A polynomial

$$k_2\beta^2 + k_4\beta^4$$

has an extremum at

$$\beta = \left(-\frac{k_2}{2k_4}\right)^{1/2} \quad , \tag{6-61}$$

so that in the case of (6–60), κ will have an extremum around $1/\sqrt{2} \doteq 0.7$, that is, between 0 and 1, so that it cannot be monotonic. Even if κ_1 deviates from κ_1^H only by 10^{-8},

$$\kappa_1 = 0.000\,00067 \quad ,$$

the function $\kappa(\beta)$ is readily seen to be no longer monotonic.

In this way we see that a monotonic behavior of κ is possible only for mass configurations which are extremely close to equilibrium configurations. As (6–53) shows, this is not the case for the equipotential ellipsoid, and for the real earth the situation is even "worse" by a factor of more than two! This serves as another confirmation of the validity of Ledersteger's theorem (sec. 4.2.4) for the case of the earth.

6.5 Numerical Results and Conclusions

Using the polynomial representations of sec. 6.4 we can evaluate the ellipsoidal potential anomaly $W_4(\beta)$ by (6–27) and gravity $g(\beta)$ inside the ellipsoid by (2–62). Then Bruns' theorem (6–34) gives the separation $\zeta = W_4 P_4(\cos\theta)/g$ between corresponding surfaces of equal potential and of equal density. The result, by (Moritz, 1973, pp. 44–45), with our present numerical values, is

$$W_4 = \beta^4(627 - 1072\,\beta^2 + 585\,\beta^4 - 140\,\beta^6) \times 10\,\mathrm{m^2s^{-2}} \quad , \tag{6-62}$$

$$g = \beta(21.7 - 17.9\,\beta^2 + 6.0\,\beta^4)\,\mathrm{ms^{-2}} \quad . \tag{6-63}$$

The values of Table 6.1 have been computed from these expressions.

We see that the maximum separation between surfaces of constant potential and corresponding surfaces of constant density is almost 60 m, occurring on a depth of about 1400 km. This is on the order of the geoidal heights, which is not unplausible. It is not to be expected that a more realistic earth model and an expression for κ that is more sophisticated than (6–49) will give significantly different values. The values of ζ for the real earth are even larger by a factor of more than 2, as (6–53) shows!

By methods described in (Jeffreys, 1976, Chapter VI) or (Moritz, 1973, pp. 35–40) we may also compute corresponding stress differences. They are on the order of $2 \cdot 10^7\,\mathrm{dyn/cm^2}$, which is considerably less than the stress differences that may occur in the actual earth (Jeffreys, 1976, p. 270; we are using the old cgs unit here in order to facilitate the comparison).

Summarizing we may say (Marussi et al., 1974): To find an earth model consistent with an equipotential ellipsoid such as represented by the Geodetic Reference System 1980, the following procedure may be used. From the given value of the

TABLE 6.1: Potential anomaly W_4 and separation ζ of surfaces W = const. and ρ = const. for the equipotential ellipsoid

β	$W_4[10\,\mathrm{m}^2\mathrm{s}^{-2}]$	$\zeta[\mathrm{m}]\,/\,P_4(\cos\theta)$
0.0	0.0	0.0
0.1	0.1	0.3
0.2	0.9	2
0.3	4	7
0.4	12	16
0.5	25	28
0.6	40	42
0.7	54	54
0.8	59	58
0.9	45	45
1.0	0	0

spherical-harmonic coefficient J_2, the flattening follows from the theory of the external ellipsoidal field, without needing information on the internal structure, obtaining $f = 1/298.257\ldots$ (cf. Moritz, 1984).

Starting from this surface value of f one may, on the basis of an appropriate earth model such as PREM (sec. 1.5), compute the functions: ellipticity $e(\beta)$ and deviation $\kappa(\beta)$ using the theory of equilibrium figures to second (Chapter 4) or higher order. This gives an earth model in hydrostatic equilibrium.

To obtain an earth model whose boundary surface is strictly an equipotential ellipsoid of revolution, the function $e(\beta)$ can be left the same as for the hydrostatic model, whereas $\kappa(\beta)$ will be different since it must be zero at the surface and satisfy the condition (6–35), with J_4 given by (6–37). In this way one gets an ellipsoidal density model that is very close to an equilibrium configuration, the deviations from hydrostatic equilibrium being only of second order in f.

The coefficient J_4 for the ellipsoid (-237×10^{-8}) lies about halfway between the hydrostatic value (-299×10^{-8}) and the actual value for the earth as obtained from satellites (-162×10^{-8}). Therefore it appears possible, by an appropriate choice of the function $\kappa(\beta)$, to incorporate part of the stress differences inside the earth into the ellipsoidal model. Thus such a model may perhaps be suited as a reference for deviatoric stresses in the earth's interior. Thus it appears conceivable that such a model might be used even for special geophysical purposes, in much the same way as a spherical-harmonic expansion of finite degree may serve, for certain purposes such as collocation (Moritz, 1980, pp. 312-313), as a reference for the gravity field.

Nevertheless, the standard reference model for geodynamical purposes will be an equilibrium configuration, in the same way as the level ellipsoid is the standard reference for the purposes of geometric and physical geodesy.