2.3 HOMOGENEOUS ELLIPSOID: FIRST ORDER

Consider a homogeneous ellipsoid of revolution, of density

$$\rho = \text{const.}$$
(2-63)

By first-order theory we mean, as usual, that only terms linear in f are considered, $O(f^2)$ being neglected. To this approximation, its surface is given by (2-6),

$$r = R \left[1 - \frac{2}{3} f P_2(\cos \theta) \right] \quad . \tag{2-64}$$

This equation may be interpreted geometrically as in Fig. 2.3: the ellipsoid consists of

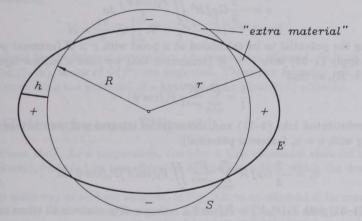


FIGURE 2.3: Ellipsoid and mean sphere

a "basic sphere" of radius R and "extra material" (plus or minus). Thus its potential is given by

$$V = V_{sphere} + v \quad . \tag{2-65}$$

Here v denotes the potential due to the "extra material", which to our approximation may be considered compressed into a surface layer on the sphere, of surface density

$$\mu = \rho h \quad , \tag{2-66}$$

where ρ is the volume density and h the thickness of the layer (Fig. 2.3). The potential of this layer is given by (1-5):

$$v = G \iint_{S} \frac{\mu}{l} dS = G\rho \iint_{S} \frac{h}{l} dS \quad , \qquad (2-67)$$

in view of (2-63). Putting

$$dS = R^2 d\sigma \quad , \tag{2-68}$$