the ascending procedure of maximum dynamic responses of dam and tower in the more difficult reservoir sediments. This means that increasing this distance can reduce the effects of sediments on the dam and tower responses. On the other hand, the maximum responses of dam and tower increase by increasing distance between them. This ascending procedure by changing the dam and tower distance from 10 meters to 20 meters is more tangible than the change from 20 meters to 30 meters. Also, changes of the tower responses are more obvious than the dam responses for increasing the dam and tower distance. It is concluded from above tables that stresses increase in dam and tower in reservoirs with more stiff sediments. Displacements of dam crest and principal stresses of dam heel increase in rigid sediments because reflective waves are more and they influence dam in one face but sediment type has fewer effects on tower responses specially on displacements it can be because tower is embedded in reservoir.

For distinguishing difference between effects of sediments types in time history responses, horizontal displacement of tower crest under Tabas earthquake compare for rigid and soft sediments in the model that distance between dam and tower is 20 meter, see Figures 2. It is included that the aggregation of sediments in reservoir bottom may amplify dam and tower responses.



Figure 2. Horizontal displacement of tower crest under Tabas earthquake

#### 6. CONCLUSIONS

Dam-reservoir-intake tower system with interior water of tower are analyzed using finite element method considering sediments absorption and distance between dam and tower as a parameter study. Displacements of dam and tower crest and principal stresses of dam heel and tower bottom have been extracted and results define in the following:

- 1. Increasing distance between dam and tower, can increase frequencies of dam-reservoir-intake tower system with interior water of tower. Also, in models with rigid sediments, the frequencies are a little more than model with two other kind of sediments, which can be due to greater stiffness.
- 2. Results show that the dam responses are far less than the tower's response, it can because hardness and thickness of dam and the enclosure of the tower with the reservoir.
- 3. Dynamic responses show that increasing the distance between dam and tower can disorder the ascending procedure of maximum dynamic responses of dam and tower in the more difficult reservoir sediments. This means that increasing this distance can reduce the effects of sediments on the dam and tower responses.
- 4. It is concluded that the maximum displacements of dam crest and principal stresses of dam heel increase by increasing distance between them. This ascending procedure by changing the dam and tower distance from 10 meters to 20 meters is more obvious than the change from 20 meters to 30 meters.
- 5. In time domain responses, displacements of dam crest and principal stresses of dam heel increase in rigid sediments because reflective waves are more and they influence dam in one face but sediment type has fewer effects on tower responses specially on it's displacements it can be because tower is embedded in reservoir.

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