theories of inverse problems for isostasy are treated, which nowadays enjoy considerable popularity in the geophysical community since none of the classical models is completely satisfactory from the geophysical point of view.

8.1 Classical Isostatic Models

From geodetic measurements performed around 1850 in India, J.H. Pratt in 1854 and 1859, and G.B. Airy in 1855 realized that the visible topographic masses of the Himalayan massif must somehow be compensated by mass deficiencies below sea level. According to Pratt, the mountains have risen from the underground somewhat like a fermenting dough. According to Airy, the mountains are floating on a fluid lava of higher density, so that the higher the mountain, the deeper it sinks; this behavior is rather similar to that of an iceberg floating in the ocean. In the next two subsections, we shall be following (Heiskanen and Moritz, 1967), using a plane approximation to the earth's surface or rather to the geoid.

8.1.1 The Model of Pratt–Hayford

This model of compensation was outlined by Pratt and put into a mathematical form by J.F. Hayford, who used it systematically for geodetic purposes.

The principle is illustrated by Fig. 8.1. Underneath the level of compensation there is uniform density. Above, the mass of each column of the same cross section is equal. Let D be the depth of the level of compensation, reckoned from sea level, and let ρ_0 be the density of a column of height D. Then the density ρ of a column of height $D+h$ (h representing the height of the topography) satisfies the equation

$$(D+h)\rho = D\rho_0 \quad , \tag{8–1}$$

which expresses the condition of equal mass. It may be assumed that

$$\rho_0 = 2.67\,\mathrm{g/cm^3} \quad . \tag{8–2}$$

According to (8–1), the actual density ρ is slightly smaller than this normal value ρ_0. Consequently, there is a density deficiency which, according to (8–1), is given by

$$\Delta\rho = \rho_0 - \rho = \frac{h}{D+h}\rho_0 \quad . \tag{8–3}$$

In the oceans the condition of equal mass is expressed as

$$(D-h')\rho + h'\rho_w = D\rho_0 \quad , \tag{8–4}$$

where

$$\rho_w = 1.027\,\mathrm{g/cm^3} \tag{8–5}$$

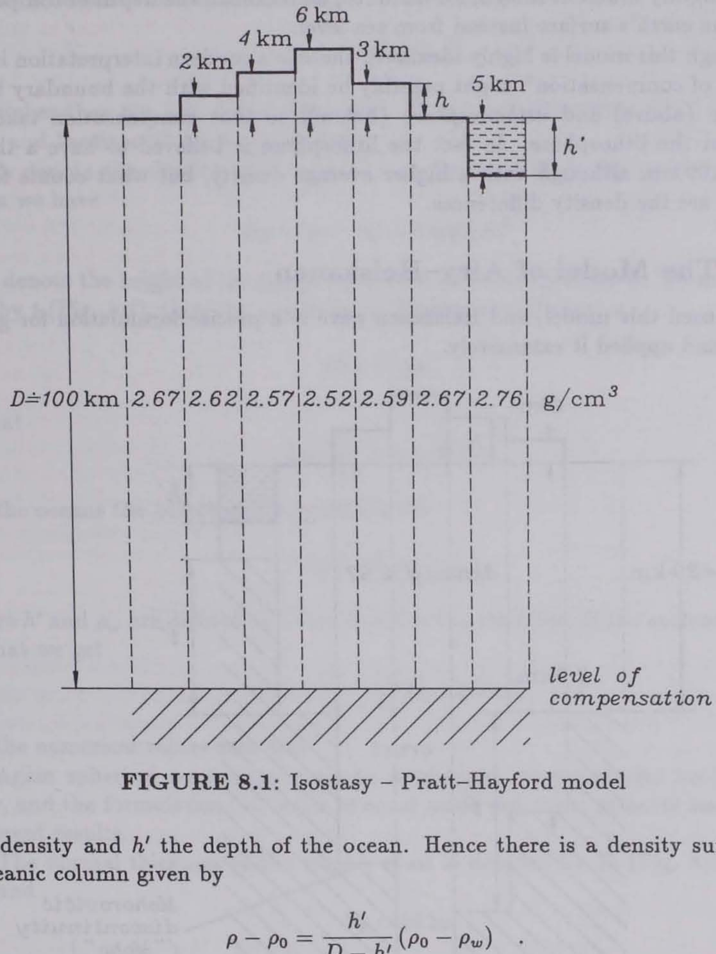

FIGURE 8.1: Isostasy – Pratt–Hayford model

is the density and h' the depth of the ocean. Hence there is a density surplus in a suboceanic column given by

$$\rho - \rho_0 = \frac{h'}{D - h'}(\rho_0 - \rho_w) \quad . \tag{8–6}$$

As a matter of fact, this model of compensation can be only approximately fulfilled in nature. Values of the depth of compensation around

$$D = 100\,\text{km} \tag{8–7}$$

are assumed.

For a spherical earth, the columns will converge slightly towards its center, and other refinements may be introduced. We may postulate either equality of mass or equality of pressure; each postulate leads to somewhat different spherical refinements. It may be mentioned that for computational reasons Hayford used still another, slightly different model; for instance, he reckoned the depth of compensation D from the earth's surface instead from sea level.

Although this model is highly idealized, there is a modern interpretation in which the "level of compensation" might possibly be identified with the boundary between *lithosphere* (above) and *asthenosphere* (below), so that compensation takes place throughout the lithosphere. In fact the lithosphere is believed to have a thickness of about 100 km, although with a higher average density, but what counts for compensation are the density differences.

8.1.2 The Model of Airy–Heiskanen

Airy proposed this model, and Heiskanen gave it a precise formulation for geodetic purposes and applied it extensively.

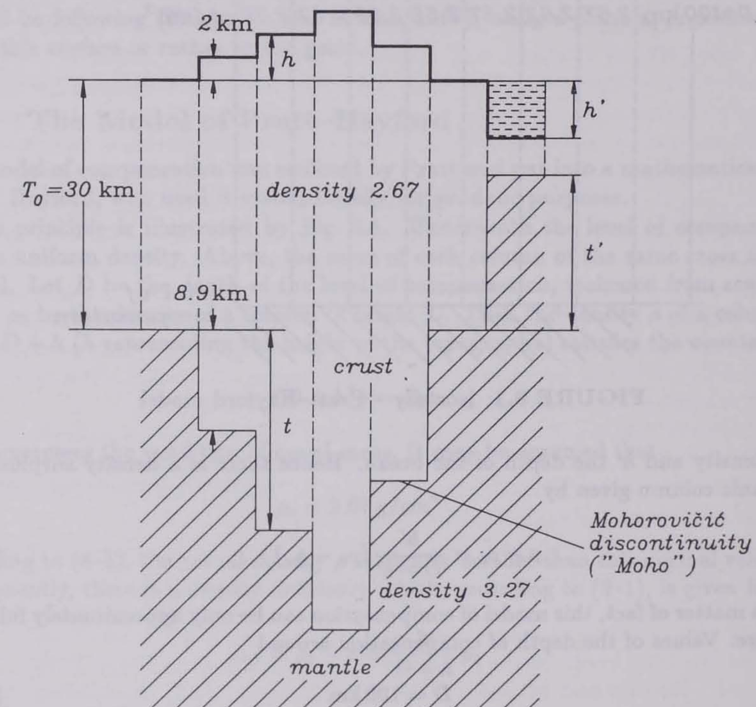

FIGURE 8.2: Isostasy – Airy–Heiskanen model

Figure 8.2 illustrates the principle. The mountains, of constant density (say)

$$\rho_0 = 2.67\,\mathrm{g/cm^3} \quad , \tag{8–8}$$